Worked Solutions for

INTRODUCTION TO ENGINEERING MECHANICS

Schlenker/McKern

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INTRODUCTION

This book contains a set of worked solutions for the practice problems found at the end of each Topic Chapter in the Jacaranda Press Publication:

INTRODUCTION TO ENGINEERING MECHANICS

Schlenker/McKern

It is intended that the book be used by both the teacher/lecturer and the student as a means of developing method and logical step sequence in problem solving.

Whilst the correct answer is the obvious aim in solving all mathematical problems, it cannot be ignored that many students find this object somewhat elusive. It is equally true that total marks awarded for questions, set in assessment tasks and examinations, are apportioned for both method and correct solution and students may often score good results, yet fail to find the right answer.

In providing this book, the author is mindful of the difficulties teachers may experience when setting assessment work which includes problems from the original text. The decision to publish was taken knowing that teachers will, with a minimum of time and effort, be able to amend either the given data or required answer to, as it were, "Keep the Students Honest".

Despite all efforts by authors, publishers and printers, some errors are inevitable in all new publications and while I hope that this edition will contain a minimum, any information regarding errors found, different methods of problem solving or indeed suggestions for improving the aim of the book would be gratefully accepted for consideration.

In conclusion I wish to record my appreciation for the co-operation of Jacaranda Press and the authors (Schlenker/McKern), the assistance of many of my Industrial Arts teacher colleagues, particularly my staff at Picnic Point High, Mick Eccleston of Grantham High, Bruce Longfoot and the forbearance of my wife Barbara during the preparation of this work.

R. HOLDEN

FOREWORD

Finally a long need in Engineering Science has been fulfilled by the publication of this book of solutions.

Mr Holden has taught Engineering Science since its inception in 1966 and has always had a passion for the mechanics section of the syllabus. His membership of both syllabus and examination committees greatly add to his understanding of the subject.

To place this work in print, Mr Holden has obviously spent many hours in the preparation of the solutions and their format.

I recommend this book as a valuable resource in Engineering Science to both teachers and students.

John A. Gray
Inspector of Schools
(Industrial Arts)

1 Introduction

1 INTRODUCTION
Analysis. Engineering Mechanics: *Newton's Laws*. Units. Scalar and Vector Quantities. Accuracy. Order of Magnitude.

1/1
Explain the difference between mass and weight.

<u>Mass</u> is the amount of matter a body contains and remains constant for that body. Unit- kg. (Scalar)
<u>Weight</u> is the attractive (gravitational) force between two bodies and varies according to their masses and the distance^2 between them. Usually refers to the weight-force of a body relative to the earth. Unit- Newton (Vector)

1/2
All stationary objects are in equilibrium.
(a) Explain the meaning of the term "equilibrium" as used in this sense.
(b) Describe a situation in which a moving object is in equilibrium.

(a) A body is in equilibrium when there is <u>NO NETT</u> force (or moment) acting on it.
(b) A body moving with constant velocity is consid--ered to be in equilibrium since no nett force is operating. Any nett force would produce acceleration.
Newton's 2nd. Law of Motion.

1/3
Briefly explain the meaning of the term *inertia* and its significance to Newton's second law.

A body will naturally continue to do what it is doing. If at rest it will remain so, if moving with constant velocity it will continue to do so. This is known as a body's <u>inertia</u>. To change a body's state of inertia, an unbalanced system of forces must apply resulting in acceleration
Newton's 1st and 2nd. Laws of Motion

1/4
Express the following quantities in their simplest forms by converting them to multiples or sub-multiples of SI units, either base or derived:

0.001 2 metres	56 000 kilograms
12 600 grams	0.2 centimetres
1 minute 6 seconds	10^7 newtons

Metres - mm	: 1·2 mm
Grams - kg	: 12·6 kg
Minutes - S	: 66 s
Kilograms - Tonnes	: 56 Tonnes
Centimetres - mm	: 2 mm
Newtons - MN	: 10 MN

1/5
Express the following quantities in terms of base SI units using the exponential system:

1 tonne	6 millimetres
2 kilometres	4 hours
1 meganewton	

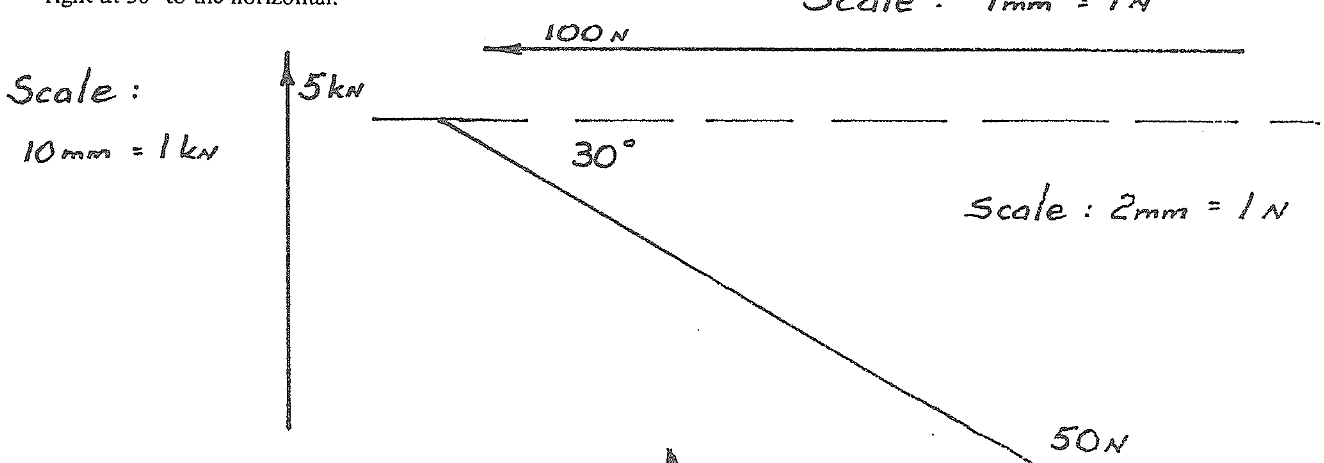
Base unit kg   1 tonne – 10^3 kg
Base unit m   2 km – 2×10^3 m
Base unit N   1 MN – 10^6 N
Base unit m   6 mm – 6×10^{-3} m
Base unit s   4 hrs – $14{\cdot}4\times10^3$ s

1/6
Classify the following quantities as vectors or scalars:

50 kilograms	9.8 metres per second^2
70 cubic metres	7 metres
30 newtons	7 metres south-east
1 second	

50 kg – Scalar
$9{\cdot}8\ m\,s^{-2}$ – vector
70 cubic metres – Scalar
7 metres – Scalar
30 Newtons – Vector
7 metres S-E – Vector
1 second – Scalar

1/7
Draw vectors to represent forces of 5 kN vertically up, 100 N horizontally to the left, 50 N down to the right at 30° to the horizontal.



1/8
Determine both graphically and analytically the resultant displacement of a helicopter which flies 100 km west and then 200 km north.

Displacement : $\sqrt{100^2 + 200^2}$

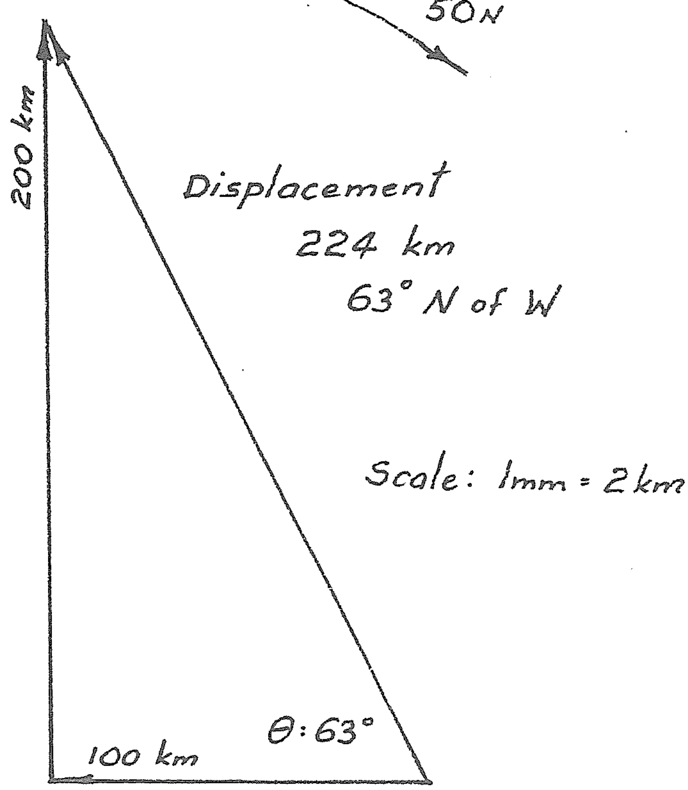
$= \underline{223{\cdot}6 \text{ km}}$

Direction

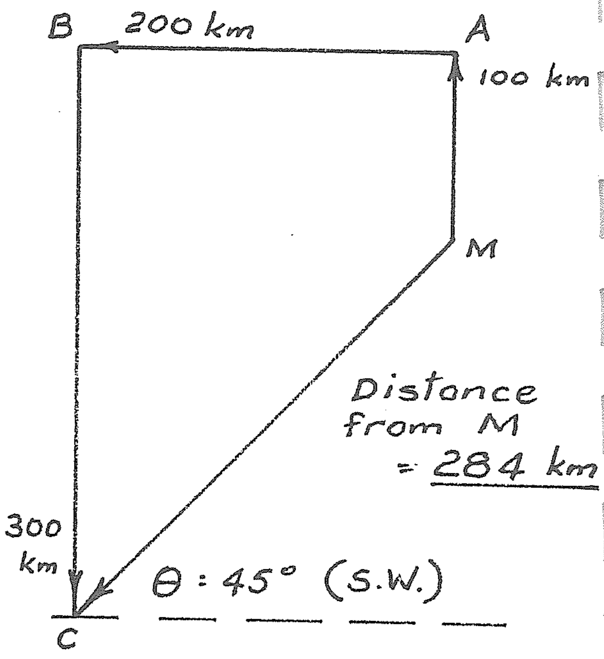
$\tan\theta = \frac{200}{100}$

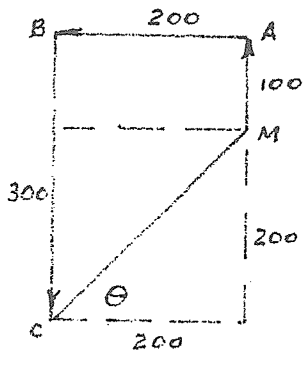
$= 2$

$\therefore \theta = \underline{63{\cdot}4° \text{ N of W}}$



1/9
A pilot on a cross-country navigational exercise has been instructed to fly 100 km north to point *A*, then 200 km west to point *B*, then 300 km south to point *C* and return to base. Using both graphical and analytical methods, determine:
(a) How far is he from base when he is over point *C*?
(b) What is the bearing he will need to set on his compass in order to fly from point *C* to base *M*?





$MC = \sqrt{200^2 + 200^2}$

$= \underline{282 \cdot 8 \text{ km}}$

$\tan\theta = \dfrac{200}{200}$

$\theta = 45^\circ$

Scale : 1mm = 4 km

He is <u>282·8 km S.W.</u> of base and his compass bearing will be <u>N.E.</u>

1/10
Determine the order of magnitude of the following calculations using approximation techniques and then obtain the significant figures for each calculation using the slide rule.

(a) $0.794 \times 386 \times 184$; (b) $\dfrac{980 \times 110 \times 8}{0.002 \times 127}$

(a) Using Significant figures and order of magnitude.

Approx. $(8\times10^{-1}) \times (4\times10^{2}) \times (2\times10^{2})$

Significant figures — $8\times4\times2$ or 64

Order of magnitude — $10^{(-1+2+2)}$ or 10^3

∴ Approx. Answer — 64000

By calculator — <u>56,393·056</u>

(b) Approx. $\dfrac{(1\times10^{3}) \times (1\times10^{2}) \times (1\times10^{1})}{(2\times10^{-3}) \times (1\times10^{2})}$

Significant figures — $\dfrac{1\times1\times1}{2\times1}$ — ·5

Order of magnitude — $10^{(3+2+1+3-2)} = 10^7$

∴ Approx. Answer — 5,000,000

By Calculator — <u>3,395,275·6</u>

2 Force

2 FORCE
Gravity. Weight-force. Equilibrium of Bodies. *Using g in Calculations.* Mass and Force. Centre of Mass. Characteristics of a Force. *Pressure and Stress.*

$g: 10\ m/s^2$

2/5
What is the SI unit of force? What other units can be used? Why is it incorrect to use the old gravitational unit, kg f?

The NEWTON (N)

kilonewton (kN), meganewton (MN) etc.

"kgf" is not an S.I. unit and therefore cannot be used in the S.I. formulae. (see P. 22)

2/6
Describe two effects of a force. Explain the relationship between mass and weight-force. How can the mass of a body change? How can the weight-force of a body change?

A FORCE can: Keep a body in equilibrium or give a body acceleration.

The mass of a body is the amount of matter it contains, its weight-force is the force with which it is attracted to the earth. Its mass does not change but its weight-force changes by altering its distance from the earth's centre (e.g. spacecraft) or by bringing it into close proximity to another body of greater or less mass than the earth (e.g. Jupiter or the moon)

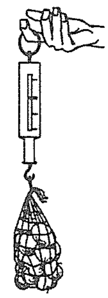
2/7
What is the weight-force produced by masses of 1 kg; 10 kg; 10.2 kg; 27.3 kg; 2000 kg? (Use $g = 9.8\ m/s^2$ first; then use $g = 10\ m/s^2$.)

	$9{\cdot}8\ m/s^2$	$10\ m/s^2$
1 kg	9·8 N	10 N
10 kg	98 N	100 N
10·2 kg	99·96 N	102 N
27·3 kg	267·54 N	273 N
2000 kg	19,600 N or 19·6 kN	20,000 N or 20 kN

2/8
Suggest values for g suitable for use in each of the following calculations. Justify your answer.
(i) Time taken for a pencil to fall from the desk to the floor.
(ii) Time taken for a stone to fall down a deep mine shaft.
(iii) Thickness of a suspended concrete floor slab.
(iv) Tension in a crane cable.
(v) Time of rocket burn for a satellite launch.

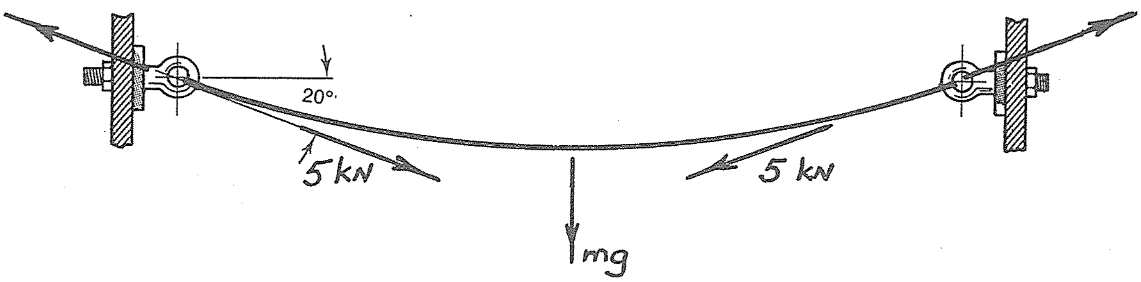
(i) $10\ m/s^2$ – Short time span, small error.
(ii) $9{\cdot}8\ m/s^2$ – Long time involved hence large error.
(iii) $10\ m/s^2$ – Floor supports other masses so slight error is covered by Safety Factor.
(iv) $10\ m/s^2$ – Large Safety Factor involved.
(v) Precise value of "g" - Otherwise orbit etc. will be affected.

2/9
A bag containing 10 apples is hung from a spring balance which then shows a reading of 1 kg. What force is the hook supporting?



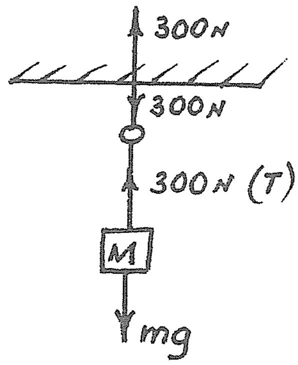
$$F = ma$$
$$= 1 \times 10$$
$$= \underline{10\ N}$$

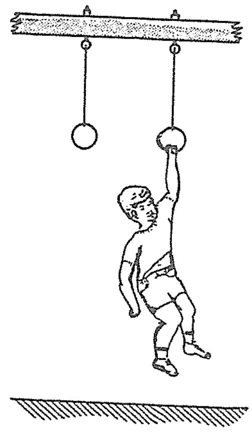
2/10
Completely describe the force acting on each anchor bolt holding the ends of a suspended cable if the tension in the cable is 5 kN.



5 kN on each bolt at 20° to horiz.

2/11
A child hangs at rest from a rope. Determine the approximate mass of the child if the tension in the rope is 300 newtons. What is the magnitude of the force tending to withdraw the eyebolt from the overhead beam?

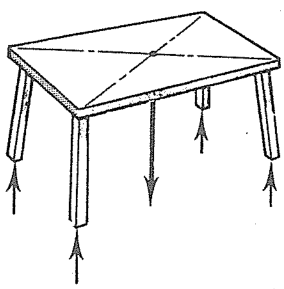




$$mg = 300$$
$$m = 300/g$$
$$= \underline{30\ kg}$$

Force on eyebolt = $\underline{300\,N \downarrow}$

2/12
What is the magnitude of the force exerted by the floor on each leg of a table of mass 50 kg? What are such forces termed?



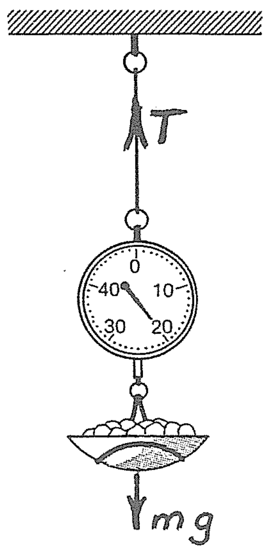
Mass force of table = 50×10
= 500 N
∴ Each leg supports $500/4$
or $\underline{125\,N}$

Reactive forces offered by the floor to maintain equilibrium.
(Compressive.)

2/13
Determine the tension in the cable supporting the spring balance. The spring balance itself has a mass of 2 kg and it is showing a reading of 20 kg.

Total Mass = 22 kg
∴ Mass force = 220 N

and $T = \underline{220\,N}$

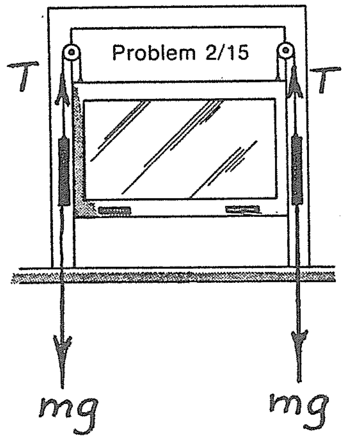


2/14
A shopper tries out his new bathroom scales in the elevator while leaving the shop. He finds that the reading on the scale varies considerably. Why? If the shopper has a mass of 80 kg, suggest likely readings during three stages of the descent:
(a) while leaving the top floor;
(b) halfway down;
(c) nearing the ground floor.

The reading on the scales will depend on the magnitude and direction of the elevator's acceleration.

(a) As the elevator starts with a downward acceleration the reading will be <u>less than 80 kg</u>.
(b) At uniform velocity (up or down) the scales will read <u>80 kg</u>.
(c) As the elevator decelerates on approaching the ground the reading will be <u>greater than 80kg</u>.

2/15
The window shown is open and in equilibrium. What is the tension in the sash cords if each sash balance has a mass of 5 kg?

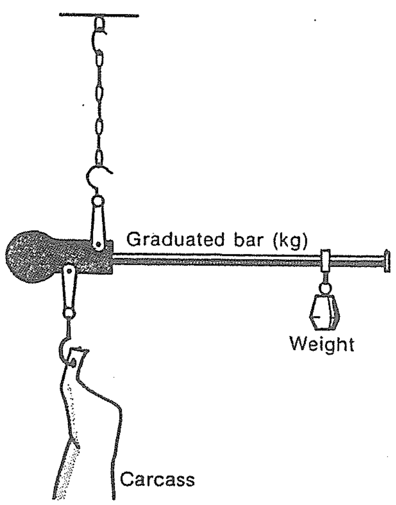


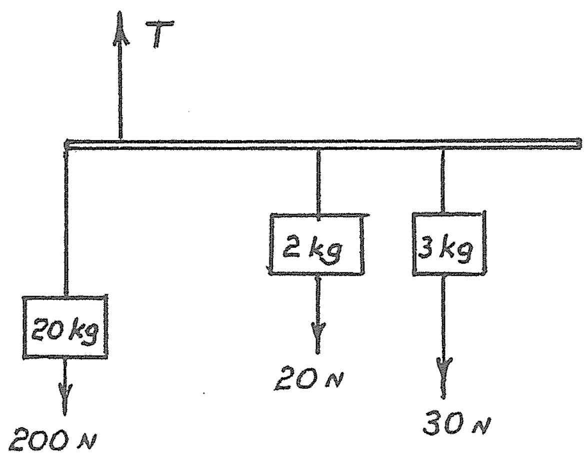
Forces vertically

$$2T = 2mg$$

$$T = \underline{50\ \text{N}}$$

2/16
Determine the force in the chain supporting the steelyard. The balance weight has a mass of 3 kg and the beam a mass of 2 kg. The steelyard is horizontal when the balance weight is positioned on the 20 kg graduation.



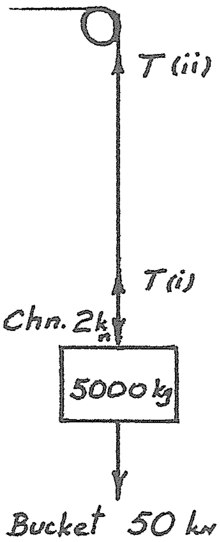


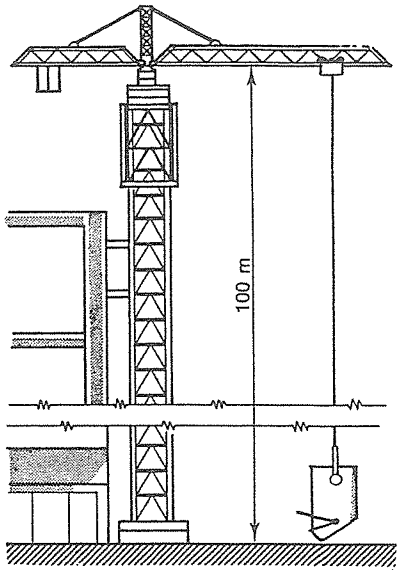
Forces vert.

$$T = 200 + 20 + 30$$

$$= \underline{250\ \text{N}}$$

2/17
The tower crane is 100 m high and is supporting a load of 5 tonnes at ground level. Determine the approximate tension in the cable
(i) near the hook, and
(ii) at the winding drum,
if the cable has a mass of 2 kg/m.





Cable mass – 200 kg
Cable mass force – 2 kN

(i) At hook :
Mass – 5000 kg
∴ Mass force (T.i) = <u>50 kN</u>

(ii) At drum :
Mass – 5000 kg + 200 kg = 5,200 kg
∴ Mass force (Tii) = <u>52 kN</u>

2/18
Billiard tables have an extremely flat surface, made of 75-mm thick, polished slate which is covered with green cloth. Determine the approximate force in each leg of a billiard table, given that the slate has a mass of 750 kg and a full-size table is 4 m × 2 m and has eight legs.

If the maximum point loading the floor can support is 1 kN, should a slate table be installed, or will it be necessary to choose a lighter table with a chipboard top?

Slate mass = 750 kg.
∴ Mass force = 7·5 kN
or approx. <u>0·94 kN per leg</u>

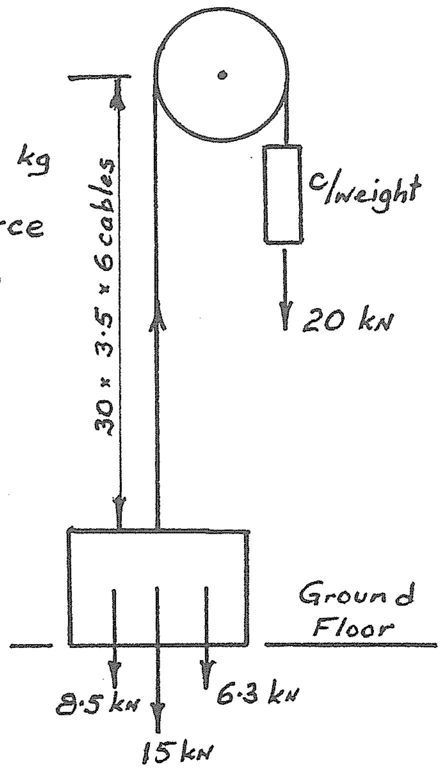
Mass of Slate – 750 kg
∴ Mass force = 7·5 kN

<u>No</u> : If the force per leg cannot exceed 1 kN, the total force the floor can support is 8 kN. This leaves only 0·5 kN or 50 kg mass for the table frame, legs etc. This is not sufficient so the slate table can't be installed.

