

# SLATE **Demonstration Curricula** Contextualizing Mathematics and Automotive Education



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# **SLATE Demonstration Curricula**

Strategic Linking of Academic and Technical Education (SLATE)

# **Contextualizing Mathematics and Automotive Education** Electric Vehicles: Are They Worth Getting Charged Up About?

Prepared by Santa Barbara Mathematics and Automotive Contextualized Learning Council



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# Contextualizing Mathematics and Automotive Education

Electric Vehicles: Are They Worth Getting Charged Up About?

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# **SLATE:** Strategic Linking of Academic and Technical Education

#### PREFACE

n 2011, the James Irvine Foundation generously committed to funding two years of SLATE with the following objectives: Each of the councils has its own personality and motivations, and the curriculum reflects that.

- Establish English and mathematics cross-discipline, intersegmental faculty councils called Contextualized Learning Councils (CLCs) to create teaching materials and methodologies that provide context and links to real-world applications;
- 2. Develop, publish, and disseminate eight contextualized curricular units, four English and four mathematics, connected to technical education and other academic disciplines; and
- 3. Develop a model of faculty professional development.

To achieve the objectives, CLCs were established across California in early 2011. In addition to English and mathematics, the disciplines represented were bio-science, business, environmental science, industrial technologies, mechatronics/manufacturing and product design, public health, public safety, social science, and statistics. Each of the councils had its own personality and motivations, and the curriculum reflects that. The contextualized learning councils were:

- Contra Costa English, Mathematics, and Environmental Science
- Los Angeles English and Social Science
- Placer-Nevada English and Public Safety
- Placer-Nevada Mathematics, Engineering, and Manufacturing
- San Bernardino West English and Environmental Science
- San Francisco Mathematics and Public Health
- Santa Barbara English, Journalism, and Media Arts
- Santa Barbara Mathematics and Automotive
- Shasta English and Small Business
- Shasta Mathematics and Industrial Technology

In addition to creating field-test ready curricula through an interdisciplinary and linked approach to improve student learning, SLATE improved professional learning for faculty via the same strategy. The SLATE curriculum design process, involving regional faculty members working across disciplines and segments, proved to be a powerful form of professional development. Participants had the advantage of long-term, ongoing support in a venue where they gained in-depth content knowledge informed by a cross-discipline.

The teaching strategies developed through SLATE will be extremely valuable as SLATE high school faculty prepare students with 21st century skills that meet the rigor and relevance demanded by the Common Core State Standards. At the same time, their postsecondary partners have a better understanding of these new standards: what they mean in terms of high school students' preparation and what adjustments colleges may need to make regarding aligning curricula, programs, and services to ensure students' continued progress.

Overall, the game-changing cross-disciplinary curriculum and assessments SLATE participants developed have moved them to the forefront of educational leadership. As evidence grows regarding the link between quality professional development and improved student achievement—and school reform—SLATE stands out as an exemplar of how dialog and reflection in a learning community of colleagues turn into achievement in the classroom.

Sandra Scott, Project Director

#### COUNCIL BACKGROUND

he Santa Barbara Mathematics Contextualized Learning Council considered a few alternatives before choosing the right cross-curricular partner for the SLATE project. SLATE really stretches the imagination because it is ahead of the times with regard to authentic implementation of the Common Core Standards.

#### **Council Participants**

Lauren Wintermeyer, SLATE Regional Coordinator

Brianna Adam, Bishop Garcia Diego High School

James Ashlock, San Marcos High School

Lisbeth Ceaser, California Polytechnic State University, San Luis Obispo

Wayne Cole, Santa Barbara Unified School District

Russell Granger, San Marcos High School

Steve Hunter, Sierra College (Emeritus)

Anuroopa Kalbag, Dos Pueblos High School

Chris Ograin, University of California, Santa Barbara

A link to the transportation sector—specifically automotive technology—emerged as a natural connection with mathematics that would provide real-world application and appeal for students. With a mathematics and automotive technology teacher from the same high school on this learning council, it allowed other participants to see how cross-curricular links could be formed on a high school campus.

The automotive technology instructor was selected as CTE teacher of the year last year at his high school. When invited to join the Santa Barbara Mathematics CLC, he eagerly came on board and was able to list the multitude of mathematics and science concepts he was already teaching in his automotive courses. Similarly, his mathematics colleague noted that his algebra students are often enrolled in automotive technology and, until this project, he had not thought about the natural connections that already existed for the students they had in common. It was a rewarding

experience to watch this cross-curricular collaboration take shape and grow into what will hopefully serve as a demonstration for other faculty across disciplines.



# Electric Vehicles: Are They Worth Getting Charged Up About?



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#### INTRODUCTION

#### Grade Level:

high school

#### Time:

Allow at least 30 minutes, each, for the 12 exercises and pre-lesson. In an automotive class, exercises could be used one at a time over a semester as coursework dictates. In an algebra class, present the exercises as examples of how algebra is used to solve a real-world issue.

#### **Cross-Disciplines:**

- Algebra I
- CTE: transportation industry sector courses

#### **Instructional Materials:**

- scientific calculator
- handouts: pre-lesson (Dimensional Analysis);
  1A (Fundamentals of Electricity); 1B (Mathematical Relationships for Electricity);
  2 (Analyzing Common Household Appliances); 3 (Electric Motor); 4 (Charging the Vehicle)
- compass to draw circles
- flashlight or other object to illustrate the principles of electricity
- battery charger
- cardstock

#### **Unit Overview**

Electric vehicles are becoming increasingly popular as an environmentally friendly choice for short, intercity transportation. Analyzing their operation provides an excellent opportunity to apply mathematics principles in Algebra I, such as:

- algebraic terms
- basic algebraic laws and rules
- fractions and percentage
- graphing
- mathematical precedence
- rearranging formulas
- simplifying equations
- substitution
- truncating and rounding
- scientific and engineering notation

This lesson is intended to enhance students' understanding and retention of algebraic concepts as they are applied to real-world situations. This lesson is intended to enhance students' understanding and retention of algebraic concepts as they are applied to real-world situations. Required electrical concepts are addressed in Lesson 1, Fundamentals of Electricity, and are sufficiently detailed to allow an instructor to proceed with the rest of the lesson. The curriculum is written so that an instructor can choose to teach all of the segments or just some lessons to best meet course requirements and student interests. Each lesson has lead-in material and then one or more student exercises.

#### **Essential Question**

What factors will determine the future of electric vehicles?

We suggest that teachers post the essential question in a prominent place in the classroom and refer to it frequently. Have students discuss/respond to this question throughout the lessons, as appropriate. Note how answers change, or don't change, over time, based on experiences and new knowledge and skills. Answering this question should lead to more questions as students create their own deep knowledge, understandings, and transferable skills.

#### **Learning Objectives**

Students can analyze an electrical vehicle's travel range using the following processes and make a recommendation regarding whether or not to use one.

- Manipulate the Ohm's Law formula and Watt's Law formula to isolate variables.
- Use formulas to calculate currents, power, and cost of operating an appliance.
- Calculate energy efficiency.
- Interpret data from a graph.
- Calculate voltage and amp-hour capacity of a battery pack.
- Calculate the amount of time for an electric vehicle battery to charge.
- Analyze and understand the structures and functions of different types of circuits.
- Calculate the rate of charge and discharge.