A table with a particle board top would be satisfactory.

2/19
Each 10-person elevator in a 30-storey building has a mass of 1500 kg when empty, and is supported by six cables, each with a mass of 1 kg/m. The counterweight has a mass of 2000 kg.
(a) Determine the maximum static tension in each cable.
(b) Under what set of conditions will this occur?
(c) An elevator car with three persons aboard is stationary at the tenth floor. Determine the approximate tension in each cable: (i) near the elevator; (ii) near the counterweight; (iii) near the winding drum.
(d) Is the value for tension obtained in (a) suitable for all conditions of operation? Justify your answer.
Floor separation in the building can be taken as 3.5 m and 85 kg as the mass of an average adult.

Cable mass
$= 30 \times 3.5 \times 1 \times 6$ kg
Cable mass force
$= \frac{30 \times 3.5 \times 1 \times 6 \times 10}{1000}$
$= 6.3$ kN



10 persons mass force : 8.5 kN

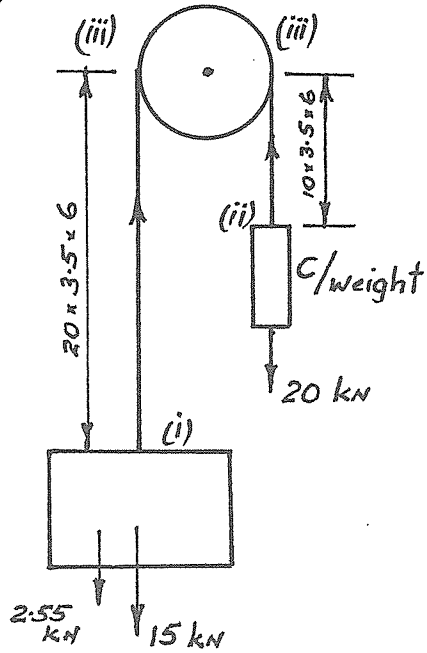
(a) Max. tension occurs when the car is close to the ground floor, where the force is 29.8 kN
∴ Tension/cable = 4.96 kN

(b) At winding drum when fully loaded car is just above ground floor.

(c) Mass force of 3 persons is 2.55 kN
(i) Total force at (i) = 15 + 2.55 kN
∴ Tension/cable = 2.925 kN
(ii) Total force at (ii) = 20 kN
∴ Tension/cable = 3.33 kN
(iii) Near DRUM.

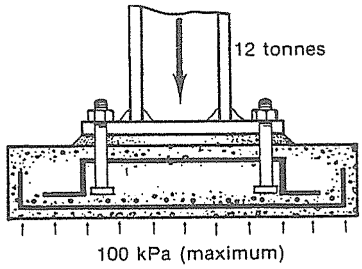
C/weight side
Mass force of cable 2.1 kN
∴ Total force = 22.1 kN
and Tension/cable = 3.68 kN

Elevator side
Total force = 15 + 2.55 + 4.2 = 21.75 kN
and Tension/cable = 3.625 kN



(d) 4.96 kN is for equilibrium. Acceleration will vary this.

2/20
Determine the approximate size of the square concrete foundation base required to support the steel column of a building. The calculated load on the column is 12 tonnes and field and laboratory tests have indicated a suitable bearing pressure for the ground on which the base is to rest would be 100 kPa (0.1 MPa).

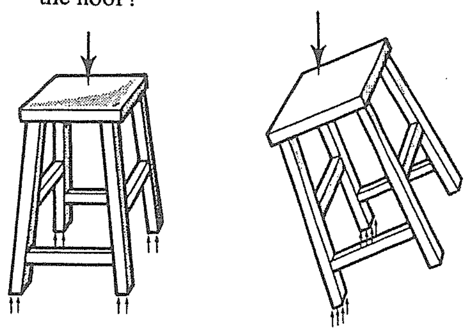


Area = Load/Pressure

$$= \frac{12 \times 1000 \times 10 \times 10}{1000 \times 1000}$$

= 1.2 sq. metres or 1.095 m sq.

2/21
You are sitting on a drawing room stool which has a mass of 4 kg. The total area of the feet of the stool in contact with the floor is 1600 mm^2.
(i) What is the pressure exerted by each leg on the floor?
(ii) What is the stress produced in the floor under each leg?
If you now tilt the stool back, the area of contact with the floor is reduced to about 80 mm^2.
(iii) What pressure does each leg now produce on the floor?

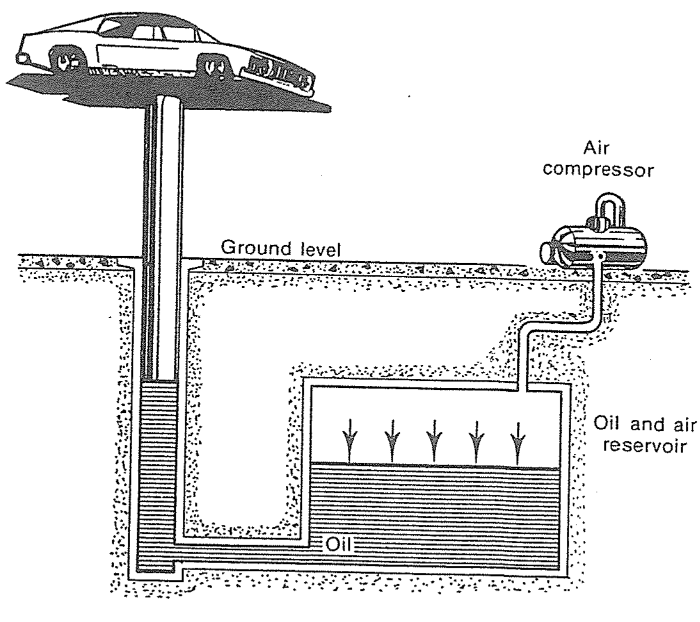


Assume a mass of 80 kg
(i) Total mass supported – 84 kg
∴ Pressure $= \frac{84 \times 10}{1600}$ MPa
or Pressure/leg = 0.13125 MPa
or 131.25 kPa

(ii) The same – an equal reaction.

(iii) Pressure/leg $= \frac{84 \times 10}{80 \times 2}$ = 5250 kPa
(40 times as much as in (i))

2/22
A car is raised on a single column hoist for service. The car has a mass of 1 tonne and the hoist has a mass of 0.5 tonne.
(i) Determine the compressive force in the base of the column of the hoist.
(ii) What air pressure will be required to just support the car if the column has a diameter of 250 mm?



(i) Total mass – 1500 kg
∴ Mass force = 15 kN

(ii) Pressure to balance this force

$$= \frac{15000 \times 4 \times 10^3}{\pi \times 250 \times 250}$$

= 306 kPa

3 Concurrent Forces

3 CONCURRENT FORCES

Resultants of Two Concurrent Forces: *Graphical Solution; Analytical Solution.* Resultant of Three or More Concurrent Forces. Components of a Force. *Analytical Resolution of Three or More Concurrent Forces.* Equilibrant Force. *Method of Solving Problems.*

$g : 10\,m/s^2$

3/5

Determine the magnitude and direction of the resultant of the two concurrent forces shown.

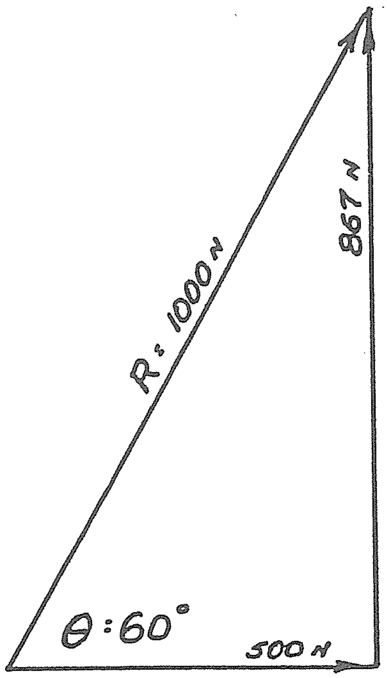
$R = \sqrt{867^2 + 500^2}$

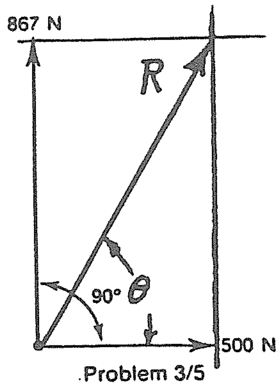
$= \underline{1000 \cdot 84\ N}$

$\tan\theta = \dfrac{867}{500}$

$= 1 \cdot 734$

$\theta = \underline{60 \cdot 03^\circ}$



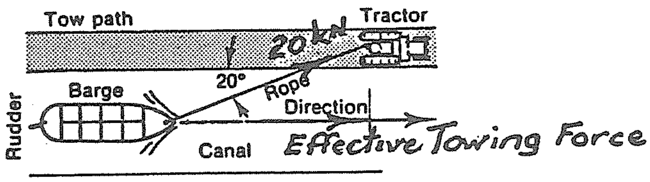


Problem 3/5

Scale : 1mm = 10 N

3/6

A barge is towed along a canal by means of a rope attached to a tractor. The direction of movement of the barge is kept parallel to the canal wall by off-setting the rudder. The rope makes an angle of 20 degrees with the canal. If the tension in the rope is 20 kN what is the effective towing force?

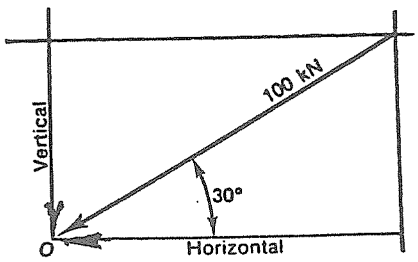


Problem 3/6

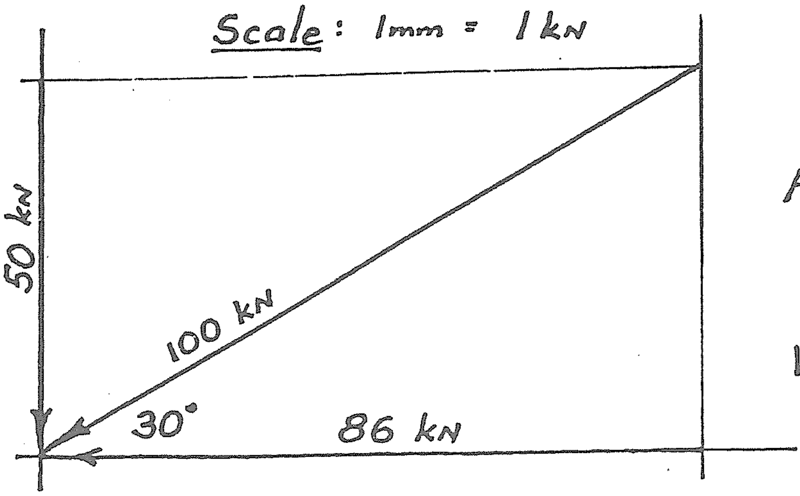
Effective Towing Force $= 20\cos 20^\circ$ or $\underline{18 \cdot 794\ kN}$

3/7

Determine graphically and analytically the horizontal and vertical components of the force acting at *O*.



Scale : 1mm = 1 kN



Horizontal comp.

$= 100\cos 30^\circ$

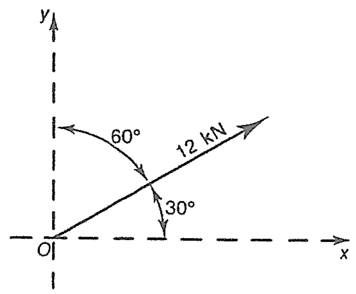
$= \underline{86 \cdot 6\ kN}$ ←

Vertical comp.

$= 100\sin 30^\circ$

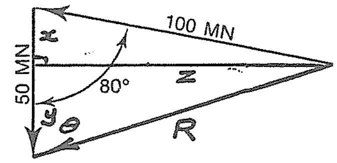
$= \underline{50\ kN}$ ↓

3/8
What are the components of the 12 kN force in directions Ox and Oy?



$$\text{Horiz. Comp.} = 12\cos 30^\circ$$
$$= \underline{10{\cdot}39\ kN}$$
$$\text{Vert. Comp.} = 12\sin 30^\circ$$
$$= \underline{6\ kN}$$

3/9
Replace the system of forces shown in the diagram by a single force which will have the same effect.

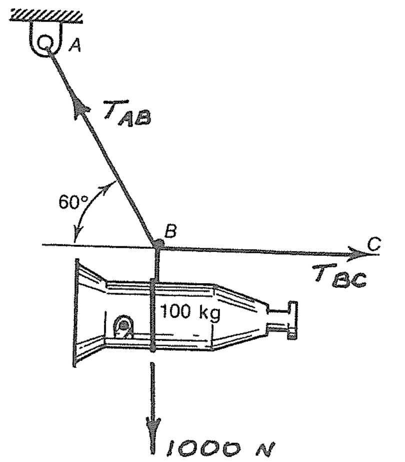


$$x = 100\cos 80^\circ = 17{\cdot}36\ MN$$
$$z = 100\sin 80^\circ = 98{\cdot}48\ MN$$
$$y = 50 - 17{\cdot}36 = 32{\cdot}64\ MN$$
$$R = \sqrt{98{\cdot}48^2 + 32{\cdot}64^2}$$
$$= \underline{103{\cdot}75\ MN}$$

$$\tan\theta = 98{\cdot}48 / 32{\cdot}64$$
$$\therefore\ \theta = \underline{71{\cdot}66^\circ}$$

Can also be solved by sin rule, cos rule & graphically

3/10
The figure shows a gearbox which has a mass of 100 kg. During a lifting operation it is supported in the position shown by a rope AB and a rope BC. Determine the tension in each rope.

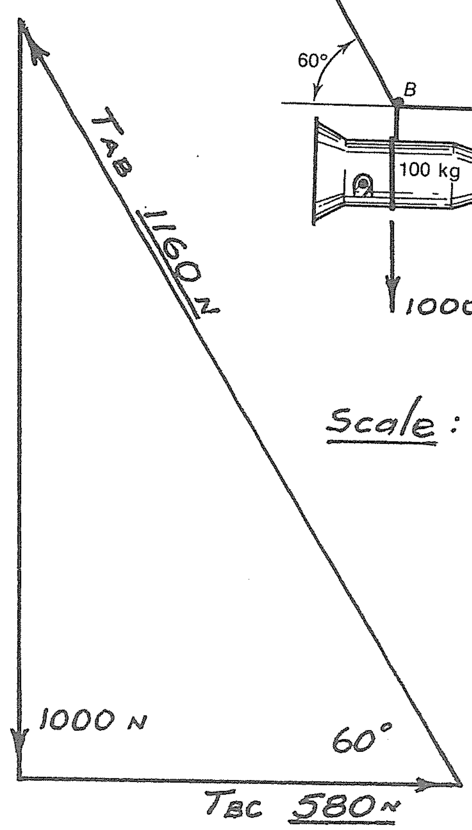


Resolve vert.

$$T_{AB}\cos 30^\circ = 1000$$
$$T_{AB} = \underline{1154\ N}$$

Resolve horiz.

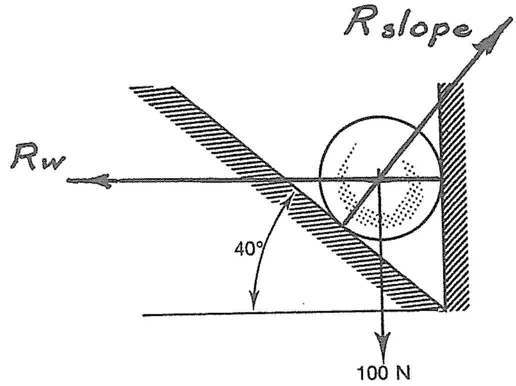
$$T_{BC} = T_{AB}\cos 60^\circ$$
$$= \underline{577\ N}$$



Scale : 1 mm = 10 N

3/11
A steel ball rests in a groove the sides of which are smooth. One side of the groove is vertical, while the other side is at 40° to the horizontal.

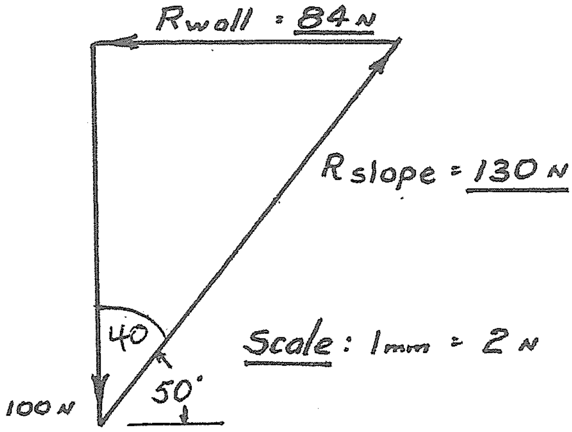
If the ball has a mass of 10 kg, find the reaction on each wall of the groove.



Resolve vert.

$R_{slope} \sin 50^\circ = 100$

$R_{slope} = \underline{130 \cdot 55\ N}$ ∠50°



Resolve horiz.

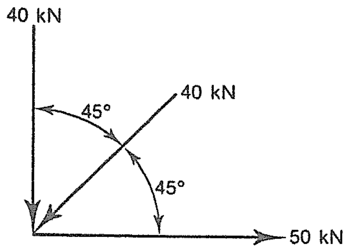
$R_W = 130 \cdot 55 \cos 50^\circ$

$= \underline{83 \cdot 915\ N} \leftarrow$

$\cos 40 = \dfrac{N}{R_{slope}}$

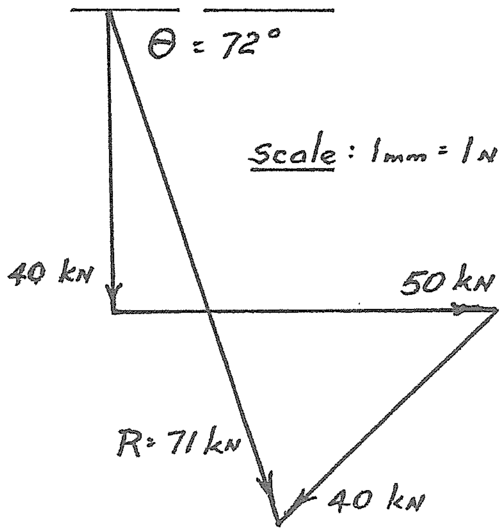
$\dfrac{R_{slope}}{\cancel{\cos 40 / 100}} = \dfrac{100}{\cos 40}$

3/12
Find the magnitude and direction of the resultant of the three forces shown in the diagram.



Resolve horiz.

$50 - 40 \cos 45^\circ = \underline{21 \cdot 72\ kN} \rightarrow$



Resolve vert.

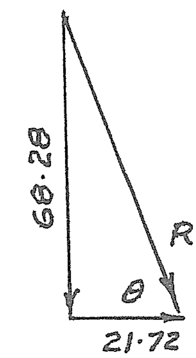
$40 + 40 \sin 45^\circ = \underline{68 \cdot 28\ kN} \downarrow$

$R = \sqrt{21 \cdot 72^2 + 68 \cdot 28^2}$

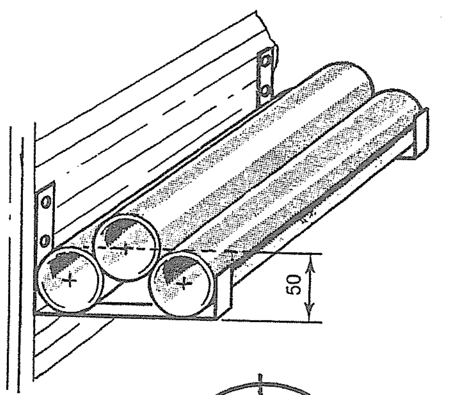
$= \underline{71 \cdot 65\ kN}$

$\tan \theta = \dfrac{68 \cdot 28}{21 \cdot 72}$

$\therefore \theta = \underline{72 \cdot 35^\circ}$



3/13
Three greasy steel pipes of 50 mm outside diameter are stacked in a wall rack as shown. Each pipe has a mass of 25 kg. Discover the forces acting on each bracket.

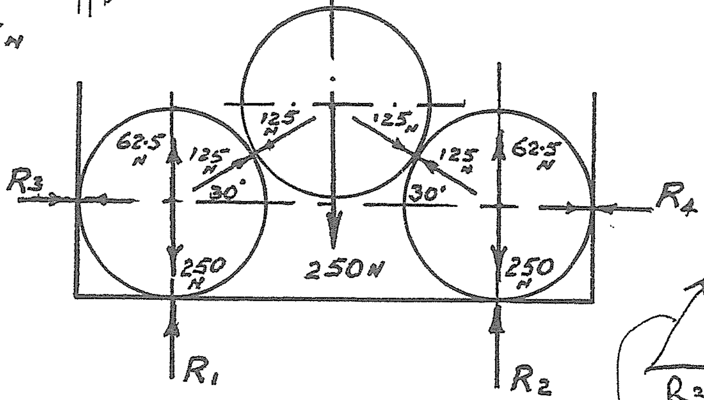


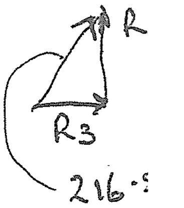
Resolve mass force of middle pipe at 30° to horiz. to give reaction between pipes of 125 N
Resolve vert.

$R_1 + 62{\cdot}5 = 250$

$\therefore \underline{R_1 = 187{\cdot}5\,N} \uparrow$

Similarly $\underline{R_2 = 187{\cdot}5\,N} \uparrow$



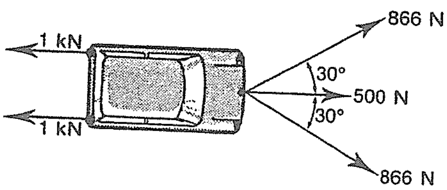


Resolve horiz.

$R_3 = 125 \cos 30°$

$= \underline{108{\cdot}25\,N} \rightarrow$ Similarly $R_4 = \underline{108{\cdot}25\,N} \leftarrow$

3/14
The car shown in the diagram is held in a bog by forces of 1 kilonewton on each side. Three ropes are attached to the car and forces as shown are applied to the ropes. Will these forces remove the car from the bog? Justify your result.

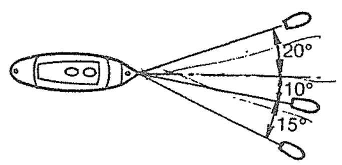


Resolve horiz. $8{\cdot}66 \cos 30° + 500 + 8{\cdot}66 \cos 30° = 2$

$2 = 2$

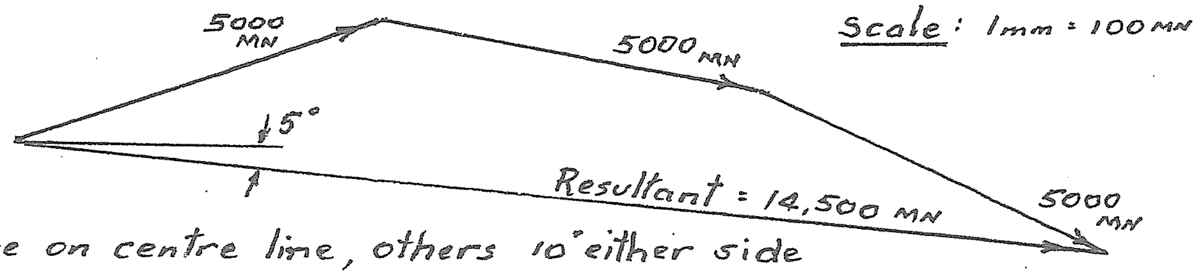
__No.__ The system is in equilibrium

3/15
An ocean liner is being towed by three tugboats as shown. The tension in each cable is 5000 MN. (a) Determine graphically the resultant force acting on the bow of the liner. (b) If the tugboats cannot operate safely when the angle between the cables is less than 10°, where should the tugboats be located in order to produce the largest resultant force? (c) What is the magnitude of this force?