#### Prior Knowledge/Prerequisite Skills

Students must understand linear functions and know how to represent mathematical relationships using variables. Students must be able to isolate a given variable in an equation. Students must be proficient using basic mathematic operations (add, subtract, multiply, divide). If students do not know how to do unit conversions, it would serve students to pre-teach this skill first (see Pre-Lesson: Dimensional Analysis).

#### Standards

#### National Common Core State Standards for Mathematics\*

#### **Standards for Mathematical Practice:**

- SMP 1. Make sense of problems and persevere in solving them.
- SMP 2. Reason abstractly and quantitatively.
- SMP 4. Model with mathematics.
- SMP 6. Attend to precision.
- SMP 7. Look for and make use of structure.

#### Mathematics, Grade 8:

8-EE 5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

#### **Mathematics Standards for High School**

Number and Quantity:

- N-Q 1. Use units as a way to understand problems and to guide the solution of multistep problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- N-Q 2. Define appropriate quantities for the purpose of descriptive modeling.
- N-Q 3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Algebra:

- A-SSE 1. Interpret expressions that represent a quantity in terms of its context.
- A-CED 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- A-CED 4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Functions:

- F-IF 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. (Extending the Lesson)
- F-BF 1. Write a function that describes a relationship between two quantities.
- F-BF 1a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- F-LE 1b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- \* National Governors Association Center for Best Practices, Council of Chief State School Officers, Common Core State Standards for Mathematical Practice, (Washington, D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010), http://www.corestandards.org/assets/CCSSI\_ Math%20Standards.pdf.

#### California Career Technical Education Model Curriculum Standards—Grades 7–12\*

## Transportation Industry Sector, Foundation Standards (Academics); 1.1 Mathematics Specific Applications of Number Sense Standards (Grade 7):

- 1.2. Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.
- 1.3. Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.

#### Specific Applications of Measurement and Geometry Standards (Grade 7):

2.4. Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 144 square inches or [1 ft<sup>2</sup>] = [144 in<sup>2</sup>], 1 cubic inch is approximately 16.38 cubic centimeters or [1 in<sup>3</sup>] = [16.38 cm<sup>3</sup>]).

#### Specific Applications of Mathematical Reasoning Standards (Grade 7):

- 2.6. Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.
- 2.8. Make precise calculations and check the validity of the results from the context of the problem.

#### 1.2 Science

#### Specific Applications of Physics Standards (Grades 9–12):

5.b. Students know how to solve problems involving Ohm's law.

#### (C) Vehicle Maintenance, Service, and Repair Pathway:

C3.5. Understand the basic principles of electricity, electronics and electrical power generation, and distribution systems.

#### Assessments

- performance on student exercises 1–11; type: formative
- performance on student exercise 12; type: summative

<sup>\*</sup> Sheila Bruton, ed., California Career Technical Education Model Curriculum Standards: Grades Seven through Twelve, (Sacramento: California Dept. of Education, 2006), http://www.cde.ca.gov/ci/ct/sf/documents/ ctestandards.pdf.

## PRE-LESSON Dimensional Analysis

#### Setup

If students are not familiar with dimensional analysis, also referred to as unit conversion, it would be appropriate to teach this lesson first. Handout 1 (Pre-Lesson: Dimensional Analysis) is included for this purpose. Students may question why it is necessary to use unit conversion, which seems like more work, when they can simply multiply or divide to solve for an answer. Doing calculations in this manner will help students to

- label their answers accurately, and
- understand when something is in square units vs. cubic units (one example).

#### Introduction

One way to begin this pre-lesson is to give the students small cardstock pieces with a line down the middle, like blank dominos. Ask them to fill in three of the dominos to show how to convert 40 cm. to inches. Their pieces should look like this:

 $\frac{40 \text{ cm}}{1 \text{ in}} \left( \frac{1 \text{ in}}{2.54 \text{ cm}} \right) = \frac{40}{2.54} \approx 15.75 \text{ in.}$ 

For further examples, refer to the Dimensional Analysis Worksheet.

If students really struggle with the notation, you might want to consider starting out this way:

- 1. 200 watts  $\times$  4 hours/day = 800 Wh/day.
- 2.  $800 \text{ Wh/day} \times 120 \text{ days/year} = 96,000 \text{ Wh/yr}.$
- 3. 96,000 Wh/yr  $\div$  1,000 Wh/kWh = 96 kWh/yr.
- 4.  $96 \text{ kWh/yr} \times 0.11/\text{kWh} = 10.56/\text{yr}$ .

#### Close

Eventually, guide students to use the method shown in the centimeters (cm) to inches (in) example above.

Common conversion factors can be found at http://www.infoplease.com/encyclopedia/science/common-weights-measures-table.html.

## LESSON 1 Fundamentals of Electricity

#### Setup

Have a flashlight that requires two C batteries or use a different object to demonstrate the principles. If you use something other than a flashlight, adjust the sample exercise accordingly. Prepare a class set of handouts (Handouts 1A–4, which include Exercises 1–12) for the entire unit and the Fundamentals of Electricity.

#### Introduction

Ask students what kind of car they think they'll be driving in the future. Generate a discussion about the pros and cons of electric, hybrid, and gas-fueled vehicles. Video clips from cars or commercials could be shown. Let students know that in this unit, they will be learning about electricity and calculating the advantages and disadvantages of running an electric car.

Explain an electric circuit to illustrate electrical principles, including efficiency, so that students can later relate these concepts to the more involved electric car exercise. (Key vocabulary is in bold for emphasis.)

Explain about the minimum requirements for a complete electric circuit:

- 1. **Source** of electric energy (in this exercise, two batteries).
- 2. Load to use the electric energy (e.g., lightbulb, motor, heater coils).
- 3. Conductor: Wires that connect the source to the load.

Explain about typical enhancements to electric circuits:

- 1. Switch: to turn the circuit on and off.
- 2. Fuse or circuit breaker: to protect the circuit in case of an electrical fault.

Have a class discussion about the benefits of the device enhancements, e.g., without a **switch**, you would have to unplug an appliance you want to turn off; or without a **circuit breaker**, you could set the house on fire if you decided to decorate your yard with a million lights.

#### Amps:

For a typical flashlight, the **source** of electric energy is two C batteries connected in a series-aiding configuration. Figure 1 below includes conducting wires and the optional **switch** typical in a flashlight.



Figure 1. Batteries connected in (end-to-end) series-aiding configuration.

From the information contained in the illustration, it can be determined that the final output voltage is 3 volts, the sum of the voltage of the two batteries. However, connecting the batteries does not extend the life of the battery pack; the energy (amp-hour capacity) is equal to that of a single cell, in this case 2 amp-hours.

#### **Guided practice:**

Check for understanding by providing the following scenario: A police officer often carries an aluminum flashlight that uses four to six D batteries connected in a series. Have students draw a diagram of this and determine the final output voltage and amp-hour capacity of these configurations.

In these examples, the **load** is an incandescent lightbulb. The flashlight is rated at 3 volts and draws ¼ ampere when in operation. Hold up the battery from the flashlight and tell students that ideally, it can use 2 amps for 1 hour before being exhausted, thus the 2 amp-hour rating.

If it is possible to draw more current than the 2 amps, the battery would be exhausted in less than 1 hour. Conversely, if it is possible to draw less than 2 amps, the battery life would be extended for longer than 1 hour. One amp-hour implies you can provide 1 amp of current for 1 hour before exhausting the battery. A **load** requiring more than 1 amp of current would result in a shorter battery life, while a **load** requiring less than 1 amp of current would allow for more hours of usage.

The flashlight draws ¼ amp per hour, so we can determine that the flashlight would remain useful for 8 hours.

$$2\div\frac{1}{4}=8.$$

In reality, this flashlight would operate for less than 8 hours. This fact is due to how the 1 amp-hour was determined by the manufacturer (amp-hour capacity) vs. how someone extracts the energy. Whenever we apply a **load** that is not the same as the manufacturer, the battery is not as efficient as the manufacturer states. A de-rating factor has to be applied.

Pose this question to students: If the de-rating factor for the batteries used in the flashlight is 85 percent, calculate the number of hours you can use the flashlight. Students will relate this issue to why their cell phone batteries never last as long as they expect. An investigation of battery life could be an extension of the lesson. For more on de-rating, see: http://data.energizer.com/PDFs/non-rechargeable\_FAQ.pdf.

#### Watts:

#### **Guided practice:**

Explain that sometimes power in watts is used to specify electrical circuit parameters. The power in watts can be determined by multiplying voltage and

current.

In the flashlight example, the battery pack is rated at 3 volts and 2 amp-hours. You could determine the source energy in watt-hours by the following:

3 volts × 2 amp-hours = 6 watt-hours of energy.

You can also determine the energy in watts to run the bulb in a similar fashion:

$$3 \text{ volts } \times \frac{1}{4} \text{ amp} = \frac{3}{4} \text{ watt.}$$

Once these values are known, the time the flashlight will operate is found by dividing the energy of the source by the demand of the load as shown below:

6 watt-hours 
$$\div \frac{3}{4}$$
 watt = 8 hours.

Note that 8 hours of operation is the same as found using the amp-hour rating of the battery and the amp-draw of the bulb in the battery section above.

#### Series-Aiding Configurations: Consider the Alternatives

In many calculators, the batteries lie side by side. Note that an electrical series-aiding configuration does not always reference a particular physical layout. Point out how the configuration below is different from the flashlight configuration (fig. 1).



Figure 2. Batteries connected in (side-by-side) seriesaiding configuration.

Provide guided practice by posing the following question: A package with two C batteries costs \$4.99. Calculate the cost per hour to operate the flashlight.

#### **Exercises 1–3**

Direct students to the Fundamentals of Electricity Handout (Handout 1A) and guide them through it as students take turns reading sections aloud. Distribute Handout 1B, Exercises 1–3. Remind them that there are two fundamental relationships that define most electric vehicle circuit operations: Ohm's Law (George Sigmund Ohm, 1789–1854), which states  $E = I \times R$ , and Watt's Law (James Watt, 1836–1919), which states  $P = E \times I$ .

Have students do Exercises 1 and 2. After completing Exercise 1, students will have a total of six mathematic equations. Because *E* and *I* appear in both Ohm's and Watt's Law, students can then use the substitution process to generate an additional six equations. This process will net a total of 12 interrelated mathematic equations that can be used to analyze and predict electric circuit operation.

For Exercise 3, have students create a tool to organize the 12 equations so that they are userfriendly. By developing a visual aid, it will assist in the understanding and retention of new mathematically related subject matter. Below is an example (see fig. 3). The algebraic procedures are commonly used in business, industry, and education. **Remember that the symbol for voltage is sometimes V, and the symbol for current is sometimes A.** If your students could have been exposed to these terms in a science or CTE class on your campus, find out which symbols are used in textbooks or trade documents and be consistent with using them to avoid confusion.



Figure 3. P, E, I, R formula wheel.

Hand each student a quarter sheet of cardstock and a compass. Have them draw two concentric circles. Show them a sample template like the one above with *P*, *E*, *I*, and *R* filled in but nothing else. Ask them to use their 12 equations to fill in the rest of the wheel. Have students check and correct, if necessary, before moving on to the rest of the worksheet.

## LESSON 2 Common Electric Appliance Application

#### Introduction

Students will now mathematically apply what they have learned to a common electric appliance and then to an electric vehicle.

#### **Exercises 4-6**

(Handout 2)

In Exercise 4, students find the current (I) in amps and the power consumed by the hair dryer.

For Exercise 5, students may need to be reminded that kilo means 1,000, and that they will need to divide by 1,000 to convert watts to kilowatts.

In Exercise 6, students apply the mathematic principles of electric circuits, including voltage, current, resistance, and power. They also apply the concepts of time, cost, and efficiency.

An extension to the lesson could be to have the students research the wattage of their home refrigerator and compute the cost of operation for one 30-day month with the assumption that it runs one-third of each day.

These exercises are a good foundation for examining the electrical principles of an electric car.

## LESSON 3 Electric Motor

#### Introduction

Over a 3-year period, students in the San Marcos High School Automotive Technology course, under the direction of their teacher, Russell Granger, converted a gasoline-powered Porsche 914 into an electric vehicle. The project was finished in the spring of 2012 and displayed during the Green Car Show as part of the 2012 Earth Day Festival.

For more information, see:

http://www.sbsdk12.org/news/2010/04/21SMHSautotechgreencaraward.shtml http://www.cecsb.org/earthday/images/stories/pdf/SB\_News\_Press\_April\_18-\_Electric\_Porsche.pdf

#### Exercises 7-11

(Handout 3)

In Exercise 7, have students review the worksheet and work through problems a-c.

For Exercise 8, the graph in figure 5 comes directly from the motor manufacturer and is typical of industrial machine graphs. It is a bit hard to read because of the multiple variables on the vertical axis. To find the amps required for 20 HP, go to the HP (horsepower) label on the right side of the graph and find 20 HP; follow that line to the left until this 20 HP line intersects with the HP (mechanical) line; next move vertically to the intersection of the amps line; and finally, follow that line to the left, which should be 237 amps.

Explain to students how to use the graph, or give them the information they need:

$$\mathbf{20}_{\textit{HPamps}}\cong\mathbf{237}_{\textit{amps}}$$

For Exercise 9a, students will need to divide their answer from 7c by their answer to problem 8. For Exercise 9b, students may need to be reminded that  $D = R \times T$ .

Let students work in groups for Exercises 10 and 11. Have them debate the necessary quantities and calculations before giving assistance.

## LESSON 4 Charging the Vehicle

#### **Exercise 12**

(Handout 4)

Exercise 12 could be a stand-alone summative assessment. Have students read the introductory material in Exercise 12 and work the problems.

#### **Extending the Lesson**

Have the students graph time of charge vs. kWh of charge for charging a totally discharged battery and note the slope of the charge line. Next, have the students graph time of charge vs. kWh of charge for charging a totally discharged battery using a battery charger. Then have students compare the slope of the battery charger with the slope of the charge line noted earlier.

Another extension could be to provide students with an activity to elicit deeper learning. Have students research the costs of operating an electric vehicle and a gas-powered vehicle, such as cost per mile. In addition to cost per mile, they should investigate purchase price, upkeep (see information on battery life below), insurance, resale value, etc. Students should prepare a report, including diagrams, graphs, and calculations, explaining what type of consumer would benefit from each purchase.

#### **Additional Battery Information**

The capacity (performance) of all batteries decreases with charge/discharge cycling and time. This is why the battery in a conventional car has to be replaced every four to seven years. For a deep discharge lead-acid battery pack like the one used in the Porsche 914 project, this needs to be taken into account because it affects the distance the car can travel when fully charged. A typical performance curve for such a battery pack is shown in figure 4. Note that the capacity of the battery actually increases slightly from new, remains at peak for some time, and then starts to decline. Electrical vehicle (EV) batteries are normally replaced when their capacity declines below 70 percent of peak capacity.