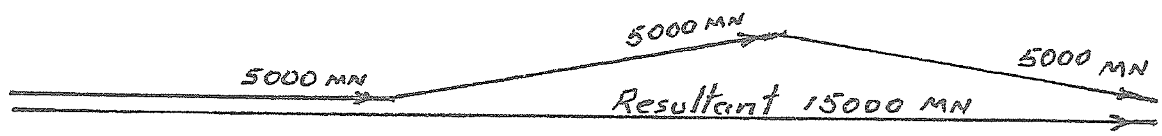
__Scale__: 1 mm = 100 MN

(a)

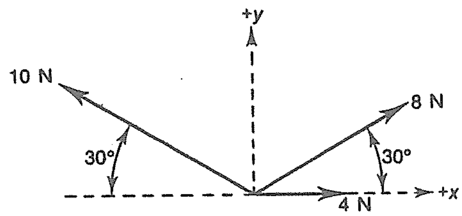


(b) One on centre line, others 10° either side

(c)



3/16
Find the resultant x component and the resultant y component of the force system shown.



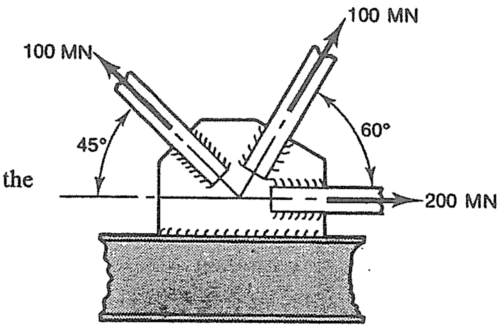
Resolve horiz. "x"

$$\text{Comp.} = +4 + 8\cos 30^\circ - 10\cos 30^\circ$$
$$= +\underline{2{\cdot}27\ N}$$

Resolve vert. "y"

$$\text{Comp.} = +10\sin 30^\circ + 8\sin 30^\circ$$
$$= \underline{+9\ N}$$

3/17
Determine the resultant of the three forces on the welded joint shown in the diagram.

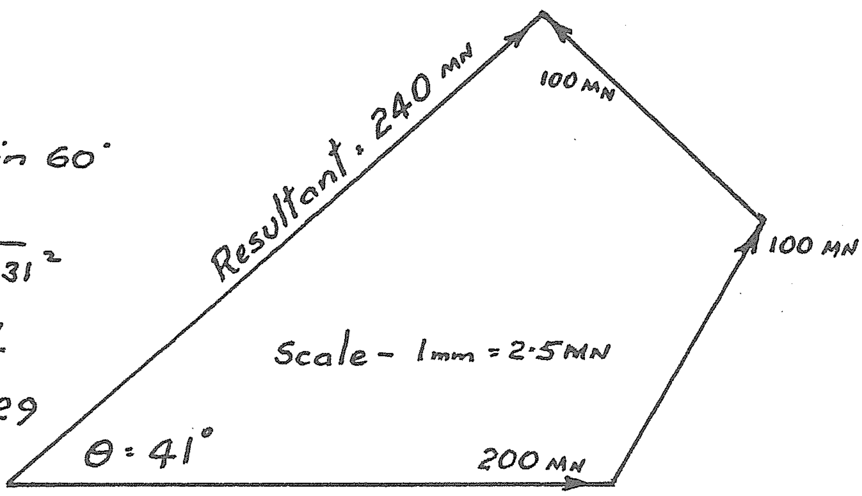


Resolve horiz.

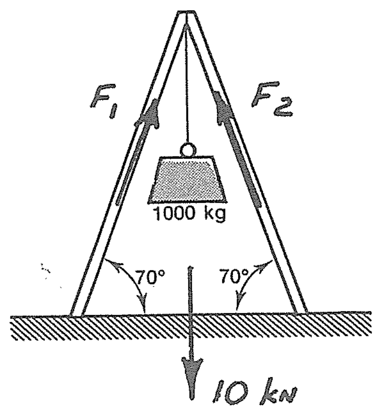
$$200 + 100\cos 60^\circ - 100\cos 45^\circ$$
$$= 179{\cdot}29\ MN$$

Resolve vert.

$$100\sin 45^\circ + 100\sin 60^\circ$$
$$= 157{\cdot}31\ MN$$
$$R = \sqrt{179{\cdot}29^2 + 157{\cdot}31^2}$$
$$= \underline{238{\cdot}52\ MN}$$
$$\tan\theta = 157{\cdot}31/179{\cdot}29$$
$$\therefore \theta = \underline{41{\cdot}26^\circ}$$



3/18
A pair of shear legs is used to support a load of 1000 kg. Determine the force in each of the legs. (The legs and load rope are in the same vertical plane.)



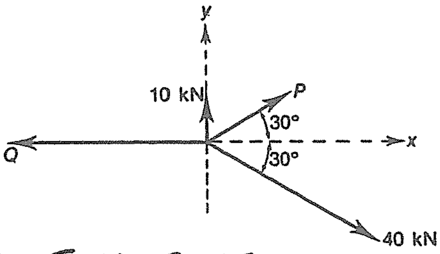
Resolve horiz.

$$F_1\cos 70^\circ = F_2\cos 70^\circ \quad \therefore F_1 = F_2$$

Resolve vert.

$$10 = F\sin 70^\circ + F\sin 70^\circ$$
$$F = \underline{5{\cdot}322\ kN \quad (\text{in each leg})}$$

3/19
State the conditions of equilibrium of any coplanar system of forces.
If the force system shown is in equilibrium, find the value of the forces P and Q.



For equilibrium : $\Sigma x = 0$, $\Sigma y = 0$, $\Sigma M = 0$

Resolve vert.

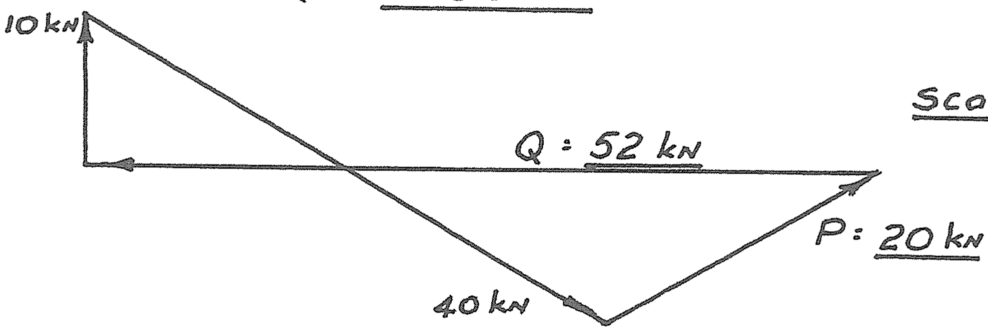
$$10 + P \sin 30^\circ - 40 \sin 30^\circ = 0$$

$$P = \underline{20 \text{ kN}}$$

Resolve horiz.

$$P \cos 30^\circ + 40 \cos 30^\circ - Q = 0$$

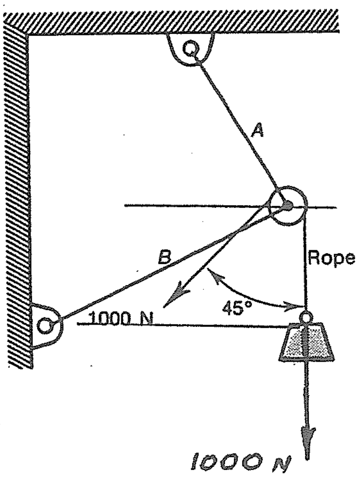
$$Q = \underline{51 \cdot 96 \text{ kN}}$$

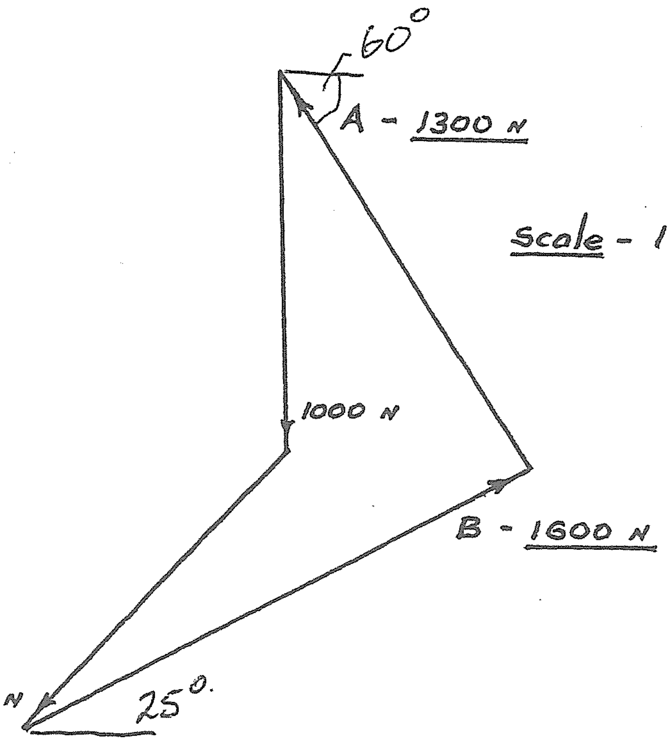


Scale : 10 mm : 5 kN

3/20
Determine the forces in each of the members A and B.

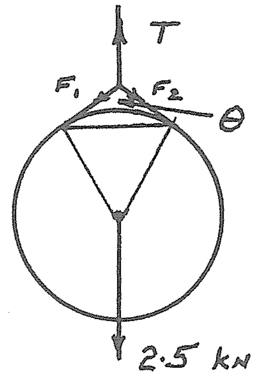
Graphically





Scale - 1 mm = 20 N

3/21
A smooth hook is attached to the end of a rope which is looped around a stormwater pipe of 500 mm diameter and 250 kg mass. Disregarding friction, determine the angle θ and the tension in the rope.



Let $\alpha = \frac{\theta}{2}$

Since there is no friction the 3 forces, T, F_1 & F_2 must be equal for equilibrium.

Resolve vert.

$$F_1 \cos\alpha + F_2 \cos\alpha = T$$

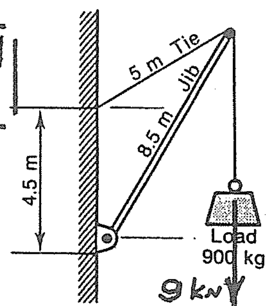
$$\cos\alpha = 0.5$$

$$\therefore \alpha = 60^\circ \text{ and } \underline{\theta = 120^\circ}$$

Rope tension = $\underline{2.5 \text{ kN}}\uparrow$

3/22
Determine the forces in the jib and tie of the jib crane shown.

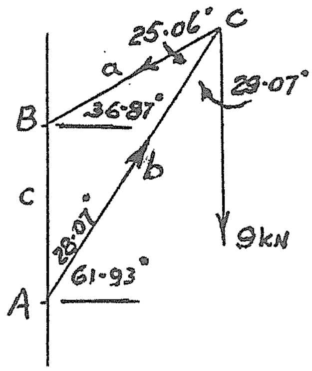
$\boxed{\text{Easier solved Graphically}}$



In ΔABC

$$\cos A = \frac{8.5^2 + 4.5^2 - 5^2}{2 \times 8.5 \times 4.5}$$

$$\therefore \hat{A} = 28.07^\circ$$



$$\frac{\sin C}{4.5} = \frac{\sin A}{5}$$

$$\therefore \hat{C} = 25.06^\circ$$

Resolve vert.

$$9 - b \sin 61.93^\circ + a \sin 36.87^\circ = 0$$

$$\therefore a = \frac{0.8824b - 9}{0.6}$$

Resolve horiz.

$$a \cos 36.87^\circ - b \cos 61.93^\circ = 0$$

$$0.8a - 0.4705b = 0$$

Subst.

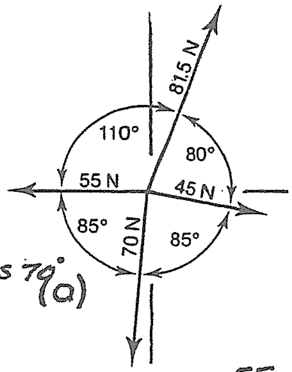
$$0.8\left(\frac{0.8824b - 9}{0.6}\right) - 0.4705b = 0$$

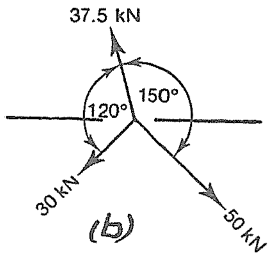
$\therefore$ b (Jib force) = $\underline{16.8 \text{ kN}}$

and a (Tie force) = $\underline{9.7 \text{ kN}}$

3/23

In each of the systems of forces shown, the condition of equilibrium may or may not exist. Determine the force necessary for equilibrium if that condition does not already exist.





(a) Resolve horiz.

$55 + 70\cos 85^\circ - 45\cos 10^\circ - 81.5\cos 70^\circ$

$= \underline{11.09\ N} \rightarrow$

Resolve vert.

$81.5\cos 20^\circ - 45\sin 10^\circ - 70\cos 5^\circ$

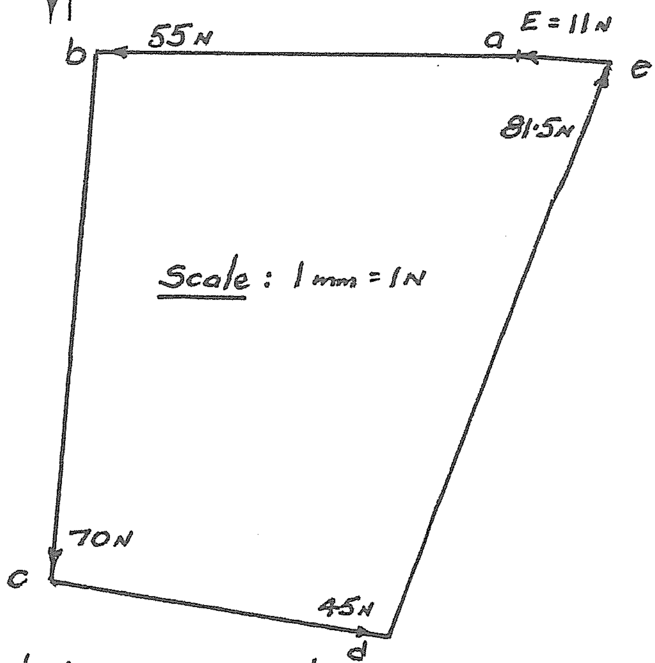
$= \underline{0.96\ N} \downarrow$

$R = \sqrt{11.09^2 + 0.96^2}$

$= \underline{11.13\ N}$

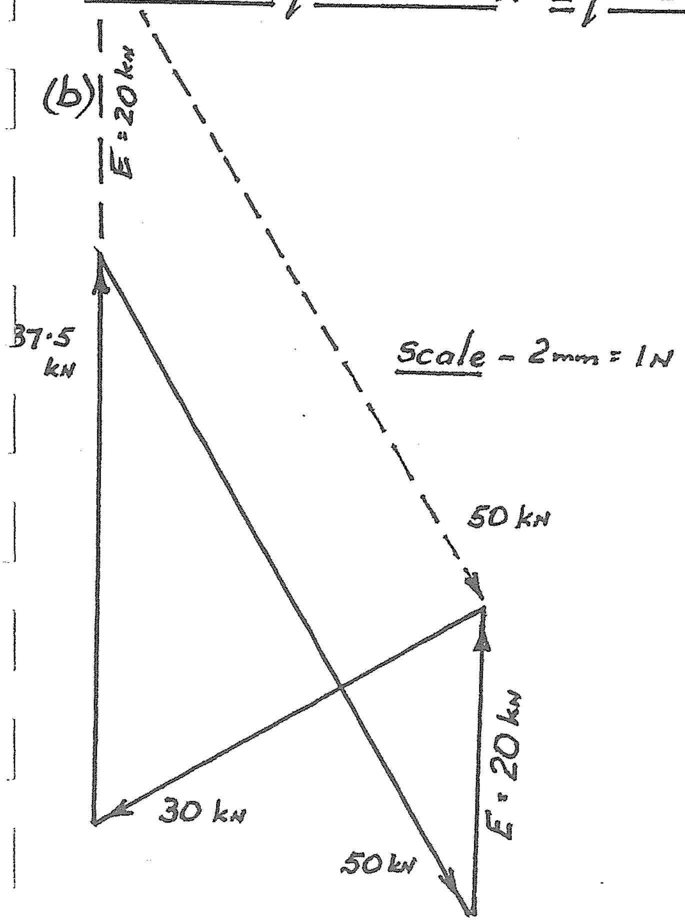
$\tan\theta = 0.96/11.09$

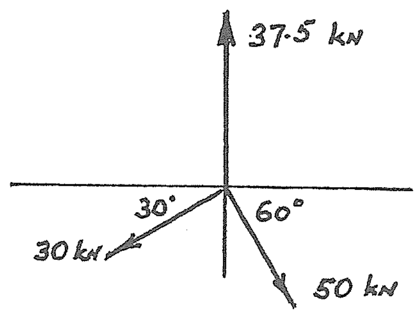
$\theta = \underline{4.94^\circ}$



Not in equilibrium. Equilibrant is 11.13 N at 4.94°

(b)





Resolve horiz.

$30\cos 30^\circ - 50\cos 60^\circ$

$25.9807 - 25 \simeq 0$

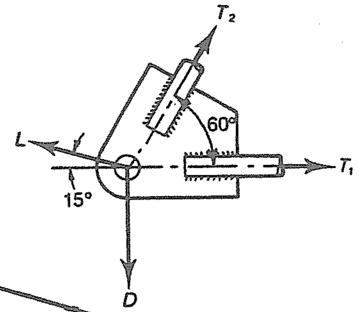
Resolve vert.

$37.5 - 30\sin 30^\circ - 50\sin 60^\circ$

$= 20.8\ kN \downarrow$

Not in equilibrium. For equilibrium increase the 37.5 kN force by 20.8 kN

3/24
Two forces, L and D, of magnitude $L = 1000$ N and $D = 1200$ N are applied to the connection shown. Knowing that the connection is in equilibrium, determine the tensions T_1 and T_2.

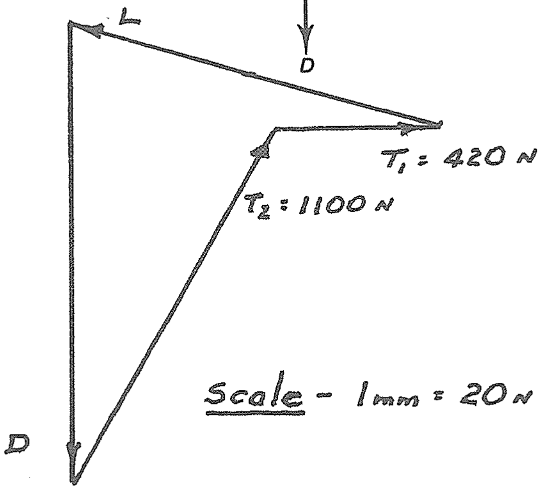


Resolve vert.

$$L \sin 15^\circ + T_2 \sin 60^\circ = D$$
$$T_2 = \underline{1086 \cdot 8 \text{ N}}$$

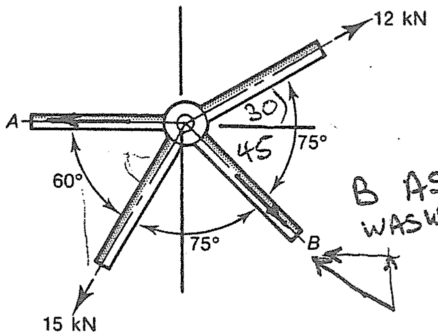
Resolve horiz.

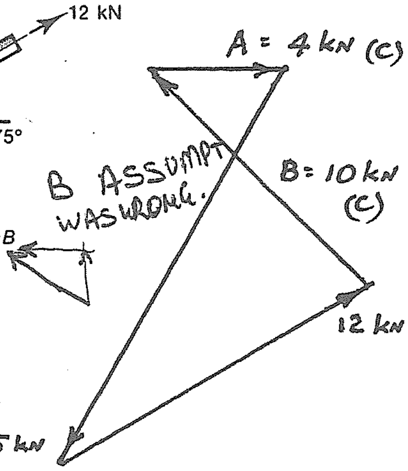
$$T_1 + T_2 \cos 60^\circ = L \cos 15$$
$$T_1 = \underline{422 \cdot 5 \text{ N}}$$



Scale - 1 mm = 20 N

3/25
One of the joints of a pin-jointed frame is shown in the diagram. Describe the forces in members A and B.





Scale - 4 mm = 1 kN

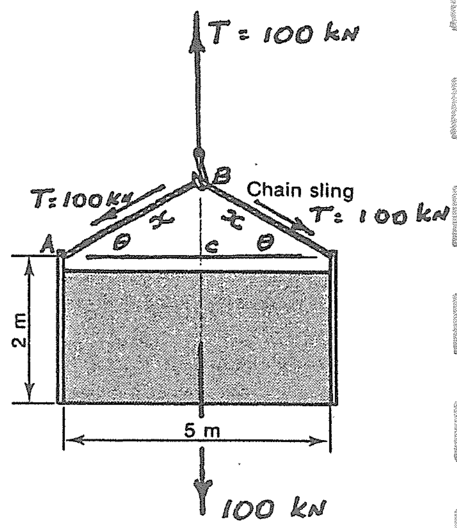
Resolve vert.

$$15 \cos 30^\circ + B \cos 45^\circ = 12 \cos 60^\circ$$
$$B = \underline{-9 \cdot 885 \text{ kN (C)}}$$

Resolve horiz

$$A + 15 \cos 60^\circ = B \cos 45^\circ + 12 \cos 30^\circ$$
7·5   −6·08   10·39
$$A = \underline{-4 \cdot 097 \text{ kN (C)}}$$

3/26
A container and its contents have a mass of 10 tonnes. Determine the shortest chain sling which may be used to lift the container if the tension in the sling is not to exceed 100 kN.



Resolve vert.

$$100 \sin \theta + 100 \sin \theta = 100$$
$$\sin \theta = 0 \cdot 5$$
$$\therefore \theta = 30^\circ$$

In Δ ABC

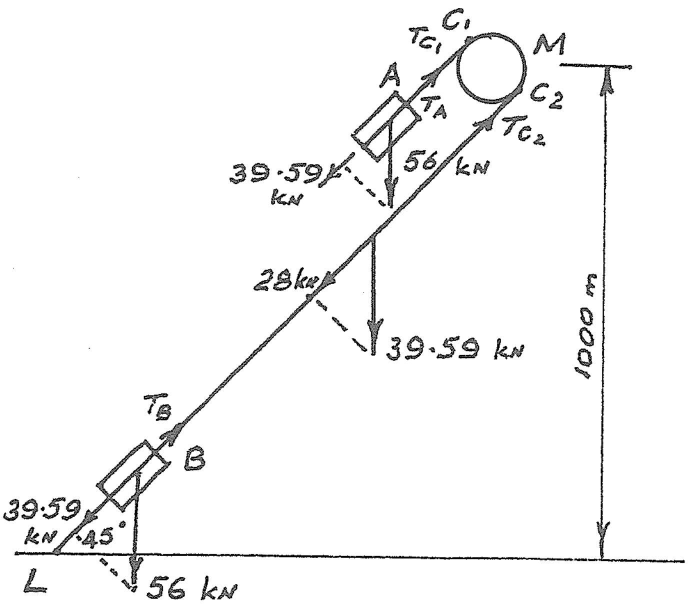
$$x = 2 \cdot 5 / \cos 30^\circ$$
$$= 2 \cdot 885 \text{ m}$$

$$\therefore \text{Sling Length} = \underline{5 \cdot 77 \text{ m}}$$

3/27

Mürren is reached from Lauterbrünnen by a funicular railway which travels up and down a slope of 45°. The cable passes from one car through guide rollers between the rails, around the winding drum, and back down to the other car. Each car acts as a counterweight for the other, thereby reducing the power needed to wind a loaded car up the mountain. The difference in elevation between the two Swiss villages is approximately 1000 m (Lauterbrünnen 646 m, Mürren 1647 m). Each loaded car has a mass of 5.6 tonnes and the cable connecting the two cars has a mass of 2.8 kg/metre.

Determine the approximate tension in the cable when there is a loaded car waiting at each village:
(a) near the car at Mürren;
(b) near the car at Lauterbrünnen;
(c) on each side of the winding drum.



Cable length $= 1000/0.7071 = 1414$ m

" mass $= 2.8 \times 1414$

$= 3959.2$ kg

" mass force $= 39.59$ kN

Component of cable force at 45° $= 28$ kN

Comp. of top car mass force at 45° $= 39.59$ kN

Comp. of bottom car mass force at 45° $= 39.59$ kN

(a) $\therefore$ Tension at A $= \underline{39.59 \text{ kN}}$

(b) Tension at B $= \underline{39.59 \text{ kN}}$

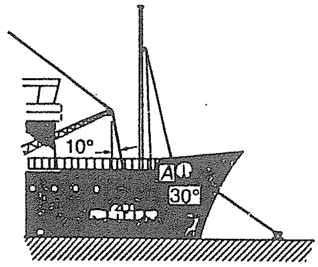
(c) (i) Tension at C_1 $= \underline{39.59 \text{ kN}}$

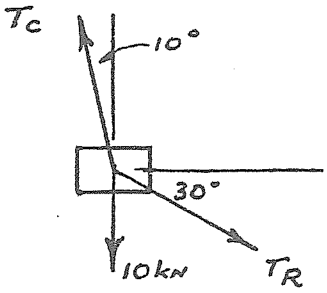
(ii) Tension at C_2 $= 39.59 + 28$

$= \underline{67.59 \text{ kN}}$

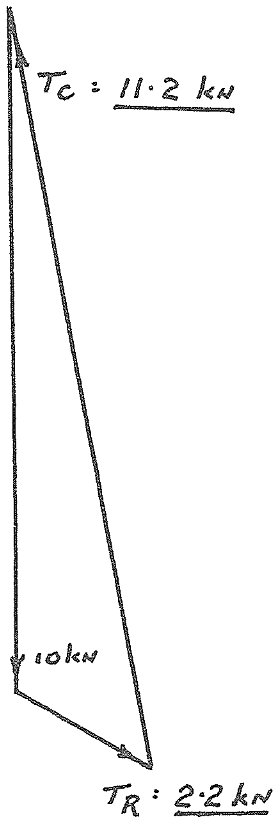
3/28

Fred's new 1-tonne car is being unloaded at the docks. A guide rope is attached to the crane hook A so that the wharf labourers can swing the car on to the dock. The crane cable is 10° off vertical and the guide rope is 30° to the horizontal. Find the tensions in cable and rope.





Scale- 10mm = 1 kN



Resolve horiz.

$$T_c \cos 80° = T_R \cos 30°$$

$$T_c = \frac{T_R \cos 30°}{\cos 80°} \quad \ldots\ldots (1)$$

Resolve vert.

$$T_c \sin 80° = 10 + T_R \sin 30°$$

subst.

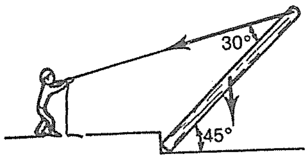
$$\frac{T_R \cos 30°}{\cos 80°} \times \sin 80 = 10 + T_R \sin 30°$$

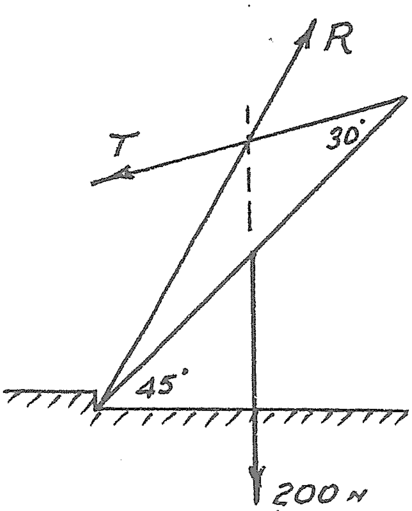
$$T_R = \underline{2 \cdot 266 \text{ kN}}$$

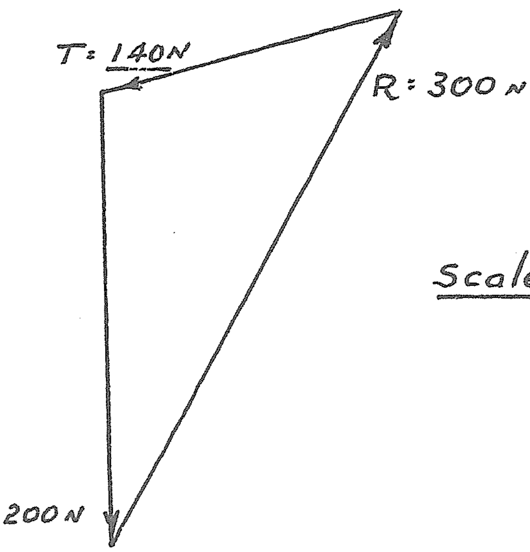
Subst. in (1) $\quad T_c = \underline{11 \cdot 3 \text{ kN}}$

3/29

A man raises a pole of mass 20 kg by pulling on a rope. Find the tension, T, in the rope.





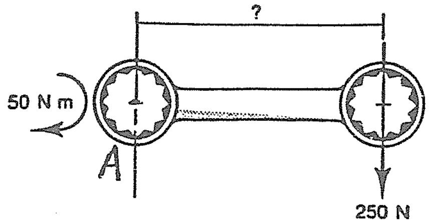


Scale – 1 mm = 3·33 N

4 Moments

4 MOMENTS
Moments. *Sign Convention.* Parallel Forces. *The Moment Arm.* $g : 10 m/s^2$

4/12
The maximum pull that a man can comfortably exert on the handle of a spanner is about 250 N. Determine how long the handle should be if the maximum torque to be exerted by the spanner is 50 N m.

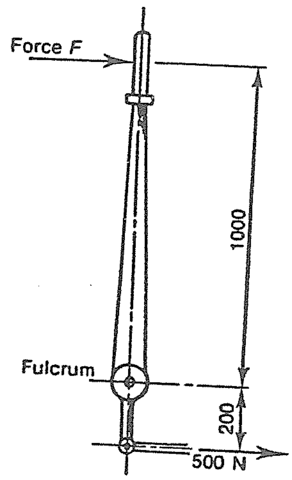


Take moments about A

$$250 \times ? = 50$$

$$? = \underline{200 \text{ mm}}$$

4/13
Determine the minimum force F required to move the brake lever. Friction in the connections may be neglected.

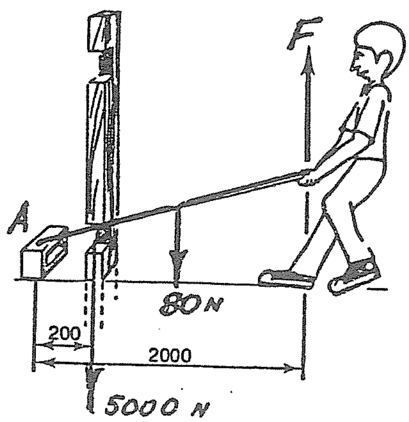


Take moments about fulcrum

$$500 \times 200 = F \times 1000$$

$$F = \underline{100 \text{ N}}$$

4/14
You are removing old fence-posts with a crowbar, which has a mass of 8 kg. The force required to lift the post is 5000 N. What force will you need to apply to the end of the crowbar to raise the post?



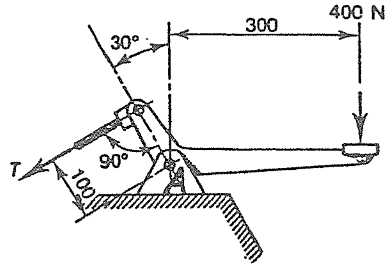
Take moments about A

$$5\cancel{0}\cancel{0}\cancel{0} \times 200 + 80 \times 1\cancel{0}\cancel{0}\cancel{0} = F \times 2\cancel{0}\cancel{0}\cancel{0}$$

$$2F = 1080$$

$$F = \underline{540 \text{ N}}$$

4/15
A 400-N force is required to operate the foot pedal shown. Determine the tension, T, in the rod.

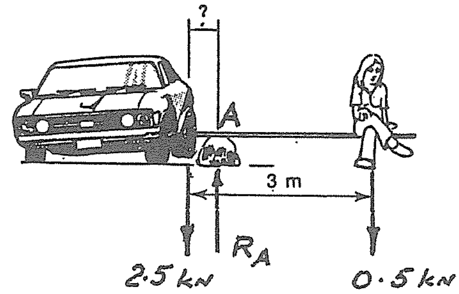


Take moments about A

$$T \times 1\cancel{0}\cancel{0} = 4\cancel{0}\cancel{0} \times 300$$

$$T = \underline{1200 \text{ N}}$$

4/16
You have suffered a flat tyre on your new car, which does not have a jack. The car has a mass of 1 tonne (equally distributed on the four wheels) and your girlfriend has a mass of 50 kg. Where should you place the rock under the lever, so that your girlfriend may sit comfortably while you change the wheel?



Mass force of car = 10 kN
Mass force per wheel = 2.5 kN
Mass force of girl = 0.5 kN

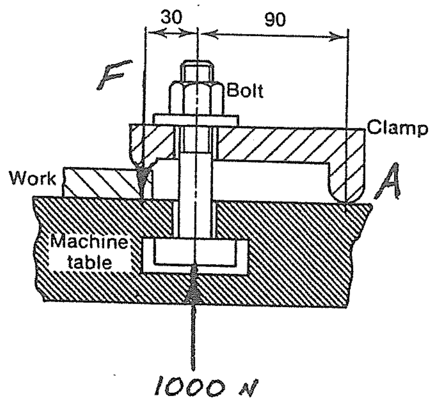
Take moments about A

$2.5 \times ? = 0.5(3 - ?)$

$? = 0.5\,m.$

Place rock closer than 0.5 m to the car

4/17
A clamp is used to secure a work piece to the table of a milling machine. If the tightening of the nut induces a force of 1000 N on the bolt, determine the clamping force exerted on the work piece.

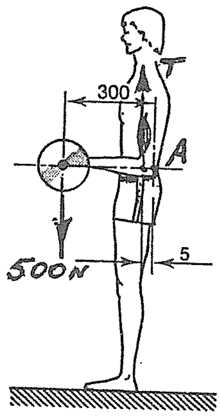


Take moments about A

$1000 \times 9\cancel{0} = F \times 12\cancel{0}$

$F = \underline{750\ N}$

4/18
The bar-bell has a total mass of 50 kg and you have raised it with both hands to the position shown. Determine the tension in your bicepses.



Take moments about A

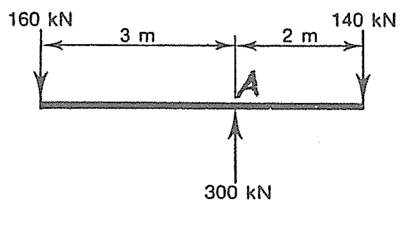
$T \times 5 = 500 \times 300$

$T = 30{,}000\ N$

$= 3\ kN$

i.e. 1.5 kN in each biceps muscle

4/19
Is the bar, loaded as shown, in a state of equilibrium? Give reasons for your answer in terms of $\Sigma F = 0$, and $\Sigma M = 0$.



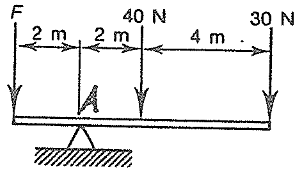
Resolve horiz. – Nil nett force
Resolve vert. – $160 + 140 - 300 = 0$

Take moments about A

$160 \times 3 - 140 \times 2 = 200\ kNm$

Not in equilibrium. ΣF_H & $F_V = 0$, but there is an anti-clockwise turning moment of 200 kNm

4/20
The lever is just balanced on the fulcrum. What is the magnitude of the force F?

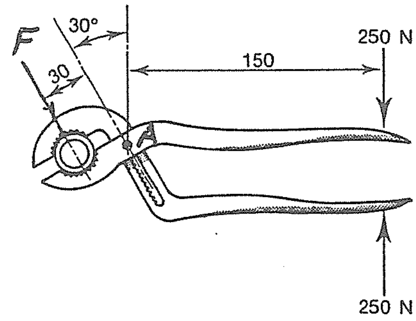


Take moments about A

$$F \times 2 = 40 \times 2 + 30 \times 2$$

$$F = \underline{130\ \mathrm{N}}$$

4/21
What pressure is exerted on the pipe when the handles of the multi-grips are squeezed as shown?



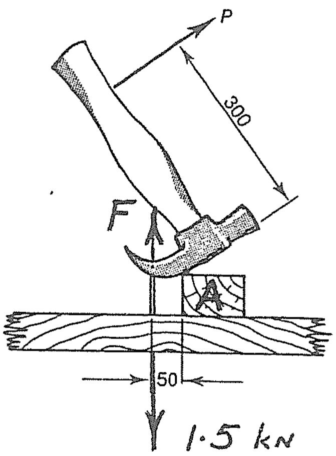
Take moments about A

$$F \times 3\cancel{0} = 250 \times 15\cancel{0}$$

$$F = 1250\ \mathrm{N}$$

i.e. $\underline{1{\cdot}25\ \mathrm{kN}}$ <u>on each side of the pipe</u>

4/22
(a) Find the pull exerted on the nail by a pull P of 200 newtons on the handle.
(b) If the nail's resistance to movement is 1.5 kN, what is the force P required on the handle of the hammer, to extract it?
(c) Describe where the block should be placed, to minimise the chances of breaking the handle of the hammer.



(a) Take moments about A

$$F \times 50 = 200 \times 300$$

$$F = \underline{1{\cdot}2\ \mathrm{kN}}$$

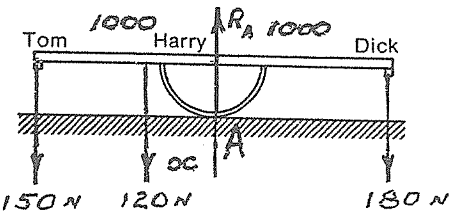
(b) Increase F from 1·2 to 1·5 kN

$$P \times 100 = 1{\cdot}5 \times 50$$

$$P = \underline{250\ \mathrm{N}}$$

(c) <u>Move block closer to the nail.</u>

4/23
The see-saw is 2 metres long. Tom (15 kg) sits on one end and Dick (18 kg) sits on the other. Where should Harry (12 kg) stand, to keep the see-saw level?



G+

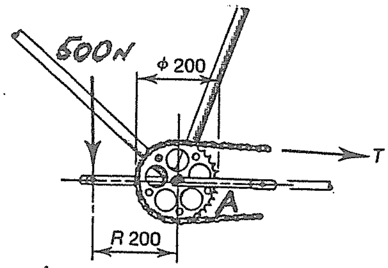
Take moments about A

$$12\cancel{0} \times x + 15\cancel{0} \times 1000 = 18\cancel{0} \times 1000$$

$$12x = 3000$$

$$x = \underline{250\ \mathrm{mm}}$$

4/24
Calculate the tension, T, in the chain of the bicycle when the 50-kg rider puts all his weight on one pedal.

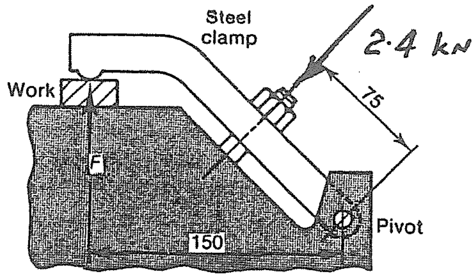


Take moments about axle A

$$T \times 100 = 500 \times 200$$

$$T = \underline{1 \text{ kN}}$$

4/25
For the pivoted clamp shown in the sketch, determine the clamping force, F, if the force exerted by the bolt when the nut is tightened is 2.4 kN.

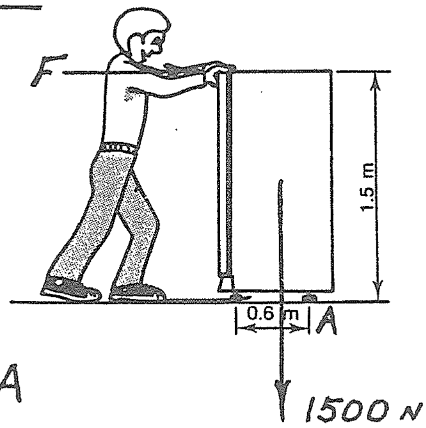


Take moments about the pivot

$$F \times 150 = 2{\cdot}4 \times 75$$

$$F = \underline{1{\cdot}2 \text{ kN}}$$

4/26
You are trying to push a piece of carpet under the front feet of the refrigerator, so you can slide it to a new place in the kitchen. The refrigerator has a mass of 150 kg. What horizontal force will you need to exert at the top of the refrigerator, to rock it back just far enough to get the carpet under? (Assume the weight of the refrigerator is equally distributed on all four feet).

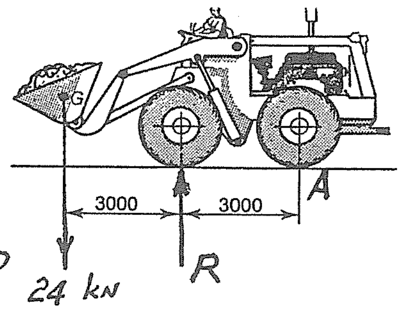


Take moments about A

$$F \times 1{\cdot}5 = 1500 \times 0{\cdot}3$$

$$F = \underline{300 \text{ N}}$$

4/27
Determine the extra load on the front wheels of the loader when it is carrying 2 m³ of earth with a mass of 1200 kg/m³.



Earth Load $= 1200 \times 2 \times 10$

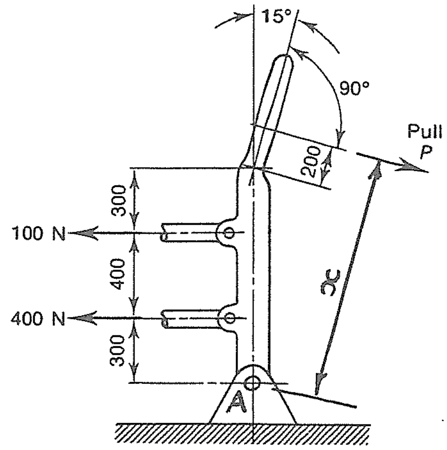
$= 24$ kN

Take moments about A

$$R \times 3000 = 24 \times 6000$$

$$R = \underline{48 \text{ kN}}$$

4/28
Determine the magnitude of the pull, P, when the forces shown in the diagram are applied to the lever.



$x = 1000 \cos 15^{\circ} + 200$

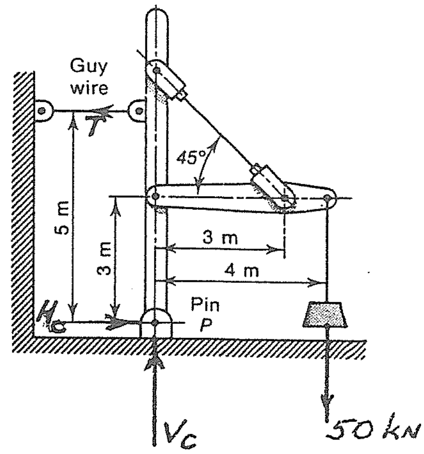
$= 1166$ mm

Take moments about A

$P \times 1166 = 400 \times 300 + 100 \times 700$

$P = \underline{163 \text{ N}}$

4/29
The pin-jointed framework shown in the diagram supports a load of 50 kilonewtons. Determine
(a) the tension in the guy wire; and
(b) the vertical and horizontal components of the force exerted on the pin P.



(a) Take moments about P

$T \times 5 = 50 \times 4$

$T = \underline{40 \text{ kN}}$

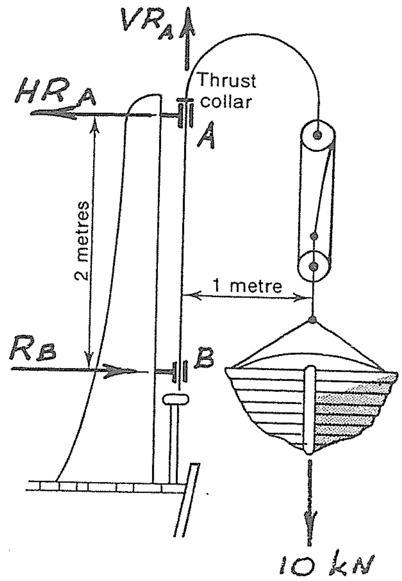
(b) Resolve horiz.

$H_c = \underline{40 \text{ kN}} \rightarrow$

Resolve vert.

$V_c = \underline{50 \text{ kN}} \uparrow$

4/30
One of a pair of life-boat davits is illustrated in the figure. If all of the vertical thrust is taken at the top pivots, determine the reactions at the supports if the boat has a mass of 1 tonne.



Take moments about A

$R_B \times 2 = 10 \times 1$

$R_B = 5 \text{ kN} \rightarrow$

Resolve horiz.

$HR_A = 5 \text{ kN} \leftarrow$

Resolve vert

$VR_A = 10 \text{ kN} \uparrow$

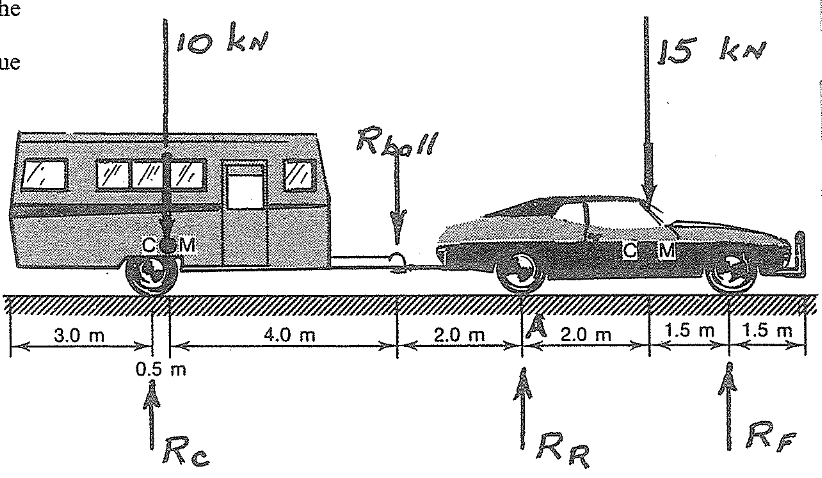
Two davits for each boat

$\therefore$ Top supports – Thrust collars : $\underline{5 \text{ kN ea. vertically}}$

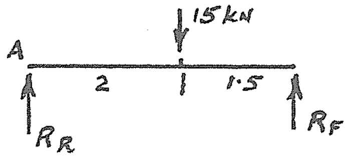
– Pivots : $\underline{2{\cdot}5 \text{ kN ea. horiz.}}$

Bottom supports – Pivots : $\underline{2{\cdot}5 \text{ kN ea. horiz.}}$

4/31
A 1-tonne caravan is attached to a 1.5-tonne car by a ball-and-socket coupling. Determine
(a) the reactions at each of the six wheels when the car and caravan are at rest;
(b) the change in load on each of the car wheels due to the caravan.



(a) Car only



Take moments about A

$15 \times 2 = R_F \times 3.5$

$\therefore R_F = 8.571$ or 4.285 kN/wheel

and $R_R = 6.429$ or 3.215 kN/wheel

Van only

Take moments about ball

$10 \times 4 = R_C \times 4.5$

$R_C = 8.889$ or 4.445 kN/wheel ; $R_{Ball\,joint}$ 1.111 kN

Car + Van

Take moments about C

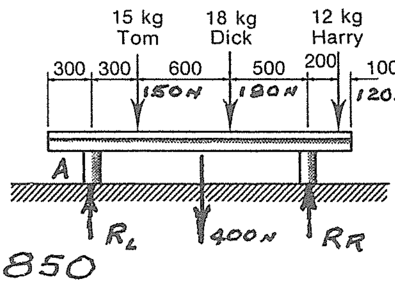
$1.111 \times 2 + R_F \times 3.5 = 15 \times 2$

$\therefore R_F = 7.937$ or 3.968 kN/wheel & $R_R = 8.174$ or 4.087 kN/wheel

(b) Difference due to van:–

(i) Front : 0.634 kN less (ii) Rear : 1.745 kN more

4/32
Tom, Dick, and Harry are sitting on the garden seat as shown. The seat has a mass of 40 kg.
(a) What is the load supported by each of the two legs of the seat?
(b) If the base of each leg is 50 mm × 300 mm, what pressure does each exert on the ground?



(a) Resolve vert. $R_L + R_R = 850$

Take moments about A

$150 \times 300 + 400 \times 700 + 180 \times 900 + 120 \times 1600 = R_R \times 1400$

$\therefore R_R = 485\,N$ & $R_L = 365\,N$

(b) Rt. leg pressure $= \frac{485}{50 \times 300} = 0.032$ MPa.

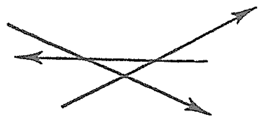
Left leg pressure $= \frac{365}{50 \times 300} = 0.024$ MPa.

5 Non-Concurrent Forces

$g = 10\ m/s^2$

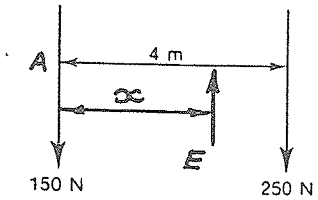
5 NON-CONCURRENT FORCES
Non-concurrent Forces. Resultants of Non-concurrent Forces. *Graphical Solution. Analytical Solution.* The Funicular Polygon. Equilibrants: *Conditions of Equilibrium, The Two-force System, The Three-force System. Reactions at Supports.*

5/10
Can the three forces, the lines of action of which are shown, be in equilibrium? (Give reasons for your answer.)



No. For a 3 force non-concurrent system ΣM cannot equal zero.

5/11
For the parallel force system shown, determine:
(i) the magnitude;
(ii) direction; and
(iii) location of the resultant force.