Figure 4. Typical performance curve of a battery.

The chart below is empirical data taken from an EV battery pack as it aged under charge/discharge cycling.

CHARGE/DISCHARGE (CYCLES)	BATTERY CAPACITY (kWh)
0	39
50	44.5
100	47.5
200	48.5
250	48.5
300	48
350	47.5
400	47
450	46
500	44
550	42
600	39
650	35.5
700	32
750	27
800	22
850	16.5
900	10

Battery performance can also be represented by a polynomial. The above empirical data closely matches the 5<sup>th</sup> degree polynomial below.

 $y(x) = 8.24E - 13(x)^{5} - 1.95E - 09(x)^{4} + 1.65E - 06(x)^{3} - .000692(x)^{2} + 0.135(x) + 39.2.$ 



# Handouts C

PRE-LESSON	
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LESSON 4	
Handout 4: Charging the Vehicle (Exercise 12)	

Name: \_\_\_\_

# Contextualizing Mathematics and Automotive Education

# **DIMENSIONAL ANALYSIS\***

#### **Length Conversions:**

1. Convert 38 inches into a measurement in centimeters, rounded to one decimal place.

2. Convert 51 inches into a measurement in millimeters, rounded to one decimal place.

3. Convert 9,456 feet into a measurement in miles, rounded to two decimal places.

<sup>\*</sup> Special thanks to Doug Gardner, Rogue Community College, for providing worksheet questions from Applied Technical Mathematics, Section 1.4 "Dimensional Analysis," pp. 41–43.

#### **Area Conversions:**

4. Convert 17 square inches into a measurement in square centimeters, rounded to two decimal places.

5. Convert 236 square inches into a measurement in square feet, rounded to two decimal places.

6. Convert 78 square feet into a measurement in square yards, rounded to two decimal places.

#### **Volume Conversions:**

7. Convert 4 cubic feet into a measurement in cubic inches.

8. Convert 5 cubic feet into a measurement in gallons, rounded to two decimal places.

9. Convert 167 cubic feet into a measurement in cubic yards, rounded to two decimal places.

#### **Rate Conversions:**

10. Convert a rate of 18 feet per second into miles per hour (mph), rounded to one decimal place.

11. Convert 23 gallons per minute (GPM) into cubic feet per day, rounded to the nearest whole number.

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# Contextualizing Mathematics and Automotive Education

# FUNDAMENTALS OF ELECTRICITY

#### Lesson 1

Electricity derives its name from the electron, a negatively charged particle of atoms. Atoms also have positively charged particles called protons. The electron is attracted to the proton due to their opposite charges. When there are an equal number of positive and negative charges, the net charge of an atom is zero and there is no electrical force. Electrical force is created when electrons move from one atom to another; this movement is called electrical current. (Key vocabulary below is in bold for emphasis.)

For more information, students can go to "Energy Story," a page of the California Energy Commission's Energy Quest website, at http://energyquest.ca.gov/story/chapter02.html.

Electricity is a force in nature that exists when there is a difference in charge between two objects.

**Voltage** is the electrical force, or pressure, that pushes electrons around an electrical circuit. Its symbol is *E* and the units are volts. Note: The symbol *V* is often used for voltage rather than *E*; either is acceptable.\*

**Current** is the amount of electrons that pass through a circuit every second. Its symbol is *I* and the units are in amperes (amps). Note: The symbol *A* is often used for current rather than *I*; either is acceptable.\*

**Resistance** is the opposition to the passage of an electric current through a circuit element. Its symbol is R and the units are ohms.

**Power** is the rate at which electric energy is generated, dissipated, converted, or stored by an electric circuit. Its symbol is **P** and the units are watts.

**Time** relates total quantity of power generated, dissipated, converted, or stored over a specified interval. Its symbol is *T* and the units can be in seconds, minutes, hours, days, months, or years.

**Efficiency** relates useful energy to the total energy used to create it. Its symbol is  $\eta$  and it is expressed as a ratio or percentage without units.

\* Instructors should use the symbol for voltage (E or V) and current (I or A) that matches their textbook and/or trade documents. This may affect the symbols used for the Mathematical Relationships for Electricity handout (1B).

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Name: \_\_\_\_

# **1B** Contextualizing Mathematics and Automotive Education

# MATHEMATICAL RELATIONSHIPS FOR ELECTRICITY

#### Lesson 1, Exercises 1-3

There are only two fundamental relationships that define most electric vehicle circuit operation. These two relationships are shown in the formulas below:

Ohm's Law (George Sigmund Ohm, 1789–1854), which states  $E = I \times R$ . and Watt's Law (James Watt, 1836–1919), which states  $P = E \times I$ .

#### Exercise 1:

a. Using Ohm's Law, rearrange the formula solving for *I* and *R*, recording your work.

b. Using Watt's Law, rearrange the formula solving for *E* and *I*, recording your work.

#### Exercise 2:

Use the equations you have generated above to complete Exercise 2. Note that *E* and *I* appear in both Ohm's and Watt's laws. By using substitution, you will be able to generate six more equations to be used in solving equations involving electricity.

a. In Ohms's Law,  $E = I \times R$ . Substitute  $\frac{P}{I}$  from Watt's Law above for *E* and solve for all variables: *P*, *R*, and *I*.

b. In Ohms's Law,  $I = \frac{E}{R}$ . Substitute  $\frac{P}{E}$  from Watt's Law above for *I* and solve for all variables: *P*, *E*, and *R*.

#### Exercise 3:

You now have twelve interrelated formulas from Ohm's and Watt's laws. These are the foundation for all electrical studies. Create a tool to help you organize these formulas, which you will use for future problems.

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# 2 Contextualizing Mathematics and Automotive Education

## ANALYZING COMMON HOUSEHOLD APPLIANCES

#### Lesson 2, Exercises 4-6

#### Exercise 4:

A standard hair dryer plugged into a 120-volt (E) outlet has 10 ohms of resistance (R) when operating.

a. Find the current (*I*) in amps using the correct algebraic relationship, and label.

b. What is the power consumed by a standard hair dryer? Label your answer in watts.

#### Exercise 5:

The cost of electrical usage is based on kilowatt-hours used over a period of one 30-day month, multiplied by the electrical rate for one kilowatt-hour. This exercise will solve for the cost of using a standard hair dryer for one 30-day month.

a. Convert the above watt value to kilowatts (watts).

b. If a standard hair dryer was used 5 minutes every day over a 30-day month, solve for the total time of operation in hours for the month.

c. Solve for the total kilowatt-hours (kWh) of electricity used by the dryer in one 30-day month.

d. Given the electrical rate is 15 cents per kilowatt-hour (kWh), solve for the total cost to operate the hair dryer for the 30-day month.

#### Exercise 6:

Not all the energy consumed by a hair dryer is used to produce hot air. Some of the energy goes into running the motor and other losses.

a. If a standard hair dryer consumed 100 of the total number of watts in loss-related functions, what is its efficiency? (Note: Divide the number of useful watts by the total watts.)

b. Efficiency is generally expressed as a percent. Convert your answer from question (6a) to a percent by multiplying the answer by 100. Round to the nearest tenth.

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# 3 Contextualizing Mathematics and Automotive Education

# **ELECTRIC MOTOR**

### Lesson 3, Exercises 7-11

Over a 3-year period, students in the San Marcos High School Automotive Technology course, under the direction of their teacher, Russell Granger, converted a gasoline-powered Porsche 914 into an electric vehicle. The project was finished in the spring of 2012 and displayed during the Green Car Show as part of the 2012 Earth Day Festival.