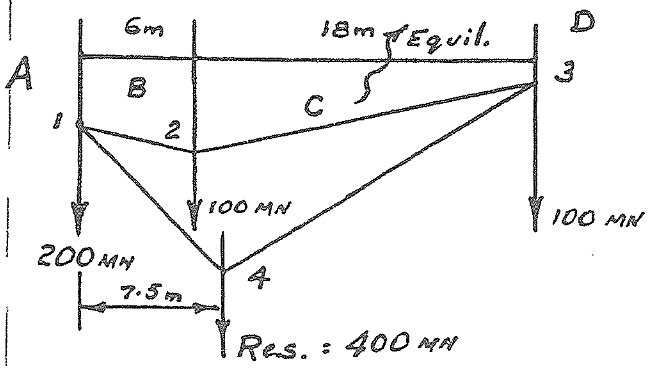


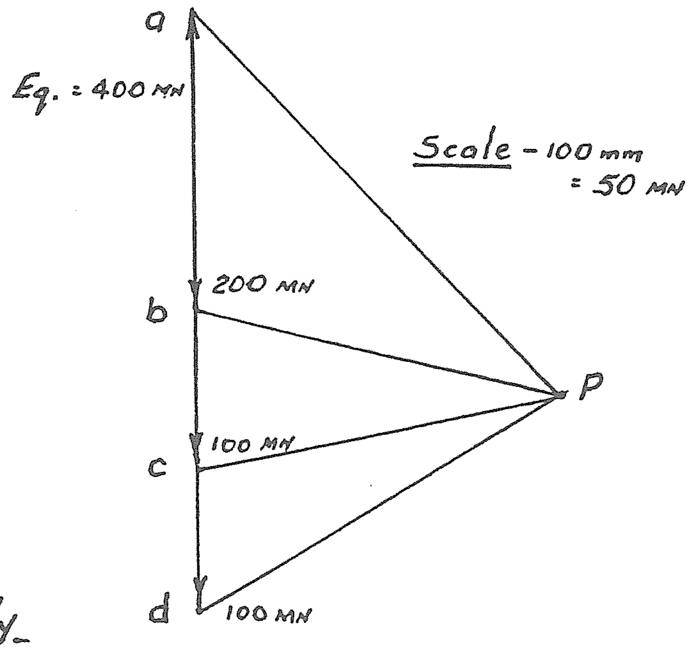
(i) Resultant = 150 + 250 = 400 N
(ii) Vertically down
(iii) Let E be the equilibrant (400 N), x metres from A
Take moments about A

$$400 \times x = 250 \times 4$$

$$x = 2.5\ m.$$

5/12
Determine graphically the magnitude and location of the resultant of the three parallel forces shown.

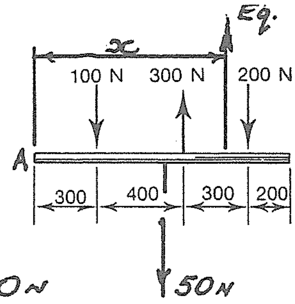




Scale - 10 mm = 4 m

Resultant = 400 MN vertically down 7.5 m to the right of the 200 MN force

5/13
A uniform bar with a mass of 5 kg has vertical forces applied to it as indicated.
(i) What additional force must be applied; and
(ii) along what line of action must it be applied to hold the bar in equilibrium?



(i) Forces down $\quad 100 + 50 + 200 = 350\text{N}$

Forces up $\quad = 300\text{ N}$

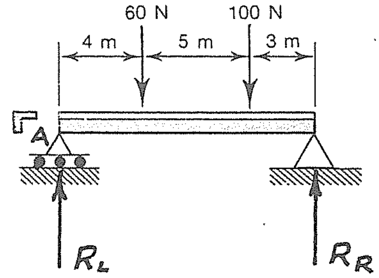
$\therefore$ An equilibrant force of 50 N up is needed

(ii) Take moments about A

$$100 \times 300 + 50 \times 600 - 300 \times 700 - 50 \times x + 200 \times 1000 = 0$$

$\therefore x = 1000\text{ mm}$ or 1 m to the right of A

5/14
Find the reactions to the beam shown in the diagram.



Resolve vert.

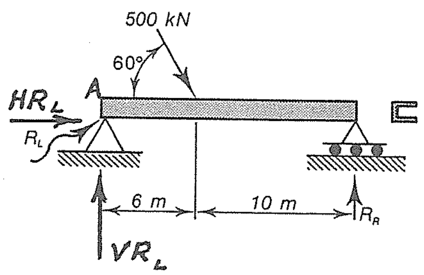
$$R_L + R_R = 160\text{N}$$

Take moments about A

$$R_R \times 12 = 60 \times 4 + 100 \times 9$$

$\therefore R_R = 95\text{N}$ & $R_L = 65\text{N}$

5/15
For the beam shown, find the reactions R_L and R_R. The reaction R_L may be expressed either in terms of its horizontal and vertical components, or in terms of its magnitude and direction.



Take moments about A

$$R_R \times 16 = 500 \sin 60° \times 6$$

$$R_R = 162.4\text{ kN} \uparrow$$

Resolve horiz.

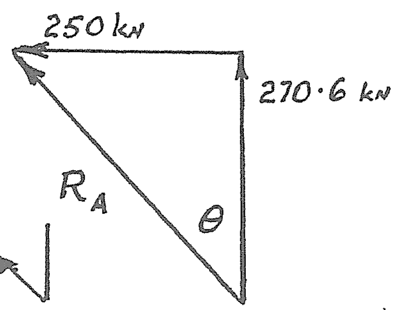
$$HR_L + 500 \cos 60° = 0$$

$$HR_L = -250\text{ kN} \leftarrow$$

Resolve vert.

$$VR_L + 162.4 = 500 \sin 60°$$

$$VR_L = 270.6\text{ kN} \uparrow$$

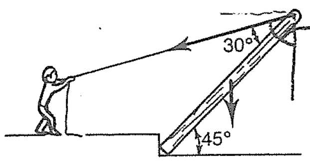


$$R_A = \sqrt{250^2 + 270.6^2}$$

$$= 368.4\text{ kN}$$

$\tan \theta = 250/270.6 \quad \therefore \theta = 42.7°$

5/16
A man raises a pole with a mass of 20 kg by pulling on a rope. Find the tension, T, in the rope and the reaction at A.



75° TENSION ANGL

Assume the pole to be 6m long

Take moments about A

$200 \times \underbrace{3\cos 45^\circ}_{\text{DIST}} + T\cos 75^\circ \times \underbrace{6\cos 45^\circ}_{\text{DIST}}$

$= T\sin 75^\circ \times \underbrace{6\sin 45^\circ}_{\text{DIST}}$

$T = \underline{142\ N}$

Resolve vert.

$200 + T\cos 75 = R_G \sin\theta$

$236{\cdot}6 = R_G \sin\theta \quad \ldots\ldots ①$

Resolve horiz.

$T\sin 75 = R_G \cos\theta$

$136{\cdot}6 = R_G \cos\theta \quad \ldots\ldots ②$

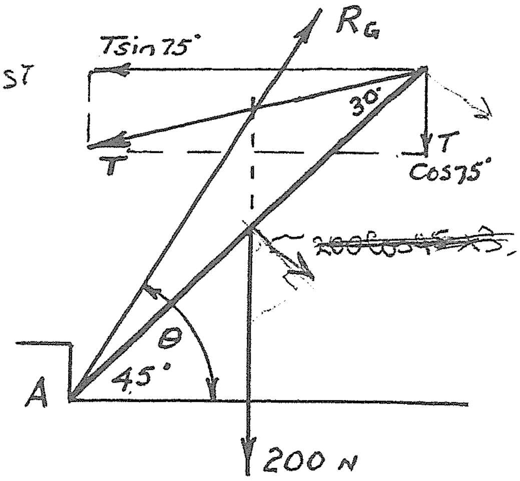
Divide ① by ②

$\tan\theta = 1{\cdot}7321$

$\theta = \underline{60^\circ}$

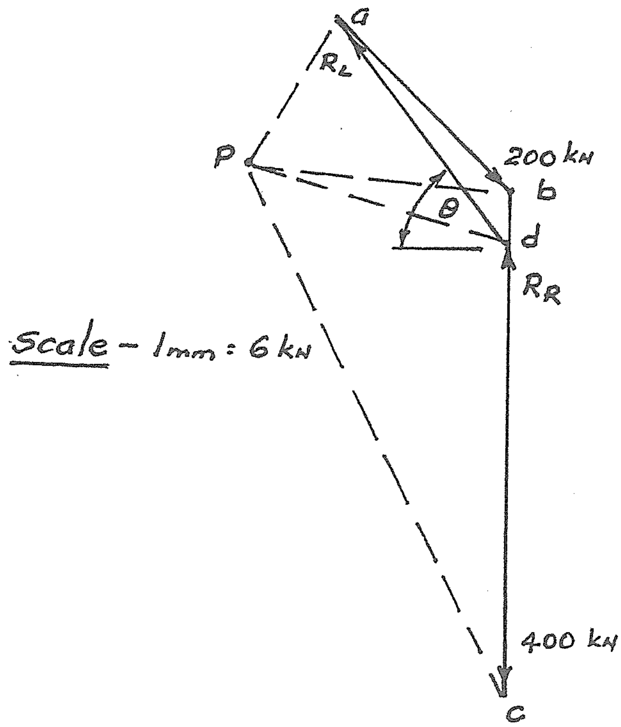
Subst. $136{\cdot}6 = R_G \times 0{\cdot}5$

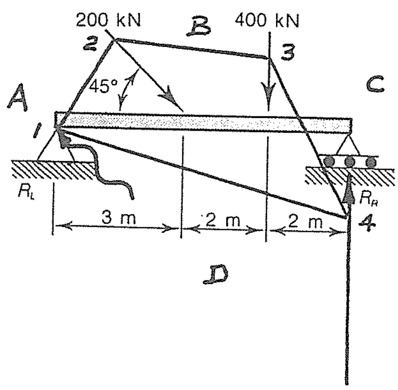
$R_G = \underline{273\ N}$



This is more easily solved graphically
See problem 3/29

5/17
Find the magnitude and direction of the reactive forces at the supports of the beam shown in the diagram.

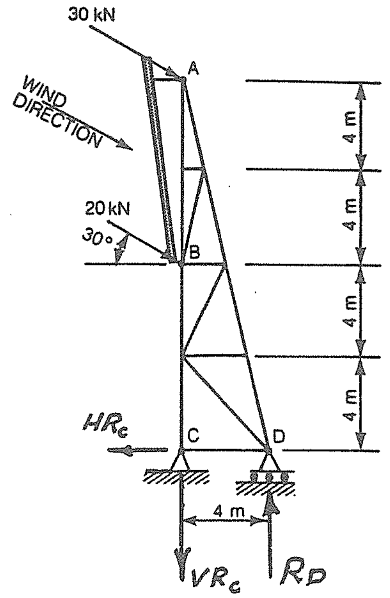




$R_R = \underline{350\ kN} \uparrow$

$R_L = \underline{235\ kN\ 53^\circ}$

5/18
The truss shown supports portion of a drive-in theatre screen. Given that the wind loadings are equal to a 30 kN force on A and a 20 kN force on B as shown, determine the reactions at the supports C and D. Ignore any other forces including the weight-force of the truss itself.



20 kN force : Vert. comp. = 20 sin 30°
Horiz. comp. = 20 cos 30°
30 kN force : Vert. comp = 30 sin 30°
Horiz. comp = 30 cos 30°

Take moments about C

$R_D \times 4 = 20 \cos 30° \times 8 + 30 \cos 30° \times 16$

$R_D = \underline{138.56 \text{ kN} \uparrow}$

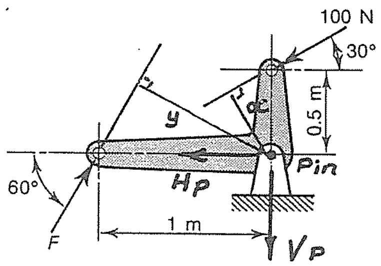
Resolve horiz.

$HR_c = 30 \cos 30° + 20 \cos 30° = 43.3 \text{ kN}$ $\quad \therefore R_c = \underline{121.54 \text{ kN}}$

Resolve vert. $\quad$ and $\theta = \underline{69.1°}$

$VR_c = -30 \sin 30° - 20 \sin 30° + 138.56 \text{ kN} = 113.56 \text{ kN}$

5/19
The bell crank shown is in equilibrium. Determine the reaction at the pivot pin, and the force *F*.



Take moment about the pin.

$F \times y = 100 \times x$

$F \times 1 \cancel{\cos 30°} = 100 \times 0.5 \cancel{\cos 30°}$

$F = \underline{50 \text{ N}}$

Resolve vert.

$F \sin 60° = 100 \sin 30° + V_P$

$V_P = -6.7$ or $6.7 \text{ N} \uparrow$

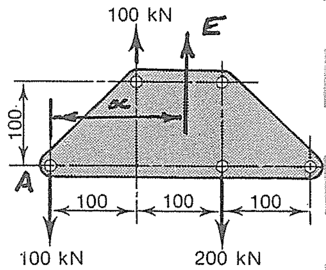
Resolve horiz.

$H_P + 100 \cos 30° = F \cos 60$

$H_P = -61.66$ or $61.66 \text{ N} \rightarrow$

$\therefore$ Resultant Pin Reaction = $\underline{62.02 \text{ N at } 6.2°}$

5/20
Find the single force which could replace the three given forces without altering the overall effect of the loading on the steel plate shown.



Let the equilibrant be E at x mm from A

Resolve vert.

$E + 100 = 100 + 200$

$E = 200 \text{ kN} \uparrow$

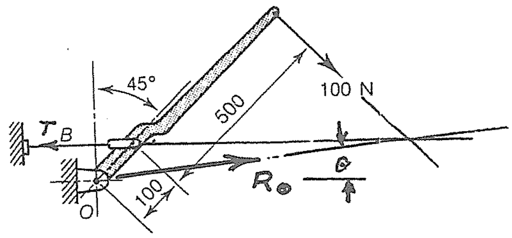
Take moments about A

$100 \times 1\cancel{00} + 2\cancel{00} \times x = 200 \times 2\cancel{00}$

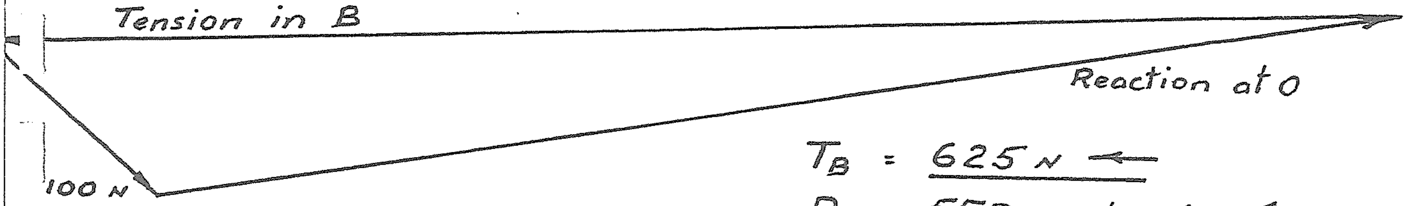
$x = 150 \text{ mm}$

$\therefore$ <u>Resultant is 200 kN down through plate centre</u>

5/21
Graphically determine the tension in the cable B, and the reaction at the pivot O.



Scale – 1mm = 3·33 N



T_B = 625 N ←

R_O = 550 N at 6°

Difficult to obtain graphical accuracy due to small angles.

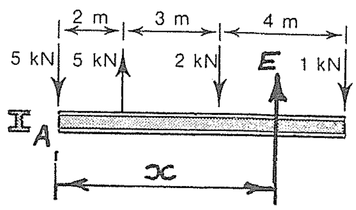
Take moments about O

$T_B \times 100 \cos 45° = 100 \times 600$

T_B : 848·5 N

Similarly, R_O analytically calculates to 780 N at 5°

5/22
The bar shown in the sketch is not in equilibrium, Determine the magnitude and location of the force that must be added to the given system of forces to achieve equilibrium.



Let E be the equilibrant at x m from A

Resolve vert.

$5 + 2 + 1 - 5 - E = 0$

E : 3 kN ↑

Take moments about A

$5 \times 2 + E \times x - 5 \times 2 - 1 \times 9 = 0$

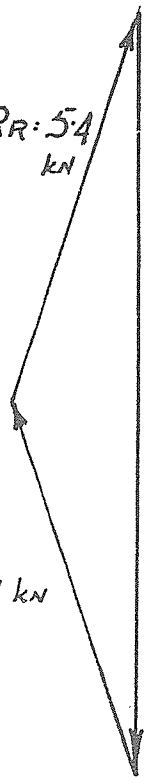
x : 3 m from L.H.end

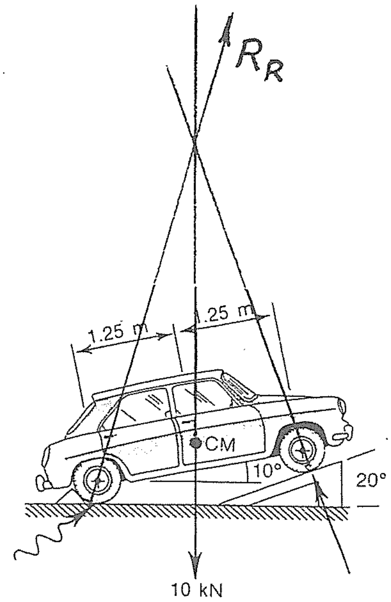
5/23
A car with a mass of 1 tonne and a wheelbase of 2.5 m is supported on ramps as shown in the diagram. The rear wheels have the handbrake applied and are chocked for safety. The front wheels are free to roll. Discover the reaction at each wheel, assuming the chocks take no load.

Scale - 10 mm : 1 kN

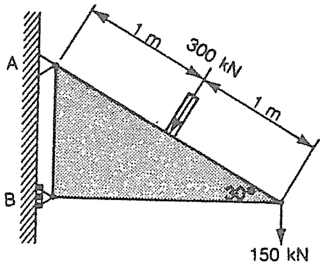
Front wheels 2·6 kN ea.

Rear wheels 2·7 kN ea. at 74°

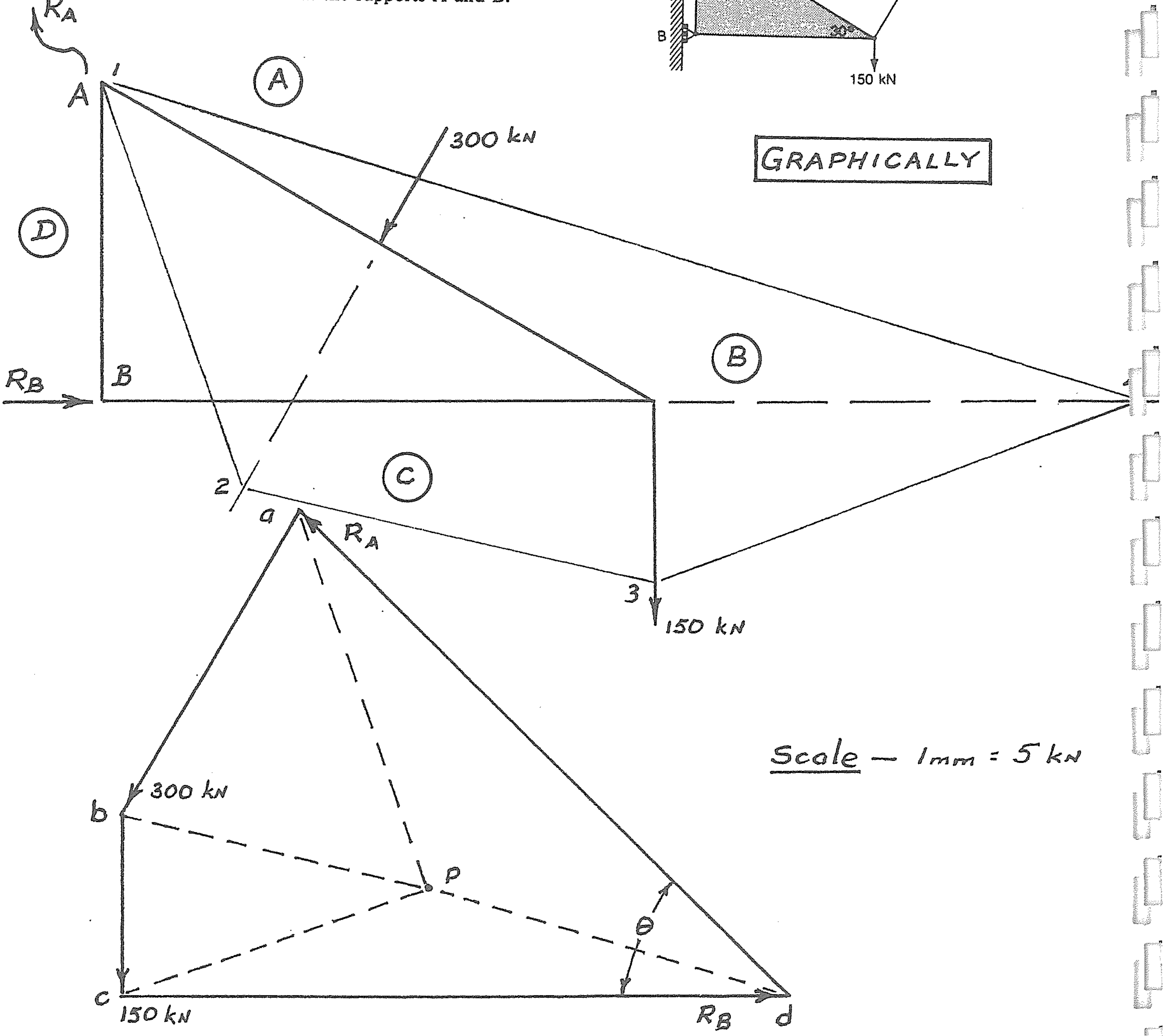




5/24
A triangular support frame in a structure is loaded as shown. Graphically and analytically determine the reactions at the supports A and B.



GRAPHICALLY

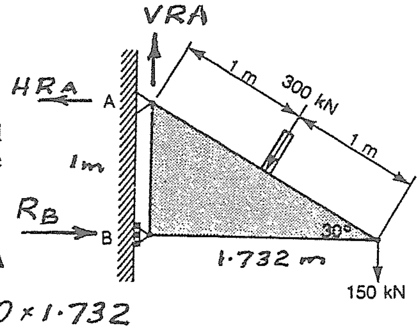


Scale — 1mm = 5 kN

R_B = 560 kN →

R_A = 582 kN at 45° θ

5/24
A triangular support frame in a structure is loaded as shown. Graphically and analytically determine the reactions at the supports A and B.



Take moments about A

$R_B \times 1 = 300 \times 1 + 150 \times 1{\cdot}732$

$R_B = \underline{559{\cdot}8 \text{ kN}} \rightarrow$

ANALYTICALLY

Resolve forces horiz.

$HR_A + 300 \sin 30 = 559{\cdot}8$

$HR_A = 409{\cdot}8 \text{ kN}$

Resolve forces vert.

$VR_A = 150 + 300 \cos 30^\circ$

$= 409{\cdot}8 \text{ kN}$

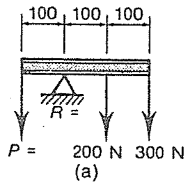
$R_A = \sqrt{409{\cdot}8^2 + 409{\cdot}8^2}$

$= \underline{579{\cdot}5 \text{ kN}}$

$\tan \theta = 409{\cdot}8 / 409{\cdot}8$

$\theta = \underline{45^\circ}$

5/25
Each of the beams shown below is in equilibrium. However, in each case not all the information is shown. Complete the diagrams by adding the missing information.



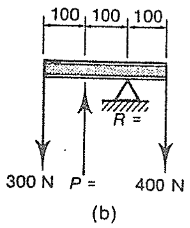
(a) Take moments about R

$P \times 100 = 200 \times 100 + 300 \times 200$

$P = \underline{800 \text{ N}}$

& $R = \underline{1300 \text{ N}} \uparrow$

(b) Take moments about R

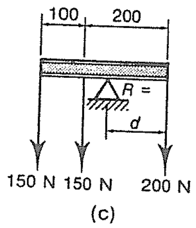


$400 \times 100 + P \times 100 = 300 \times 200$

$P = \underline{200 \text{ N}}$

& $R = \underline{500 \text{ N}} \uparrow$

(c) Take moments about R



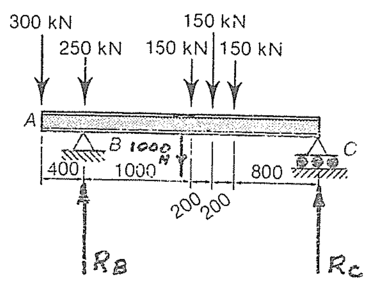
$200 \times d = 150(200 - d) + 150(300 - d)$

$200d + 150d + 150d = 75000$

$d = \underline{150 \text{ mm}}$

& $R = \underline{500 \text{ N}} \uparrow$

5/26
Determine the reactions at B and C for the beam and loading shown. The beam has a mass of 100 kg.



Take moments about B

$300 \times 400 + R_c \times 2200 = 1 \times 900 + 150 \times 1000 + 150 \times 1200 + 150 \times 1400$

$22 R_c = 4209$

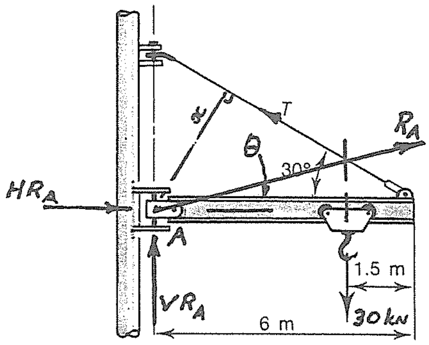
$R_c = \underline{191{\cdot}3 \text{ kN}} \uparrow$

Resolve vert.

$R_B = 300 + 250 + 1 + 450 - 191{\cdot}3$

$= \underline{809{\cdot}7 \text{ kN}} \uparrow$

5/27
A light crane supports a 3-tonne load as shown in the diagram. Determine the tension, T, in the cable and the reaction at the pivot A.



Take moments about A

$30 \times 4.5 = T \times x$

$T = 135/6\sin 30$

$= \underline{45\ kN}$

Resolve horiz.

$HR_A = T\cos 30^\circ$

$= 38.97\ kN$

Resolve vert.

$VR_A + T\sin 30^\circ = 30$

$VR_A = 7.5\ kN$

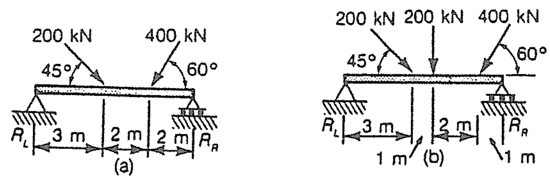
$R = \sqrt{38.97^2 + 7.5^2}$

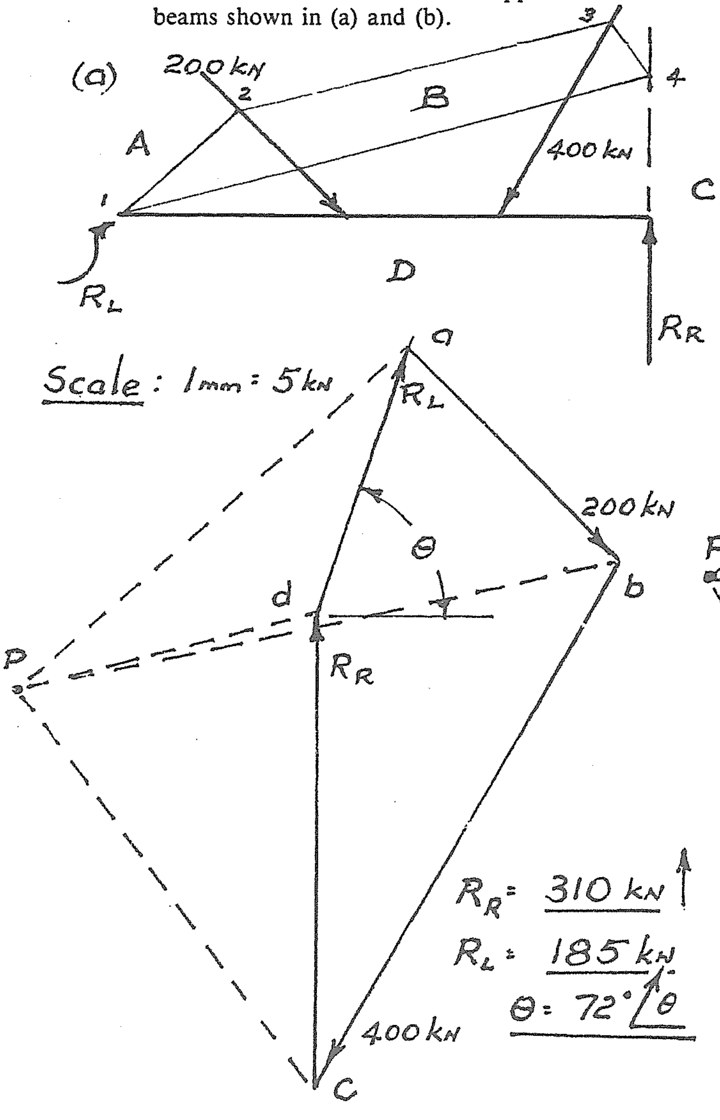
$= \underline{39.68\ kN}$

$\tan\theta = 7.5/38.97 = 0.1924$

$\theta = \underline{10.9^\circ}$

5/28
Graphically determine the magnitude and direction of the forces at the supports of the beams shown in (a) and (b).

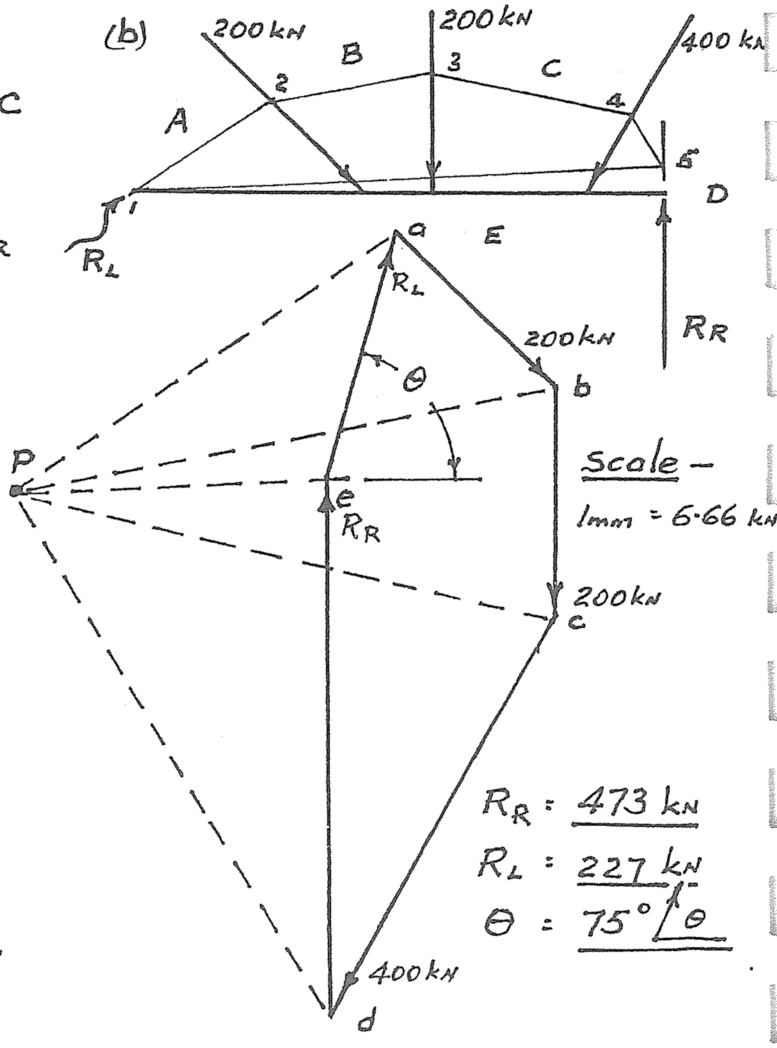




$R_R = \underline{310\ kN}$ ↑

$R_L = \underline{185\ kN}$

$\theta = \underline{72^\circ}$

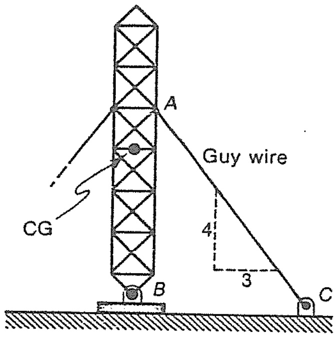


$R_R = \underline{473\ kN}$

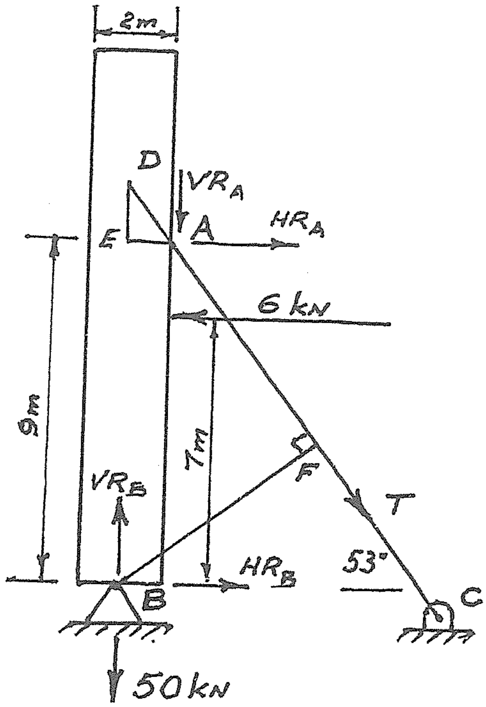
$R_L = \underline{227\ kN}$

$\theta = \underline{75^\circ}$

5/29
The radio tower shown in the diagram is 2 m square and 14 m high and it has a mass of 5 tonnes. It is supported against horizontal wind loads by four guy wires attached 9 m above the central base B. Its effective projected area of 10 m^2 is subjected to horizontal wind pressure of 600 N/m^2. When the wind blows from right to left, only guy wire AC is active. Determine the vertical and horizontal reaction components at A and B and the tension T in AC.



ANALYTICALLY



$EA = 1$

$ED = 1 \times \tan 53° = 1{\cdot}327$

$AD = 1/\cos 53° = 1{\cdot}662$

$BD = 9 + 1{\cdot}327 = 10{\cdot}327$

$BC = BD/\tan 53° = 7{\cdot}782$

$BF = BC \sin 53° = 6{\cdot}215$

Take moments about B

$T \times 6{\cdot}215 = 6 \times 7$

$T = \underline{6{\cdot}76 \text{ kN}}$ 53°

$HR_A = T\cos 53°$
$= 4{\cdot}068 \text{ kN} \rightarrow$

$VR_A = T\sin 53°$
$= 5{\cdot}398 \text{ kN} \downarrow$

Resolve horiz.

$HR_B + 4{\cdot}068 = 6$

$HR_B = 1{\cdot}93 \text{ kN} \rightarrow$

Resolve vert.

$VR_B = 50 + VR_A$

$= 55{\cdot}398 \text{ kN} \uparrow$

Reactions at A:

horiz. = $\underline{4{\cdot}068 \text{ kN}} \rightarrow$

vert. = $\underline{5{\cdot}398 \text{ kN}} \downarrow$

Reactions at B:

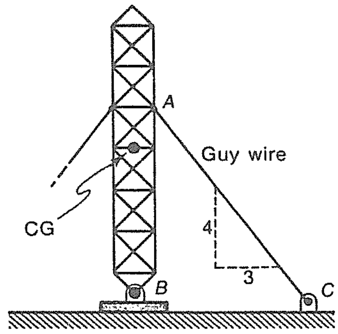
horiz = $\underline{1{\cdot}93 \text{ kN}} \rightarrow$

Vert. = $\underline{55{\cdot}398 \text{ kN}} \uparrow$

Tension in AC

$= \underline{6{\cdot}76 \text{ kN}}$ 53°

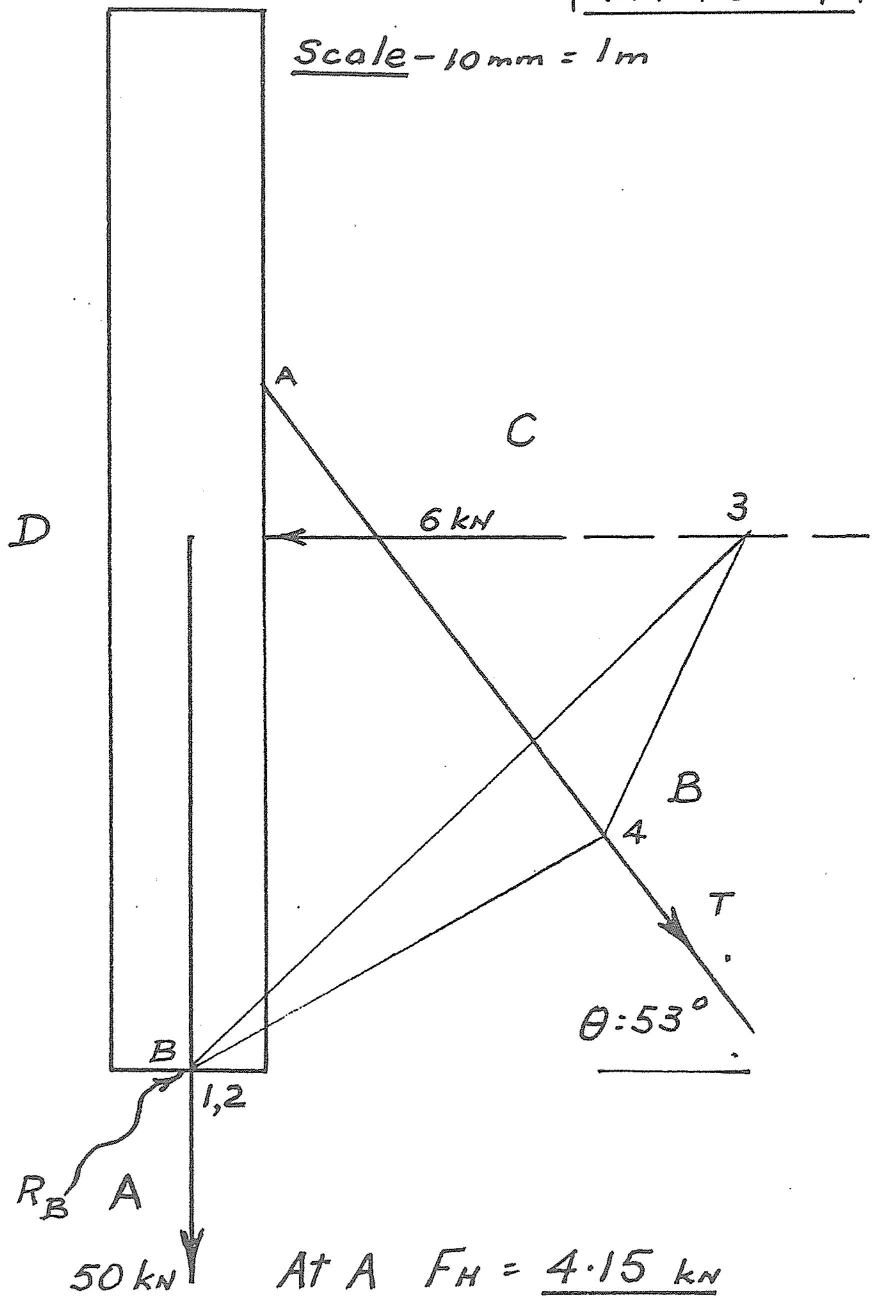
5/29
The radio tower shown in the diagram is 2 m square and 14 m high and it has a mass of 5 tonnes. It is supported against horizontal wind loads by four guy wires attached 9 m above the central base B. Its effective projected area of 10 m^2 is subjected to horizontal wind pressure of 600 N/m^2. When the wind blows from right to left, only guy wire AC is active. Determine the vertical and horizontal reaction components at A and B and the tension T in AC.

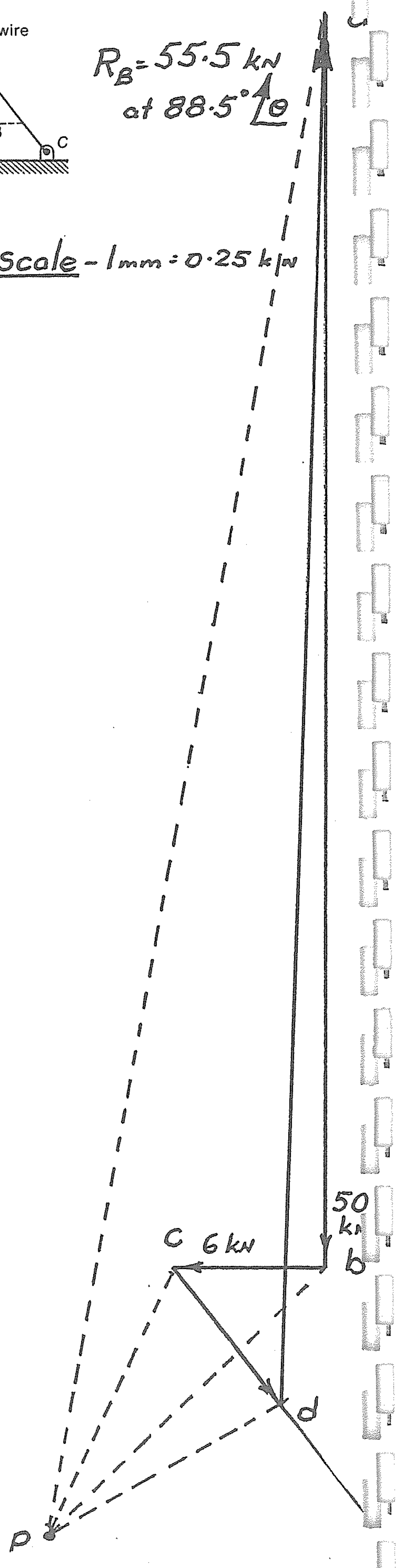


GRAPHICALLY

Scale – 10 mm = 1 m

Scale – 1 mm : 0.25 kN





At A F_H = 4.15 kN

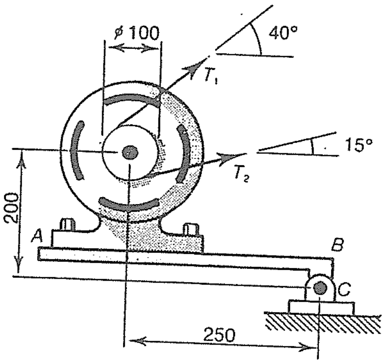
F_V = 5.51 kN

At B F_H = 1.45 kN

F_V = 55.4 kN

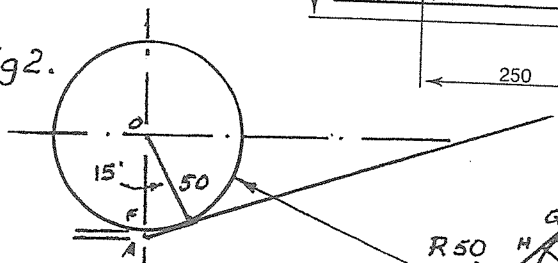
T = 6.9 kN

5/30
In the pivot drive, the weight of the motor is used to maintain tension in the belt. When the motor is at rest, the tensions T_1 and T_2 may be assumed equal. The mass of the motor is 75 kg and the diameter of the drive pulley is 100 mm. Assuming that the weight of the platform AB is negligible, determine the tension in the belt and the reaction at the pivot C when the motor is at rest.



ANALYTICALLY

Fig2.



<u>To find x</u>

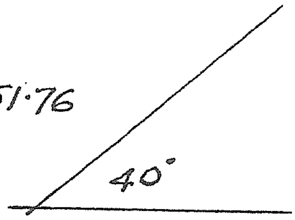
$EB = 250 \tan 15° = 66 \cdot 98$

$BD = OA$ (see fig 2)

$OA = 50/\cos 15° = 51 \cdot 76$

$BC = 200 - 51 \cdot 76$

$= 148 \cdot 24$



$CE = BC + EB = 215 \cdot 22$

In Δ EJC $x = 215 \cdot 22 \cos 15° = 207 \cdot 88$

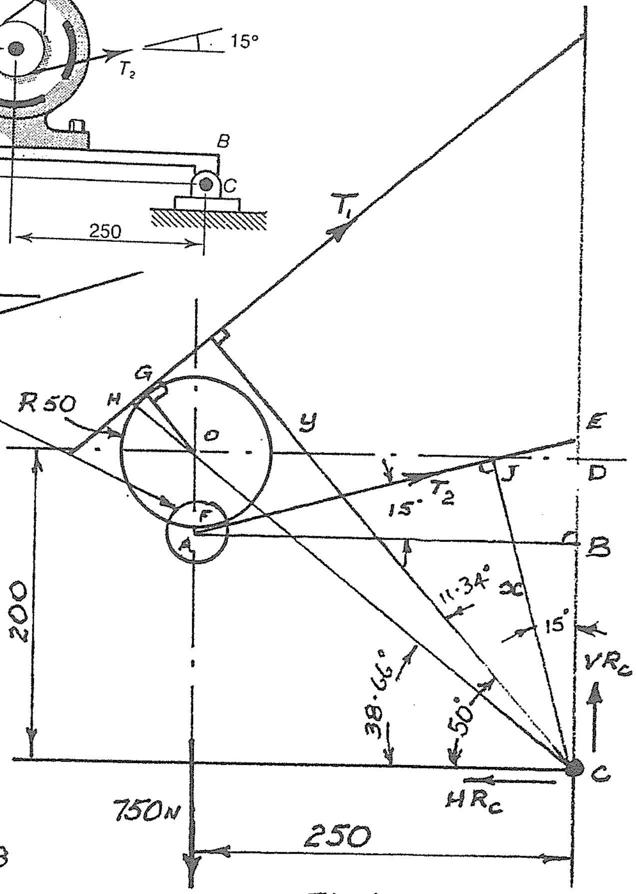


Fig 1.

<u>To find y</u>

In Δ OGH, $OG = 50$, $\widehat{GOH} = 11 \cdot 34°$ $\therefore$ $OH = 50 \cdot 99$

$OC = \sqrt{200^2 + 250^2} = 320 \cdot 16$

$\therefore CH = 371 \cdot 15$ and $y = CH \cos 11 \cdot 34° = 363 \cdot 9$

Take moments about C

$T_1 \times 363 \cdot 9 + T_2 \times 207 \cdot 88 = 750 \times 250$ but $T_1 = T_2$

$\therefore$ $T = \underline{327 \cdot 9 \text{ N}}$

Resolve horiz.

$HR_c = T \cos 15° + T \cos 40°$

$= 567 \cdot 9 \text{ N}$

Resolve vert.