#### The car's electrical specifications are:

Battery Type:	6-volt, 245-amp-hour (20-hour discharge rate; see sidebar below) lead-acid
	golf cart batteries
Battery Configuration:	20 batteries connected in series-aiding configuration
Electric Motor:	9-inch brush type dc motor (see performance graph in Exercise 8)

#### Exercise 7:

Referring to the above electric car specifications, note the batteries are connected in a series-aiding configuration. When batteries are connected in a series-aiding configuration, their voltages add together; however, the amp-hour capacity is that of one battery.

Examining the above battery information, one will note that the 245 amp-hour rating is at a 20-hour discharge rate. This fact means it would take 20 hours to discharge a charged battery. You could find the current discharge rate by dividing the amp-hour rating by the discharge rate. Also, this battery pack when new and fully charged would provide 12.25 amps of current for 20 hours. Theoretically, it should be able to provide 245 amps for 1 hour, or any product of amps × hours that equals 245, like 24.5 amps for 10 hours.

 $I_{\text{discharge}} = \frac{245 \text{ amp-hours}}{20 \text{ hours}} \rightarrow I_{\text{discharge}} = 12.25 \text{ amps.}$ 

a. Draw a diagram illustrating the series-aiding configuration for the electric car. Label volts and amp-hours.

b. Determine the final output voltage and amp-hour capacity of the entire vehicle battery pack.

c. Electric vehicles require hundreds of amps from the battery pack when operating at reasonable speed. This high discharge rate lowers the available amp-hour capacity because of losses (inefficiencies) inherent in the rapid chemical-to-electrical conversion and a condition called losses. Lead-acid type batteries in electric vehicle applications can provide only about 80 percent of their rated amp-hour capacity. Solve for the electric vehicle amp-hour capacity of the above battery pack taking into account this 80 percent efficiency factor.



#### **Exercise 8:**

The electric vehicle featured requires about 20 continuous horsepower (HP) to propel it down the road at 60 miles per hour (mph). From the ImPulse 9 motor graph above (fig. 5), find the current in amps needed to provide this 20 HP.

Figure 5. NetGain Motors Impulse 9 motor performance graph. Source: NetGain Motors, Inc. http://www.go-ev.com/PDFs/003\_07\_ImPulse\_9\_Graph.pdf

#### Exercise 9:

a. Solve for the time a totally charged battery can provide the required current for 20 HP (60 mph).

b. Solve for the number of miles the electric vehicle can travel on a fully charged battery at 60 mph.

#### Exercise 10:

#### **Electrical cost of operation:**

Consumers want to know the cost to operate a vehicle. For gasoline-powered vehicles, consumers want to know the cost to fill the tank, miles/gallon, and cents/mile related to the fuel. For electrical vehicles, this equates to the cost to charge the battery, miles on a charged battery, and cents/mile related to the electrical energy.

A battery pack's energy can also be rated in kilowatt-hours (kWh) rather than voltage and amp-hours. From this information, with the addition of charging efficiency, you can determine the cost of operation as well as how long it would take to charge the battery.

a. Convert the vehicle battery pack rating (at the 20-hour discharge rate) from voltage and amphours to kWh. Refer to the *P*, *E*, *I*, *R* formula wheel you created for the power formula given voltage and current. Remember that there are 1,000 watts in a kilowatt.

b. You have previously determined the total number of miles the vehicle can travel on a charged battery. Solve for the number of kW it will take to travel each mile at 60 mph.

#### Exercise 11:

Lead-acid batteries require about  $\frac{1}{3}$  more power, or approximately 133 percent to charge them fully, than their rated capacity. Once again, this fact is related to inefficiencies in chemical conversions and electrical resistances.

a. Given the kWh rating found in the previous problem, find the required kWh charging requirement when the battery's inefficiency is considered.

b. Knowing the above "real" kWh energy, including charging inefficiencies, compute how much it will cost to charge a fully discharged battery given an electrical rate of 15 cents per kWh.\*

c. This is also the cost to travel the number of miles determined above for a fully charged battery. Find the cost per mile by dividing this cost by the total number of miles you can drive on a fully charged battery.<sup>†</sup>

 Electrical vehicles use so much energy for charging, they can put consumers into a much higher electrical rate bracket (more than double the baseline rate) and thus significantly affect the cost of charging and cost per mile.

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<sup>\*</sup> This kWh value is the energy that the charger needs to provide to charge a fully discharged battery. It is also the energy that you have to pay for to charge the vehicle from a fully discharged state.

Name: \_

# 4 Contextualizing Mathematics and Automotive Education

## CHARGING THE VEHICLE

#### Lesson 4, Exercise 12

The time it takes to charge an electric vehicle is of major concern.

Just like filling an empty gasoline tank in a gas-powered vehicle, an electric vehicle requires drivers to charge the battery pack. Filling the tank with gasoline takes only a few minutes; charging an electric vehicle battery pack can take many hours depending on the state of the battery and the electrical connection used.

Most electrical vehicles have two ways to charge the battery pack. The standard (high current) charger resides at the home and/or workplace and is connected to a special 30-amp, 240-volt circuit. This charger can charge the vehicle's battery pack in a reasonable length of time. The other charger (onboard backup charger) is in the vehicle itself and can be plugged into a regular 15-amp, 120-volt outlet. This charger can be used as a backup when the standard charger is not available, but it takes a very long time to charge the battery. The driver of an electrical vehicle must give priority to charging the battery pack as part of normal operation.

Either type of charger is designed to only draw 90 percent of the total current available from the circuit to avoid accidentally tripping the circuit breaker.

a. Given the external 30-amp, 240-volt circuit running at 90 percent available current, solve for its charge capacity in kilowatts.

b. Solve for how long it would take to charge a fully discharged battery pack in hours using the standard (high current) charger.\*

c. Given a fully charged battery and a workplace 35 miles away, or 70 miles round trip, with a standard (high current) charger at work, if you drove at an average speed of 60 mph, would the vehicle make it home after an 8-hour workday?

- d. Let's examine a different situation: Given a fully charged battery, if you visited a friend's place 40 miles from home without a standard (high current) charger and drove at an average speed of 60 mph, would the vehicle make it home after a 4-hour stay? Work through the problems below to find your answer.
  - i. Given the onboard charger that can be plugged into a 15-amp, 120-volt circuit running at 90 percent available current, solve for its charge capacity in kilowatts.

<sup>\*</sup> The value found in question (b) is the time to charge a fully discharged battery using the standard (high current) charger. The time would be less if the battery is only partially discharged.

ii. Solve for how long it would take to charge a fully discharged battery pack in hours using the backup charger.

- iii. If you did not recharge the battery, how many miles could the vehicle travel? Given that answer, for how many more miles do you need to charge the battery in order to go 40 miles total.
  - a. First, find the energy remaining in the battery after the 40-mile trip.

b. If you did not recharge the battery, how many miles could the vehicle travel?

c. For how many more miles do you need to charge the battery in order to go 40 miles total?

iv. You can see from the previous answer (iii) that a charge will be needed to provide the energy for the additional miles to get home. What percent of a fully charged battery do these additional miles represent? Find the percent of the total fully charged miles these additional miles represent.

v. Using the results from question (ii) and the percentage found in answer (iv), how long would it take to charge the battery enough for you to get home using the onboard charger?