$VR_c = 750 - T \sin 15° - T \sin 40°$

$= 454 \cdot 37 \text{ N}$

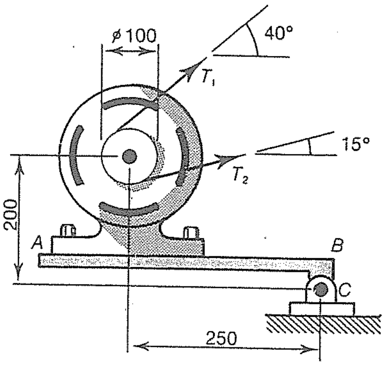
$R_c = \sqrt{567 \cdot 9^2 + 454 \cdot 37^2}$

$= \underline{727 \cdot 3 \text{ N}}$

$\tan \theta = 454 \cdot 37 / 567 \cdot 9$

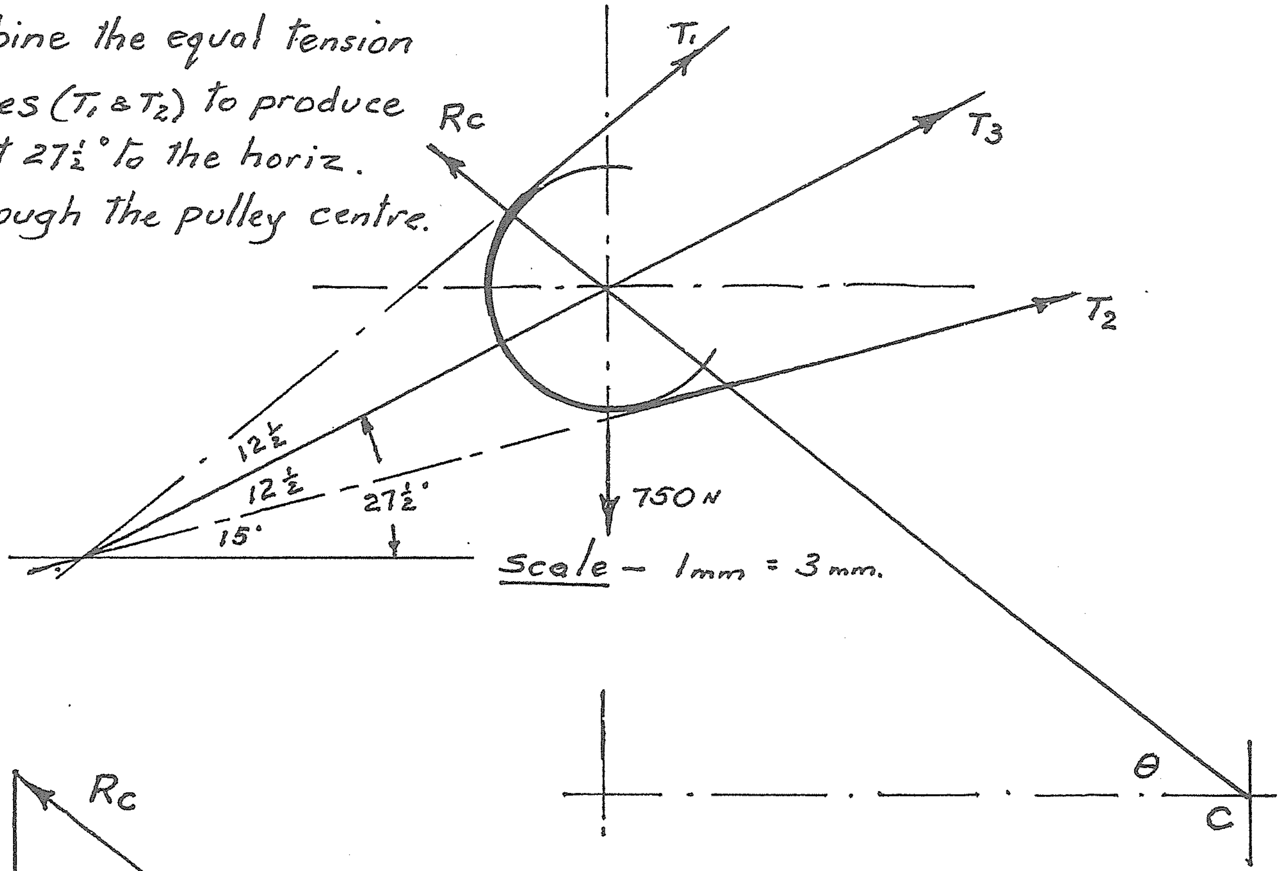
$\therefore \theta = \underline{38 \cdot 66°}$ θ

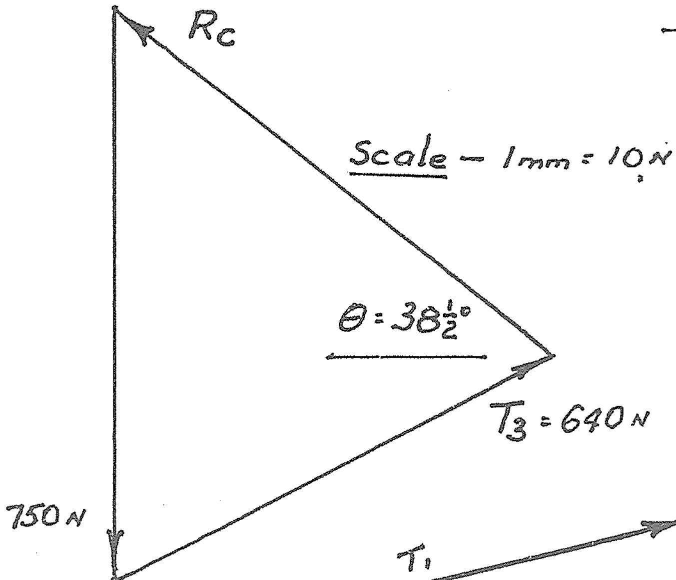
5/30
In the pivot drive, the weight of the motor is used to maintain tension in the belt. When the motor is at rest, the tensions T_1 and T_2 may be assumed equal. The mass of the motor is 75 kg and the diameter of the drive pulley is 100 mm. Assuming that the weight of the platform *AB* is negligible, determine the tension in the belt and the reaction at the pivot *C* when the motor is at rest.



GRAPHICALLY

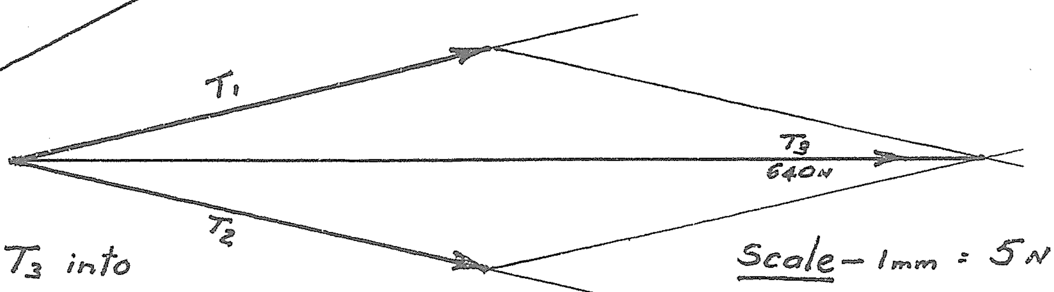
Combine the equal tension forces (T_1 & T_2) to produce T_3 at $27\frac{1}{2}°$ to the horiz. through the pulley centre.





R_c = 730 N at $38\frac{1}{2}°$ θ

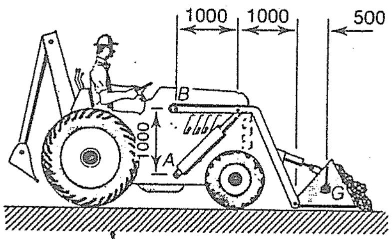
$T_1 = T_2$ = 325 N



Scale – 1mm : 5 N

Resolve T_3 into components at $12\frac{1}{2}°$ either side
These will be the valves of T_1 & T_2

5/31
Determine the force F on the pistons of the two hydraulic cylinders A necessary to start raising the load of 1500 kg. Neglect the weights of the bucket and arms compared with the load and take the centre of gravity of the load to be at G. Determine the shear forces in the pivot pins at B.



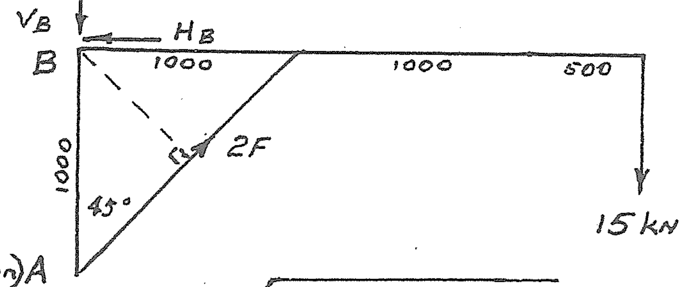
Let the force on each piston = F

Take moments about B

$2F \times 707.1 = 15 \times 2500$

$2F = 53.03$

or $F = \underline{26.5\ \text{kN}}$ (each piston)



Resolve horiz.

$H_B = 53 \cos 45°$

$= 37.48$ kN

Resolve vert.

$V_B + 15 = 53.03 \cos 45°$

$V_B = 22.48$ kN

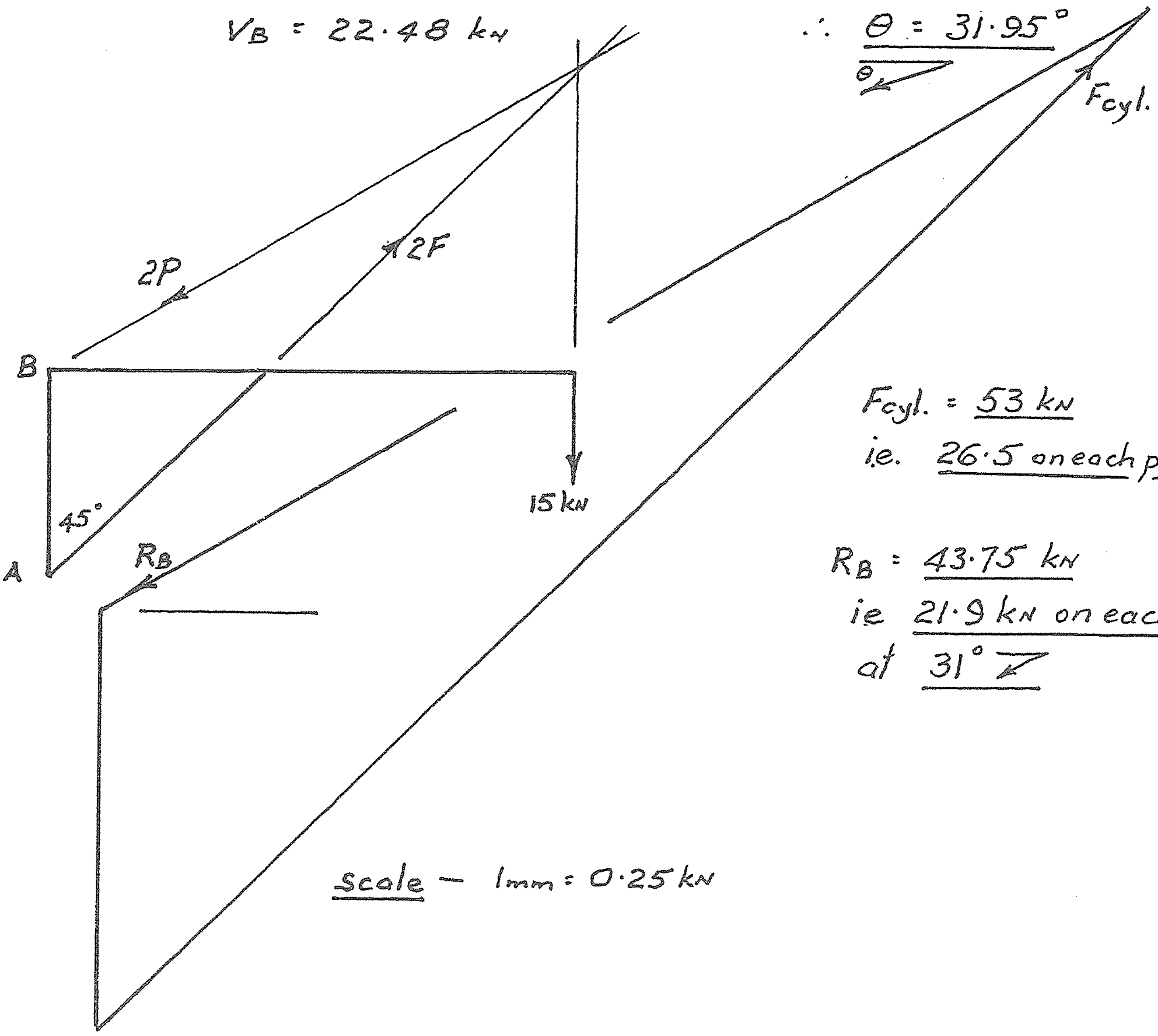
$R_B = \sqrt{37.48^2 + 22.48^2}$

$= 43.7$ kN

or $\underline{21.9\ \text{kN each pin}}$

$\tan\theta = 22.48/37.48$

$\therefore \underline{\theta = 31.95°}$



Fcyl. = $\underline{53\ \text{kN}}$

ie. $\underline{26.5 \text{ on each piston}}$

$R_B = \underline{43.75\ \text{kN}}$

ie $\underline{21.9\ \text{kN on each pin}}$

at $\underline{31°}$

<u>Scale</u> – 1mm = 0.25 kN

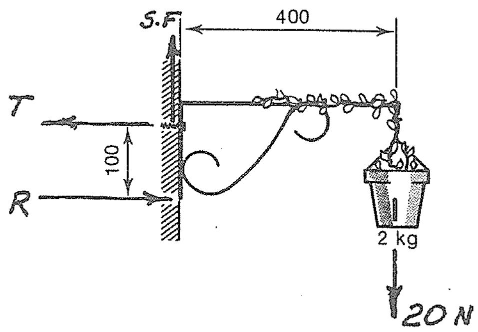
6 Couples

A couple can be balanced only by another couple

$g = 10\,m/s^2$

6 COUPLES
Couples. Resolving a Force into a Force and a Couple. Resolving a Force-couple into a Single Force.

6/6
Determine the forces in the mounting screw for the pot-plant bracket shown in the diagram.



Resolve vert.

Screw shear force $\underline{S.F = 20\,N}$

Resolve horiz.

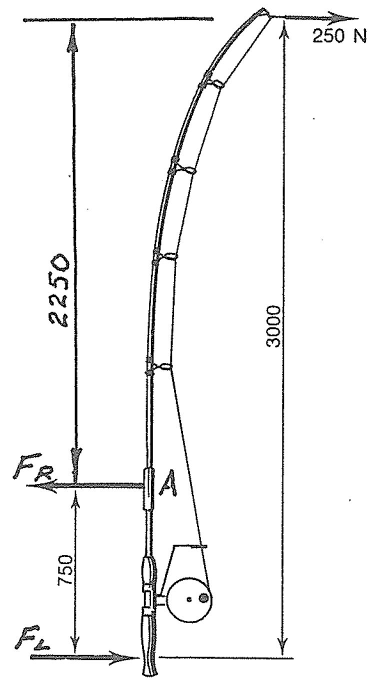
Tension T = Reaction R.

T & R form an anticlockwise couple equal to $(T \times 100)$ balanced by the clockwise moment (20×400)

$$T \times 100 = 20 \times 400$$

$$\underline{T = 80\,N} \text{ (tension)}$$

6/7
Fred's "gone fishin'" and hooked a whopper on his 3 m rod. Fred's strength and imagination estimate the fish to be pulling on the line with a 250 N force. Fred's left hand is placed on the bottom of the rod and right hand 750 mm from the bottom of the rod. What is the force in his left arm while playing the fish? What is the force in his right arm?



Take moments about A

$$F_L \times 750 = 250 \times 2250$$

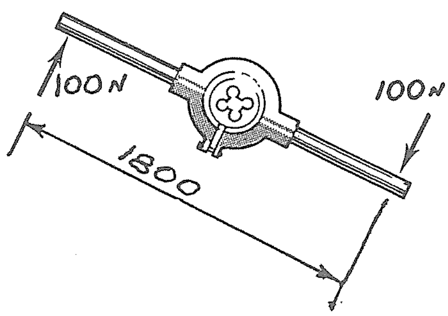
$$F_L = \underline{750\,N}$$

Resolve horiz.

$$F_R = F_L + 250$$

$$= \underline{1000\,N}$$

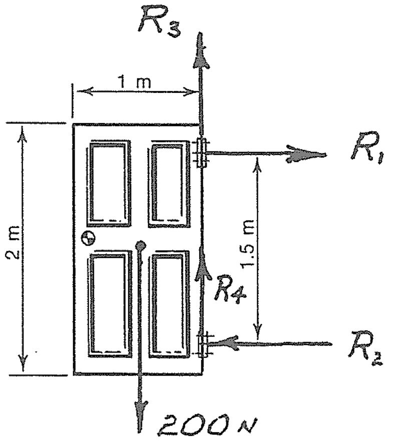
6/8
A screw thread is cut with a hand stock and die. What is the torque or moment being applied if each hand exerts a force of 100 N and the effective length of each handle to the axis of the screw is 900 mm?



Torque $= 100 \times 1800$ Nmm

$= \underline{180\ Nm.}$

6/9
The cottage door has a mass of approximately 20 kg. Determine the loads on the hinges.



Clockwise reaction couple ($R_1 + R_2$) = Anti-clockwise couple ($R_3 + R_4$ and 200 N)

Resolve horiz. $R_1 = R_2$

Resolve vert. $R_3 + R_4 = 200$

Take moments to find the values of the couples

Clockwise $= R_1 \times 1.5$ Nm

Anti-clockwise $= 200 \times 0.5$ Nm

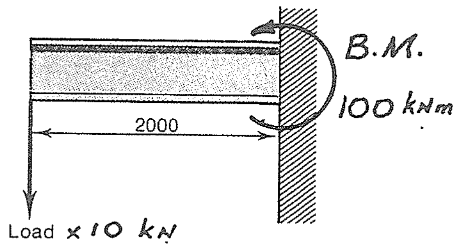
$\therefore \quad R_1 = \dfrac{200 \times 0.5}{1.5} = \underline{66.67 \text{ N}}$

Also $(R_3 + R_4) \times 0.5 = 66.67 \times 1.5$

$R_3 + R_4 = 200$

$\therefore$ Max. value for either R_3 or R_4 = $\underline{200 \text{ N}}$

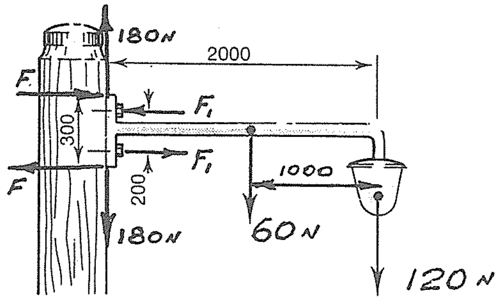
6/10
A steel I-beam is cantilevered as shown. If the couple at the support (the bending moment) is not to exceed 100 kN m, calculate the maximum load which can be supported at the end of the beam. (Neglect the mass of the beam).



$10L \times 2\text{m} = 100 \text{ kNm}$

$L = \underline{5 \text{ tonnes}}$

6/11
The street lamp has a mass of 12 kg and the bracket which supports it has a mass of 6 kg.
(a) Describe the force-couple at the pole.
(b) Determine the minimum tension in the top mounting bolt of the bracket.



Replace mass forces with force/couple

(a) $F \times 300 = 60 \times 1000 + 120 \times 2000$

$F = 1000 \text{ N}$

$\therefore$ Couple $= 1000 \times 0.3$

$= 300 \text{ Nm}$

and force/couple is 300 Nm

180 N

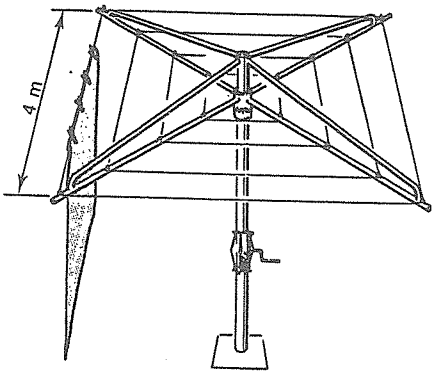
(b) Couple at pole balances Couple produced by bolts

$F \times 300 = F_1 \times 200$

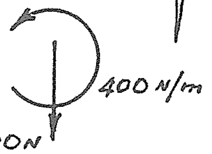
$1000 \times 300 = F_1 \times 200$

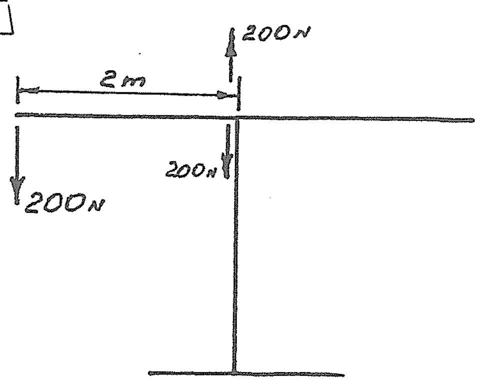
$F_1 = \underline{1500 \text{ N (T)}}$

6/12
A wet blanket with a mass of approximately 20 kg is hanging on the outermost line of the rotary clothes line as shown.
(a) Resolve the weight-force produced by this blanket into a force-couple system at the base of the column.
(b) What is the bending moment (i) at ground level? (ii) at peg level?

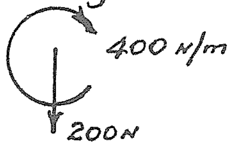


(a) At peg level:
Force/couple is





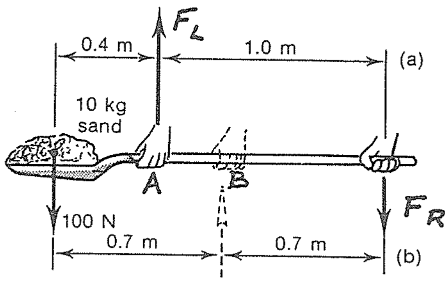
balanced at the base by:
Force/couple



(b) (i) B.M. at ground level : $\underline{400 \text{ Nm}}$
B.M. at peg level : $\underline{400 \text{ Nm}}$

NOTE : This is more appropriately 3 unit.

6/13
The long-handled shovel is used to lift 10 kg of sand.
(a) Determine the force in each hand for the position shown. What is the moment of the couple exerted by the hands?
(b) If the left hand is moved to a position halfway along the shovel (distance 0.7 m) what is the force in each hand? What is the couple?
(c) Describe the position of the hands for easier shovelling.



(a) Take moments about A

$F_R \times 1 = 100 \times 0{\cdot}4$

$F_R = \underline{40 \text{ N}}$

Resolve vert.

$F_L = \underline{140 \text{ N}}$

L.H. is lifting 100N and turning with 40N ∴ Hand couple is 40×1 or $\underline{40 \text{ Nm}}$

(b) Take moments about B

$F_R \times 0{\cdot}7 = 100 \times 0{\cdot}7$

$F_R = \underline{100 \text{ N}}$

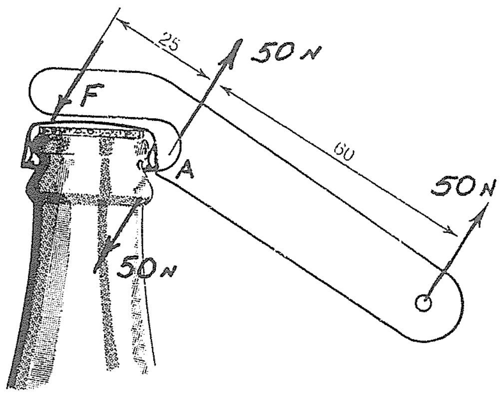
Resolve vert.

$F_L = \underline{200 \text{ N}}$

L.H. is lifting 100N an turning with 100 N ∴ Hand couple is $100 \times 0{\cdot}7$ or. $\underline{70 \text{ Nm}}$.

(c) The easiest hand position is where the couple effect is least. i.e. Where hands are furthest apart.

6/14
A 50-N pull is required to operate a certain bottle-opener. Describe the force-couple system acting on the bottle top.



Take moments about A

$F \times 25 = 50 \times 60$

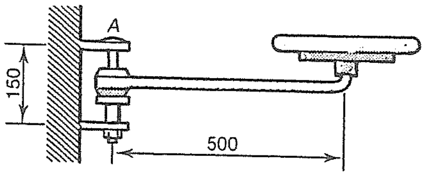
$F = 120\ N$

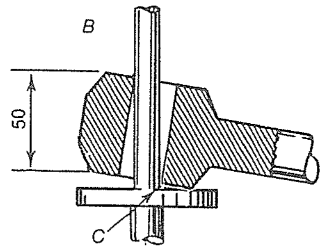
The force/couple at A is

$\underline{50\ N}$ ↗ and

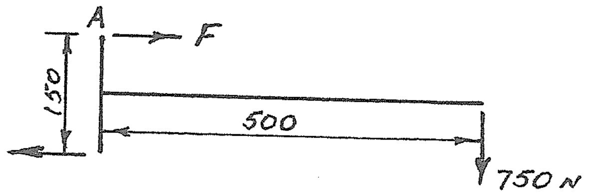
50×60 or $\underline{3\ Nm}$

6/15
A workbench seat is held in the position shown by a vertical bar and supports a 75-kg man.
(a) Determine the reaction at A.
(b) If the inside diameter of the collar is slightly larger than the bar, the collar will bear only at points B and C. Determine the magnitude of the horizontal and vertical forces developed at B and C.