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# Answer Keys

51
53
58

## Answer Key



# DIMENSIONAL ANALYSIS\*

#### **Length Conversions:**

1. Convert 38 inches into a measurement in centimeters, rounded to one decimal place.



2. Convert 51 inches into a measurement in millimeters, rounded to one decimal place.

$$\left(\frac{51\,\text{in}}{1}\right)\left(\frac{25.4\,\text{mm}}{1\,\text{in}}\right)\approx 1295.4\,\text{mm}.$$

3. Convert 9,456 feet into a measurement in miles, rounded to two decimal places.

$$\left(\frac{9456 \text{ ft}}{1}\right) \left(\frac{1 \text{ mi}}{5280 \text{ ft}}\right) \approx 1.79 \text{ mi}.$$

\* Special thanks to Doug Gardner, Rogue Community College, for providing worksheet questions from Applied Technical Mathematics, Section 1.4 "Dimensional Analysis," pp. 41–43.

#### **Area Conversions:**

4. Convert 17 square inches into a measurement in square centimeters, rounded to two decimal places.

$$\left(\frac{17 \text{ in}^2}{1}\right) \left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right) \left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right) \approx 109.68 \text{ cm}^2.$$

5. Convert 236 square inches into a measurement in square feet, rounded to two decimal places.

$$\left(\frac{236 \text{ in}^2}{1}\right)\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) \approx 1.64 \text{ ft}^2.$$

6. Convert 78 square feet into a measurement in square yards, rounded to two decimal places.

$$\left(\frac{78 \text{ ft}^2}{1}\right) \left(\frac{1 \text{ yd}}{3 \text{ ft}}\right) \left(\frac{1 \text{ yd}}{3 \text{ ft}}\right) \approx 8.67 \text{ yd}^2.$$

#### **Volume Conversions:**

7. Convert 4 cubic feet into a measurement in cubic inches.

$$\left(\frac{4 \text{ ft}^3}{1}\right)\left(\frac{12 \text{ in}}{1 \text{ ft}}\right)\left(\frac{12 \text{ in}}{1 \text{ ft}}\right)\left(\frac{12 \text{ in}}{1 \text{ ft}}\right) \approx 6912 \text{ in}^3.$$

8. Convert 5 cubic feet into a measurement in gallons, rounded to two decimal places.

$$\left(\frac{5 \text{ ft}^3}{1}\right) \left(\frac{7.48 \text{ gal}}{1 \text{ ft}^3}\right) \approx 37.40 \text{ gallons.}$$

9. Convert 167 cubic feet into a measurement in cubic yards, rounded to two decimal places.

$$\left(\frac{167 \, \text{ft}^3}{1}\right)\left(\frac{1 \, \text{yd}}{3 \, \text{ft}}\right)\left(\frac{1 \, \text{yd}}{3 \, \text{ft}}\right)\left(\frac{1 \, \text{yd}}{3 \, \text{ft}}\right) \approx 6.19 \, \text{yd}^3.$$

#### **Rate Conversions:**

10. Convert a rate of 18 feet per second into miles per hour (mph), rounded to one decimal place.

$$\left(\frac{18 \text{ ft}}{\text{sec}}\right)\left(\frac{1 \text{ mile}}{5280 \text{ ft}}\right)\left(\frac{60 \text{ sec}}{1 \text{ min}}\right)\left(\frac{60 \text{ min}}{1 \text{ hr}}\right) \approx 12.3 \text{ mph.}$$

11. Convert 23 gallons per minute (GPM) into cubic feet per day, rounded to the nearest whole number.

$$\left(\frac{23 \text{ gal}}{\text{min}}\right) \left(\frac{1 \text{ ft}^3}{7.48 \text{ gal}}\right) \left(\frac{60 \text{ min}}{1 \text{ hr}}\right) \left(\frac{24 \text{ hr}}{1 \text{ day}}\right) \approx 4428 \text{ ft}^3/\text{day}.$$

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## Answer Key

# 1B Contextualizing Mathematics and Automotive Education

## MATHEMATICAL RELATIONSHIPS FOR ELECTRICITY

Lesson 1, Exercises 1-3

There are only two fundamental relationships that define most electric vehicle circuit operation. These two relationships are shown in the formulas below:

Ohm's Law (George Sigmund Ohm, 1789–1854), which states  $E = I \times R$ . and Watt's Law (James Watt, 1836–1919), which states  $P = E \times I$ .

#### Exercise 1:

a. Using Ohm's Law, rearrange the formula solving for *I* and *R*, recording your work.

$$E = I \times R \longrightarrow \frac{E}{R} = \frac{I \times \cancel{R}}{\cancel{R}} \longrightarrow I = \frac{E}{R}, \quad E = I \times R \longrightarrow \frac{E}{I} = \frac{\cancel{R}}{\cancel{R}} \longrightarrow R = \frac{E}{I}.$$

b. Using Watt's Law, rearrange the formula solving for *E* and *I*, recording your work.

$$P = E \times I \longrightarrow \frac{P}{I} = \frac{E \times I}{I} \longrightarrow E = \frac{P}{I}, \quad P = E \times I \longrightarrow \frac{P}{E} = \frac{\not E \times I}{\not E} \longrightarrow I = \frac{P}{E}.$$

#### Exercise 2:

Use the equations you have generated above to complete Exercise 2. Note that *E* and *I* appear in both Ohm's and Watt's laws. By using substitution, you will be able to generate six more equations to be used in solving equations involving electricity.

a. In Ohms's Law,  $E = I \times R$ . Substitute  $\frac{P}{I}$  from Watt's Law above for *E* and solve for all variables: *P*, *R*, and *I*.

$$\frac{P}{I} = I \times R \longrightarrow \frac{P \times I}{I} = I \times R \times I \longrightarrow P = I^2 \times R,$$

$$P = I^2 \times R \longrightarrow \frac{P}{I^2} = \frac{J^2 \times R}{J^2} \longrightarrow R = \frac{P}{I^2},$$

$$P = I^2 \times R \longrightarrow \frac{P}{R} = \frac{I^2 \times R}{R} \longrightarrow I^2 = \frac{P}{R} \longrightarrow \sqrt{I^2} = \sqrt{\frac{P}{R}} \longrightarrow I = \sqrt{\frac{P}{R}}.$$

b. In Ohms's Law,  $I = \frac{E}{R}$ . Substitute  $\frac{P}{E}$  from Watt's Law above for *I* and solve for all variables: *P*, *E*, and *R*.

$$\frac{P}{E} = \frac{E}{R} \longrightarrow \frac{P \times \not E'}{\not E'} = \frac{E \times E}{R} \longrightarrow P = \frac{E^2}{R},$$

$$P = \frac{E^2}{R} \to P \times R = \frac{E^2 \times \cancel{R}}{\cancel{R}} \to E^2 = P \times R \to \sqrt{E^2} = \sqrt{P \times R} \to E = \sqrt{P \times R},$$

$$P = \frac{E^2}{R} \rightarrow P \times R = \frac{E^2 \times \cancel{R}}{\cancel{R}} \rightarrow P \times R = E^2 \rightarrow \frac{\cancel{P} \times R}{\cancel{P}} = \frac{E^2}{P} \rightarrow R = \frac{E^2}{P}.$$

#### Exercise 3:

You now have twelve interrelated formulas from Ohm's and Watt's laws. These are the foundation for all electrical studies. Create a tool to help you organize these formulas, which you will use for future problems.

Students will create a tool resembling the image in figure 3 (pg. 18).

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## **Answer Key**

# <sup>2</sup> Contextualizing Mathematics and Automotive Education

## ANALYZING COMMON HOUSEHOLD APPLIANCES

### Lesson 2, Exercises 4-6

#### Exercise 4:

A standard hair dryer plugged into a 120-volt (E) outlet has 10 ohms of resistance (R) when operating.

a. Find the current (*I*) in amps using the correct algebraic relationship, and label.

 $I = \frac{E}{R} \rightarrow I = \frac{120 \text{ volts}}{10 \text{ ohms}} \rightarrow I = 12 \text{ amps.}$ 

b. What is the power consumed by a standard hair dryer? Label your answer in watts.

 $P = E \times I \rightarrow P = 120$  volts  $\times 12$  amps  $\rightarrow P = 1440$  watts.

#### Exercise 5:

The cost of electrical usage is based on kilowatt-hours used over a period of one 30-day month, multiplied by the electrical rate for one kilowatt-hour. This exercise will solve for the cost of using a standard hair dryer for one 30-day month.

a. Convert the above watt value to kilowatts (watts).

$$P_{\text{kilowatts}} = \frac{P_{\text{watts}}}{1000} \rightarrow P = \frac{1440 \text{ watts}}{1000} P = 1.44 \text{ kilowatts}.$$

b. If a standard hair dryer was used 5 minutes every day over a 30-day month, solve for the total time of operation in hours for the month.

time =  $\frac{5 \text{ min/ day} \times 30 \text{ days}}{60 \text{ min/ hour}} \rightarrow \text{time} = 2.5 \text{ hours.}$ 

c. Solve for the total kilowatt-hours (kWh) of electricity used by the dryer in one 30-day month.

 $kWh = kW_{drver} \times time_{hours} \rightarrow kWh = 1.44 kW \times 2.5 hours \rightarrow kWh = 3.6.$ 

d. Given the electrical rate is 15 cents per kilowatt-hour (kWh), solve for the total cost to operate the hair dryer for the 30-day month.

 $Cost = kWh \times rate \rightarrow Cost = 3.6 kWh \times $0.15 / kWh \rightarrow Cost = $0.54.$ 

#### Exercise 6:

Not all the energy consumed by a hair dryer is used to produce hot air. Some of the energy goes into running the motor and other losses.

a. If a standard hair dryer consumed 100 of the total number of watts in loss-related functions, what is its efficiency? (Note: Divide the number of useful watts by the total watts.)

$$\eta = \frac{\text{energy}_{\text{useful}}}{\text{energy}_{\text{total}}} \rightarrow \eta = \frac{1440 \text{ W} - 100 \text{ W}}{1440 \text{ W}} \rightarrow \eta = .931.$$

b. Efficiency is generally expressed as a percent. Convert your answer from question (6a) to a percent by multiplying the answer by 100. Round to the nearest tenth.

 $\eta = .931 \times 100 \rightarrow \eta = 93.1\%.$ 

So, 93.1 percent of the energy consumed by a standard hair dryer goes directly into producing hot air. That means 6.9 percent of the energy consumed (and paid for) is lost in the system and does not produce hot air. In any system, it is desirable to have the highest efficiency (least loss) for economic and environmental reasons.

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## **Answer Key**

# 3 Contextualizing Mathematics and Automotive Education

# **ELECTRIC MOTOR**

## Lesson 3, Exercises 7-11

#### Exercise 7:

a. Draw a diagram illustrating the series-aiding configuration for the electric car. Label volts and amphours.

Student diagrams should show 20 batteries connected in a series-aiding configuration, adding to 120 volts and 245 amp-hours, labeled appropriately. Refer to battery illustrations in figures 1 (pg. 16) and 2 (pg. 17) for examples of series-aiding configurations.

b. Determine the final output voltage and amp-hour capacity of the entire vehicle battery pack.

$$E_{pack} = E_{battery} \times N_{batteries} \rightarrow E_{pack} = 6_{volts} \times 20 \rightarrow E_{pack} = 120_{volts}.$$

The amp-hour capacity remains unchanged from the specification of one battery, thus 245 amp-hours. In the answer to 7a, it states that the vehicle battery pack is rated at 120 volts at 245 amp-hours.

c. Electric vehicles require hundreds of amps from the battery pack when operating at reasonable speed. This high discharge rate lowers the available amp-hour capacity because of losses (inefficiencies) inherent in the rapid chemical-to-electrical conversion and a condition called losses. Lead-acid type batteries in electric vehicle applications can provide only about 80 percent of their rated amp-hour capacity. Solve for the electric vehicle amp-hour capacity of the above battery pack taking into account this 80 percent efficiency factor.

 $\textit{EV}_{\textit{capacity}} = 245_{\textit{amp-hours}} \times .8 \rightarrow \textit{EV}_{\textit{capacity}} = 196_{\textit{amp-hours}}.$ 

Realistically, the battery pack would be 120 volts at 196 amp-hours when used in EV applications. It is this value we will use to solve for the vehicle's performance and range.



#### **Exercise 8:**

The electric vehicle featured requires about 20 continuous horsepower (HP) to propel it down the road at 60 miles per hour (mph). From the ImPulse 9 motor graph above (fig. 5), find the current in amps needed to provide this 20 HP.

 $\mathbf{20}_{_{HPamps}}\cong\mathbf{237}_{_{amps}}.$ 

Figure 5. NetGain Motors Impulse 9 motor performance graph. Source: NetGain Motors, Inc. http://www.go-ev.com/PDFs/003\_07\_ImPulse\_9\_Graph.pdf

#### Exercise 9:

a. Solve for the time a totally charged battery can provide the required current for 20 HP (60 mph).

 $time_{total} = \frac{amp-hours_{total}}{amps_{20HP}} \rightarrow time_{total} = \frac{196 \text{ amp-hours}}{237 \text{ amps}} \rightarrow time_{total} = .827 \text{ hours}.$ 

b. Solve for the number of miles the electric vehicle can travel on a fully charged battery at 60 mph.

miles<sub>total</sub> = mph × hours  $\rightarrow$  miles<sub>total</sub> = 60 mph × .827 hours  $\rightarrow$  miles<sub>total</sub> = 49.6 miles.

#### Exercise 10:

#### **Electrical cost of operation:**

Consumers want to know the cost to operate a vehicle. On gasoline-powered vehicles, consumers want to know the cost to fill the tank, miles/gallon, and cents/mile related to the fuel. On electrical vehicles, this equates to the cost to charge the battery, miles on a charged battery, and cents/mile related to the electrical energy.

A battery pack's energy can also be rated in kilowatt-hours (kWh) rather than voltage and amp-hours. From this information, with the addition of charging efficiency, you can determine the cost of operation as well as how long it would take to charge the battery.

a. Convert the vehicle battery pack rating (at the 20-hour discharge rate) from voltage and amphours to kWh. Refer to the *P*, *E*, *I*, *R* formula wheel you created for the power formula given voltage and current. Remember that there are 1,000 watts in a kilowatt.

 $kWh = \frac{E_{battery} \times AH_{battery}}{1000} \rightarrow kWh = \frac{120 \text{ volts} \times 245 \text{ amp-hours}}{1000} \rightarrow kWh = 29.4.$ 

b. You have previously determined the total number of miles the vehicle can travel on a charged battery. Solve for the number of kW it will take to travel each mile at 60 mph.

 $\frac{kW}{mile} = \frac{kW_{total}}{miles_{total}} \rightarrow \frac{kW}{mile} = \frac{29.4 \text{ kW}}{49.6 \text{ miles}} \rightarrow \frac{kW}{mile} = .593 \text{ kW/mile}.$ 

#### Exercise 11:

Lead-acid batteries require about <sup>1</sup>/<sub>3</sub> more power, or approximately 133 percent to charge them fully, than their rated capacity. Once again, this fact is related to inefficiencies in chemical conversions and electrical resistances.

a. Given the kWh rating found in the previous problem, find the required kWh charging requirement when the battery's inefficiency is considered.

 $kWh_{charging} = kWh_{capacity} \times 1.33 \rightarrow kWh_{charging} = 29.4 kWh \times 1.33 \rightarrow kWh_{charging} = 39.1 kWh.$ 

b. Knowing the above "real" kWh energy, including charging inefficiencies, compute how much it will cost to charge a fully discharged battery given an electrical rate of 15 cents per kWh.\*

$$cost_{total} = kWh_{total} \times \frac{cost}{kWh} \rightarrow cost_{total} = 39.1 kWh \times \frac{\$0.15}{kWh} \rightarrow cost_{total} = \$5.87.$$

c. This is also the cost to travel the number of miles determined above for a fully charged battery. Find the cost per mile by dividing this cost by the total number of miles you can drive on a fully charged battery.<sup>†</sup>

 $\frac{\text{cost}}{\text{mile}} = \frac{\text{cost}_{\text{total}}}{\text{miles}_{\text{total}}} \rightarrow \frac{\text{cost}}{\text{mile}} = \frac{\$5.87}{49.6 \text{ miles}} \rightarrow \frac{\text{cost}}{\text{mile}} = \$0.118 = 11.8 \text{ cents}.$ 

- \* This kWh value is the energy that the charger needs to provide to charge a fully discharged battery. It is also the energy that you have to pay for to charge the vehicle from a fully discharged state.
- Electrical vehicles use so much energy for charging, they can put consumers into a much higher electrical rate bracket (more than double the baseline rate) and thus significantly affect the cost of charging and cost per mile.

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## **Answer Key**

4

# Contextualizing Mathematics and Automotive Education

## CHARGING THE VEHICLE

#### Lesson 4, Exercise 12

The time it takes to charge an electric vehicle is of major concern.

Just like filling a gasoline vehicle when the tank is empty, an electric vehicle requires charging the battery pack. Filling the tank with gasoline takes only a few minutes; charging an electric vehicle battery pack can take many hours depending on the state of the battery and the electrical connection used.

Most electrical vehicles have two ways to charge the battery pack. The standard (high current) charger resides at the home and/or workplace and is connected to a special 30-amp, 240-volt circuit. This charger can charge the vehicle's battery pack in a reasonable length of time. The other charger (onboard backup charger) is in the vehicle itself and can be plugged into a regular 15-amp, 120-volt outlet. This charger can be used as a backup when the standard charger is not available, but it takes a very long time to charge the battery. The driver of an electrical vehicle must give priority to charging the battery pack as part of normal operation.

Either type of charger is designed to only draw 90 percent of the total current available from the circuit to avoid accidentally tripping the circuit breaker.

a. Given the external 30-amp, 240-volt circuit running at 90 percent available current, solve for its charge capacity in kilowatts.

$$kW_{charge} = \frac{volts_{charge} \times amps_{charge}}{1000 watts/kW} \times .90,$$
$$kW_{charge} = \frac{240 volts \times 30 amps}{1000 watts/kW} \times .90,$$

 $kW_{charge} = 6.48 \, kW.$ 

b. Solve for how long it would take to charge a fully discharged battery pack in hours using the standard (high current) charger.\*

$$time_{hours} = \frac{kWh_{battery}}{kW_{charger}},$$
$$time_{hours} = \frac{39.1 \, kWh}{6.48 \, kW},$$

time = 6.03 hours.

c. Given a fully charged battery and a workplace 35 miles away, or 70 miles round trip, with a standard (high current) charger at work, if you drove at an average speed of 60 mph, would the vehicle make it home after an 8-hour workday?

By observation, the answer to the above question is YES. As determined in Exercise 9, the range of the electrical vehicle is 49.6 miles, so there would still be energy left over from the 35-mile trip into work. As we can see from question b above, it only takes about 6 hours to charge a fully discharged battery and the workday is 8 hours, allowing plenty of time to fully recharge the battery. Again, the vehicle has a range of 49.6 miles and only needs to go 35 miles. An electric vehicle would be a good choice for this type of travel.

- d. Let's examine a different situation: Given a fully charged battery, if one visited a friend's place 40 miles from home without a standard (high current) charger and drove at an average speed of 60 mph, would the vehicle make it home after a 4-hour stay? Work through the problems below to find your answer.
  - i. Given the onboard charger that can be plugged into a 15-amp, 120-volt circuit running at 90 percent available current, solve for its charge capacity in kilowatts.

$$kW_{charge} = \frac{volts_{charge} \times amps_{charge}}{1000 \text{ watts/kW}} \times .90,$$
$$kW_{charge} = \frac{120 \text{ volts} \times 15 \text{ amps} \times .90}{1000 \text{ watts/kW}},$$
$$kW_{charge} = 1.62 \text{ kW}.$$

\* The value found in question (b) is the time to charge a fully discharged battery using the standard (high current) charger. The time would be less if the battery is only partially discharged.

ii. Solve for how long it would take to charge a fully discharged battery pack in hours using the backup charger.

 $time_{hours} = \frac{kWh_{battery}}{kW_{charger}} \rightarrow time_{hours} = \frac{39.1 \, kWh}{1.62 \, kW} \rightarrow time = 24.1 \, hours.$ 

- iii. If you did not recharge the battery, how many miles could the vehicle travel? Given that answer, for how many more miles do you need to charge the battery in order to go 40 miles total.
  - a. First, find the energy remaining in the battery after the 40-mile trip.

 $kWh_{remaining} = kWh_{total} - (miles \times kWh/mile),$   $kWh = 29.4 kWh - (40 \times 0.593 kWh/mile),$ kWh = 5.68 kWh.

b. If you did not recharge the battery, how many miles could the vehicle travel?

miles<sub>remaining</sub> =  $\frac{kWh_{remaining}}{kWh/mile}$ , miles =  $\frac{5.68 \, kWh}{0.593 \, kWh/mile}$ , miles<sub>remaining</sub> = 9.58 miles. c. For how many more miles do you need to charge the battery in order to go 40 miles total?

40 - 9.58 = 30.42 miles.

iv. You can see from the previous answer (iii) that a charge will be needed to provide the energy for the additional miles to get home. What percent of a fully charged battery do these additional miles represent? Find the percent of the total fully charged miles these additional miles represent.

$$\% = \frac{\text{miles}_{\text{required}}}{\text{miles}_{\text{charged}}} \times 100,$$
$$\% = \frac{30.42 \text{ miles}}{49.6 \text{ miles}} \times 100 \rightarrow \% = 61.3\%.$$

v. Using the results from question (ii) and the percentage found in answer (iv), how long would it take to charge the battery enough for you to get home using the onboard charger?

 $time_{recharge} = \%_{time} \times time_{total},$  $time_{recharge} = .613 \times 24.1 hours,$ 

 $time_{recharge} = 14.8$  hours.

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