(a)

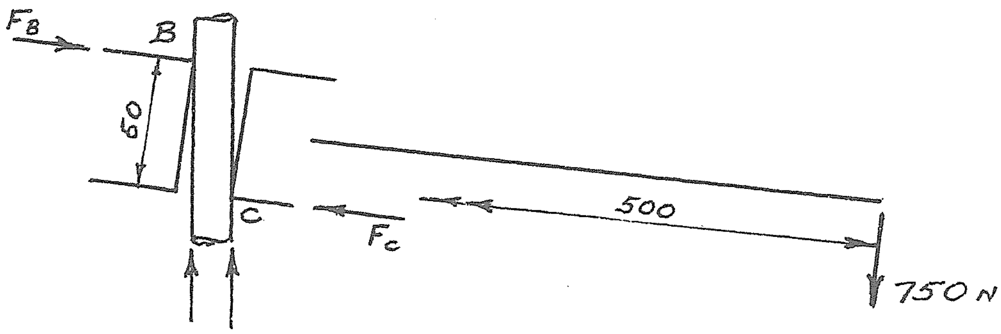


$F \times 150 = 750 \times 500$

$F = 2.5\ kN$

∴ Reaction at A $= \underline{2.5\ kN \leftarrow}$

(b)



$F \times 50 = 750 \times 500$

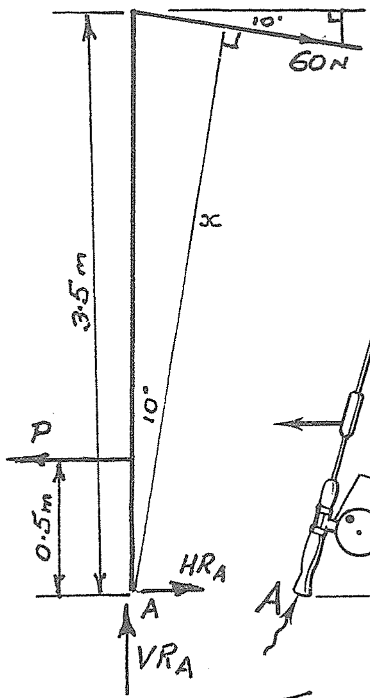
$F = \underline{7.5\ kN}$

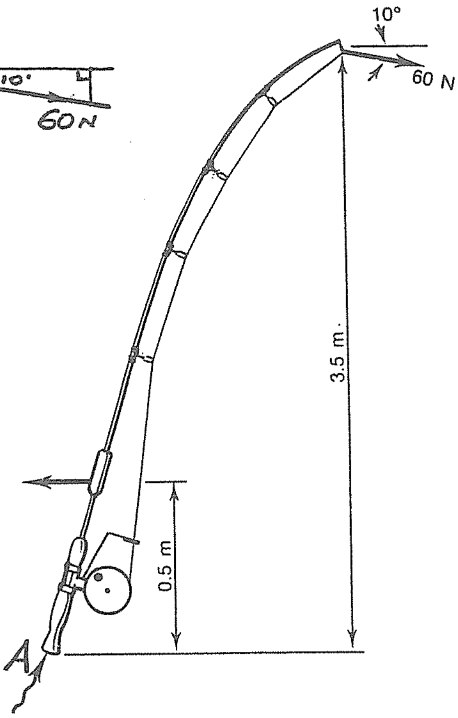
∴ Horiz. reactive force developed at B : $\underline{7.5\ kN}$ ←

" " " " " C : $\underline{7.5\ kN}$ →

Vert. reactive forces developed at B&C total $\underline{750\ N}$ ↑

6/16
A fish pulls on the line with a force of 60 N. Describe the forces the fisherman will exert while playing the fish.





Take moments about A

$P \times 0.5 = 60 \times 3.5 \cos 10^\circ$

$P = \underline{413.6\ N} \leftarrow$

At A

Resolve horiz.

$P = HR_A + 60 \sin 10^\circ$

$HR_A = 354.53\ N$

Resolve vert.

$VR_A = 60 \sin 10^\circ$

$= \underline{10.42\ N} \uparrow$

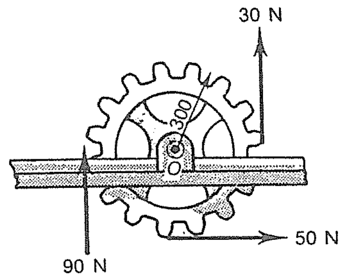
$R_A = \sqrt{354.53^2 + 10.42^2}$

$= \underline{354.7\ N}$

$\tan\theta = 10.42/354.53$

$\theta = \underline{1.68^\circ}$

6/17
Replace the three forces acting on the gear by an equivalent force-couple system at O.



Nett vert. force = 120 N

Nett horiz force = 50 N

Resultant = $\sqrt{120^2 + 50^2}$ = $\underline{130\ N}$

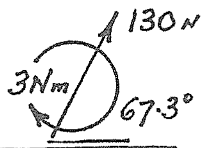
$\tan\theta = 120/50$

$\theta = \underline{67.3^\circ}$

Nett turning effect about O

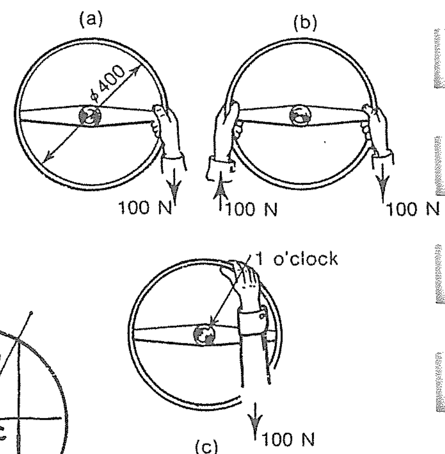
$90 \times 0.3 - 30 \times 0.3 - 5.0 \times 0.3 =$

$= \underline{3\ Nm}$



Force/couple system

6/18
Determine the torque in the steering column of the motor car for the conditions shown. Find the other force completing the couple for (a) and (c).



(a) Torque = 100×200

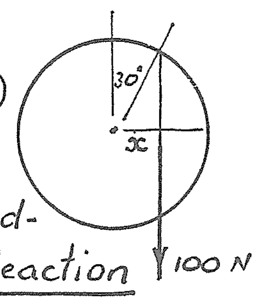
$= \underline{20\ Nm}$

(b) Torque = 100×400

$= \underline{40\ Nm}$

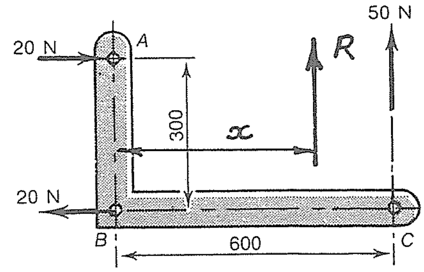
(c) Torque = $100 \times x$ ($x = 100 \sin 30^\circ$)

$= \underline{10\ Nm}$



For (a) & (c) the completing force producing a couple is Steering Column Reaction

6/19
Replace the three forces shown in the sketch by a single force acting at a point along BC. The replacing force must have the same overall effect on the member as the combined effect of the three forces shown. Find the location of the replacing force and its magnitude in its new position.

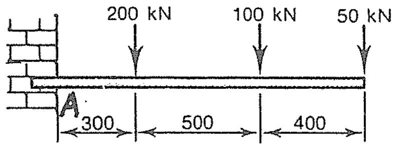


The existing system has a nett turning effect anti-clockwise of $(50 \times 600 - 20 \times 300)$ or 24000 Nmm.
The required force to balance the system (equilibrant) is 50 N ↓ so the replacement force (resultant) must be <u>50 N ↑</u> positioned to provide an anti-clockwise moment of 24000 Nmm

$$50 \times x = 24000$$

$$x = \underline{480 \text{ mm to the right of B}}$$

6/20
A cantilever beam is loaded as shown. The beam is fixed at the left end and free at the right end. Determine the reactions at the fixed end.

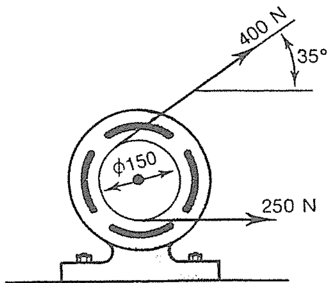


Resolve vert. — 350 kN ↓
Take moments about A

$$200 \times 300 + 100 \times 800 + 50 \times 1200 = 200 \text{ kN m.}$$

∴ Reactions for equilibrium are:-
<u>350 kN ↑ and an anti-clockwise B.M. of 200 kNm.</u>

6/21
The pulley attached to the electric motor revolves with constant velocity. The belt tensions are found to be 400 N and 250 N on the taut and slack sides of the pulley. Determine
(a) the shear force on the pulley shaft;
(b) the resultant moment produced by the motor armature.



(a) Pulley shear force is the resultant of the two forces.

Resolve vert. $R_V = 400 \sin 35°$
$= 229 \cdot 4$ N ↑

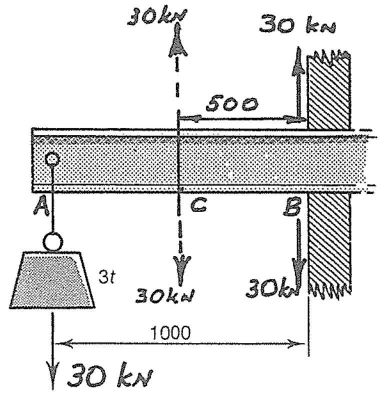
Resolve horiz $R_H = 250 + 400 \cos 35°$
$= 577 \cdot 7$ N →

Resultant $= \sqrt{229 \cdot 4^2 + 577 \cdot 7^2}$
$= \underline{621 \cdot 5 \text{ N}}$

$\tan \theta = 229 \cdot 4 / 577 \cdot 7$
$\theta = \underline{29 \cdot 65°}$ $\angle \theta$

(b) Resultant moment $= 400 \times 75 - 250 \times 75$
$= \underline{11 \cdot 25 \text{ Nm}}$

6/22
A steel I-beam is supported and loaded as shown. Determine the load on the support, and calculate the magnitude of the couple in the beam
(a) at the support;
(b) half-way between load and support.
Neglect the mass of the beam.



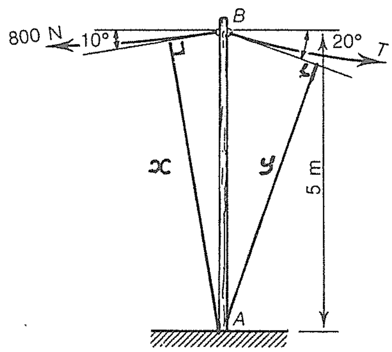
Resolving the 30 kN load force into a force/couple system at B:

The load at B will be 30 kN

(a) The couple in the beam at B will equal 30 kN m

(b) The couple in the beam at C will equal 15 kN m

6/23
A telephone pole is used to support the ends of two wires. The tension in the wire to the left is 800 N and, at the point of support, the wire forms an angle of 10° with the horizontal. Determine the largest and smallest allowable tension T if the magnitude of the couple at A may not exceed 600 N m.



For maximum Tension, the B.M. set up at A will be negative (clockwise)

$800 \times x - T \times y = -600$

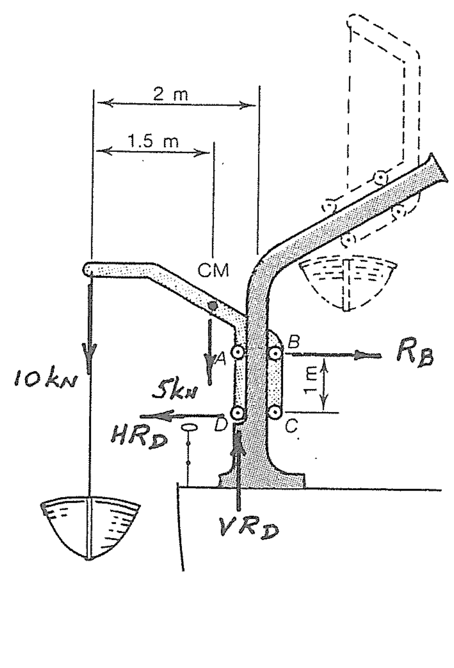
$800 \times 5\cos 10^\circ - T \times 5\cos 20^\circ = -600$

$T = 966\ N$

For minimum Tension the B.M. at A will be positive

$800 \times x - T \times y = 600 \quad \therefore T = 710\ N$

6/24
One of a pair of gravity davits is shown in the diagram. Determine the loads on its rollers A, B, C, and D, when the lifeboat is slung outboard as shown. The lifeboat has a mass of 1000 kg and each davit has a mass of 500 kg with its centre of mass (CM) as shown.



The rollers at A and C would clear the frames and so carry NO LOAD

Take moments about D

$R_B \times 1 = 5 \times 0.5 + 5 \times 0.5 + 10 \times 2$

$R_B = 25\ kN$ or $12.5\ kN$ each

Resolve vert.

$VR_D = 20\ kN \quad (10 + 2 \times 5)$

Resolve horiz

$HR_D = 25\ kN$

$R_D = \sqrt{25^2 + 20^2}$

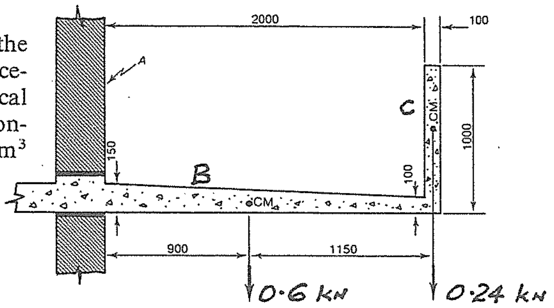
$= 32\ kN$

or 16 kN each

$\tan\theta = 20/25$

$\theta = 38.6^\circ$

6/25
The concrete deck is cantilevered beyond the supporting wall as shown. Determine the force-couple system at the wall face A for a typical module (100 mm) of the slab. (Reinforced concrete has a mass of approximately 2400 kg/m^3 giving it a weight density of 24 kN/m^3.)



Area of Section B $= 125 \times 2000 \text{ mm}^2$

Volume of Section B $= 125 \times 2000 \times 100 \text{ mm}^3$

$\therefore$ Wt. force $= \dfrac{125 \times 2000 \times 100 \times 24}{10^9}$

$= 0{\cdot}6$ kN

Area of Section C $= 100 \times 1000 \text{ mm}^2$

Volume of Section C $= 100 \times 1000 \times 100 \text{ mm}^3$

$\therefore$ Wt. force $= \dfrac{100 \times 1000 \times 100 \times 24}{10^9}$

$= 0{\cdot}24$ kN

$\therefore$ Total wt. force of module $= 0{\cdot}84$ kN $\downarrow$

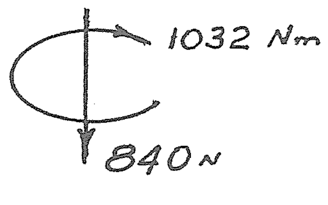
Take moments about face A

$0{\cdot}6 \times 900 + 0{\cdot}24 \times 2050$

$= 1032$ kN mm or 1032 Nm

$\therefore$ Force/couple system at wall face A

is



6/26

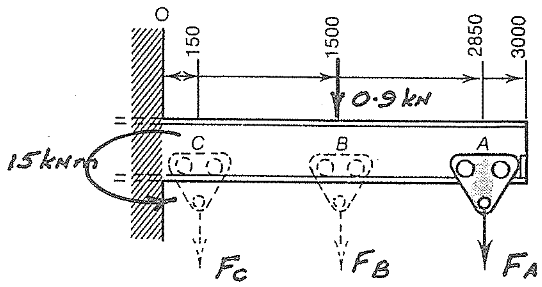
The diagram shows a monorail hoist cantilevered through the wall of a factory, and used to lift light loads with a chain-block. The rolled steel section chosen for the monorail is a 200UB30 (that is, a universal beam, with a nominal depth of 200 mm and a mass of 30 kg per metre). To avoid buckling, this beam should not be subjected to a bending moment greater than 15 kN m. Loads greater than 3 tonnes *will* cause local bending of the bottom flange of the beam, at the trolley wheels.

(a) Suggest suitable safe working load limits for the positions *A*, *B* and *C*.

(b) Will the 50 kg mass of the trolley and chain-block affect these significantly?

(c) What effect will the dead-weight of the beam have on the maximum loads?

Keep in mind that a safety factor of about 4 to 1 is used in this type of design (that is, the beam may bend appreciably (fail) at about 60 kN m) and rationalise your answers accordingly. Your results are to be painted on the side of the beam as a guide to the operators.



(a) For position A

Mass force of beam $= \dfrac{3000 \times 30 \times 10}{1000 \times 1000}$ kN

$= 0{\cdot}9$ kN

Take moments about surface O

$F_A \times 2{\cdot}85 + 0{\cdot}9 \times 1{\cdot}5 = 15$

$F_A = 4{\cdot}789$ kN

$\therefore$ Allowable Load : <u>0·5 T</u>

For position B

$F_B \times 1{\cdot}5 + 0{\cdot}9 \times 1{\cdot}5 = 15$

$\therefore F_B = 9{\cdot}1$ kN

$\therefore$ Allowable Load : <u>1 T</u>

For position C

$F_C \times 0{\cdot}15 + 0{\cdot}9 \times 1{\cdot}5 = 15$

$F_C = 91$ kN

$\therefore$ Allowable Load : <u>9 T</u>

| In each case the S.F. of 4:1 has been considered |

Note : A 9 T load at C will cause local bending of the bottom flange so load at C is limited to <u>3 T</u>

(b) With a 4:1 S.F. built in, the 50 kg. trolley mass will have no effect in any of the 3 positions

(c) The total mass of beam and trolley is 0·14 T

<u>For Pos. A</u> If the 0·14 T is ignored, the permitted load changes by only 47 kg (526 − 479) or 9·4 %

<u>For Pos. B</u> The load changes by 90 kg (1000 − 910) or 9%

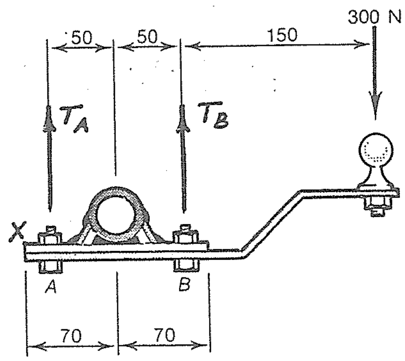
<u>For Pos. C</u> Of the allowable B.M. of 15 kNm, the beam and trolley contribute only 1·425 kNm or 9·5 %

Since 9% − 9·5% of the allowable B.M. is well within the 4:1 S.F., the trolley-beam effects will be small.

6/27

Despite the fact that towing trailers, boats, and caravans forms a regular part of our way of life, Australian and American car designers always seem to put the number plate right behind the towbar, thereby destroying the advantage of the quick-release ball coupling, and requiring the driver to remove part of the towbar each time he uncouples his trailer. This thoughtless design puts a big responsibility on the driver to reassemble his towbar safely each time.

For the recommended 300-N static load shown, calculate the minimum tension in each of the bolts, *A* and *B*, if the other bolt has worked loose. Discuss the likely shock loading conditions when towing with a loose bolt. (*Note*: Calculation of the varying loads occurring in correctly tensioned bolts is beyond the scope of this course, in which we only consider rigid bodies.)



(i) If the front bolt (A) is loose

Take moments about X

$$T_B \times 120 = 300 \times 270$$

$$T_B = \underline{675\ N} \quad \text{(satisfactory)}$$

(ii) If the rear bolt (B) is loose.

Take moments about X

$$T_A \times 20 = 300 \times 270$$

$$T_A = \underline{4050\ N} \quad \text{(Far too high)}$$

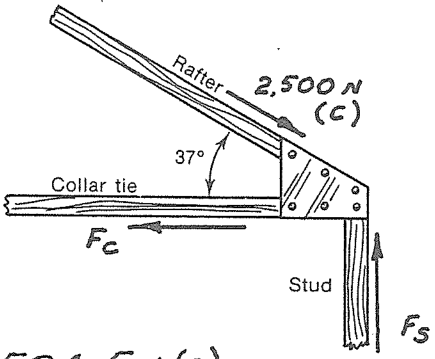
Shock or suddenly applied loads are usually calculated at 4 times the gradually applied equivalent load.

For bolt A, 4050 N is more than 6 times the designed static load.

7 Frames and Structures

7 FRAMES AND STRUCTURES
Frames. Types of Bar Assemblies. Types of Forces to Consider in Frame Analysis: *External Forces, Active Loads, Reactive Forces, Internal Forces.* Method of Joints. Method of Sections.

7/5
The corner of a roof truss is shown connected by a steel plate. The rafter has a compressive load of 2500 N. Determine the loads in the collar tie and in the stud.



Resolve vert.

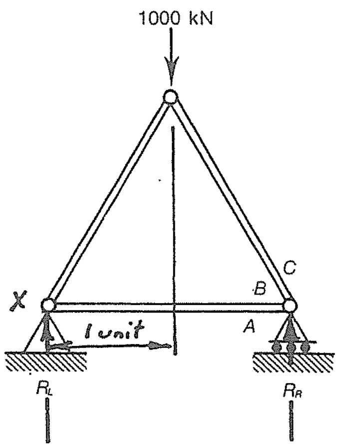
$F_S = 2{,}500 \sin 37° = \underline{1504{\cdot}5\text{ N (c)}}$

Resolve horiz.

$F_C = 2500 \cos 37° = \underline{1996{\cdot}6\text{ N (T)}}$

7/6
The structure shown in the sketch is in the form of an equilateral triangle. For the loading shown determine:
(i) the reactions R_L and R_R;
(ii) the forces in members AB and BC.

Let member length = 2 units



Take moments about X

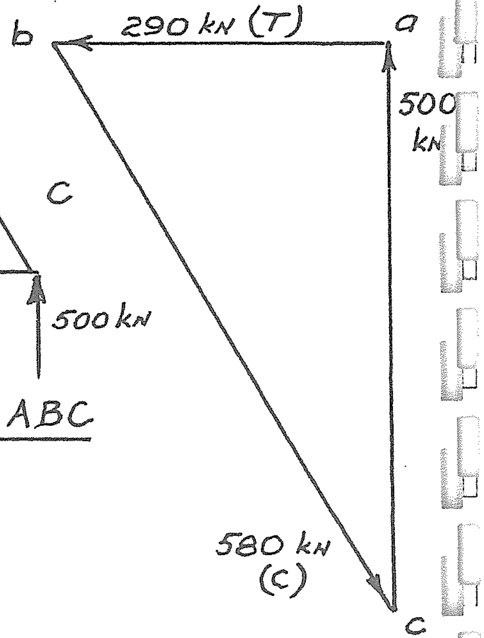
$R_R \times 2 = 1000 \times 1$

$R_R = \underline{500\text{ kN}} \uparrow$

Resolve vert.

$R_L = \underline{500\text{ kN}} \uparrow$

Scale: 1mm = 6·667 kN

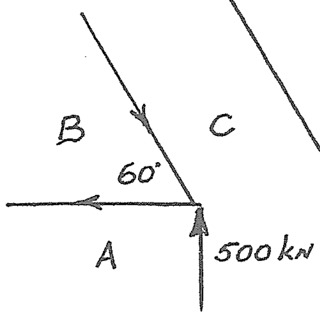


<u>Joint ABC</u>

Resolve vert.

$BC \sin 60° = 500$

$BC = \underline{577\text{ kN (c)}}$



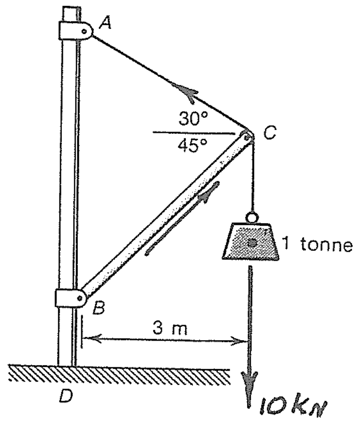
<u>Joint ABC</u>

Resolve horiz.

$AB = BC \cos 60°$

$= \underline{289\text{ kN (T)}}$

7/7
A simple derrick is shown, which supports a load of 1 tonne. Draw free-body diagrams of the load pin C, tie AC, strut CB, and king post AD, showing all forces. Calculate these forces.



Resolve vert.

$$BC \sin 45^\circ + AC \sin 30^\circ = 10$$

Resolve horiz.

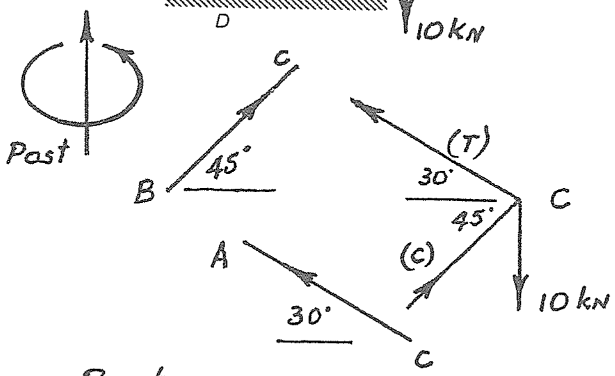
$$BC \cos 45^\circ = AC \cos 30^\circ$$

$$BC = \frac{0.866\,AC}{0.7071}$$

Subst. and solve to give:

$AC = \underline{7.32 \text{ kN (T)}}$

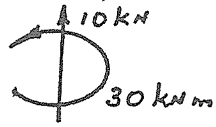
and $BC = \underline{8.96 \text{ kN (C)}}$



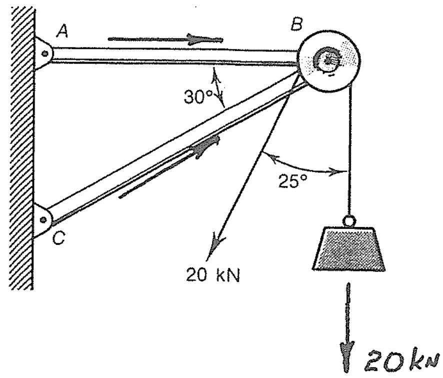
Take moments about D

$10 \times 3 = 30$ kNm

∴ Ground reaction forces on Post are a force/couple



7/8
A pin-jointed frame shown in the diagram supports a pulley which is used as illustrated to lift a load. Find;
(a) the magnitude of the force in each member; and
(b) state whether they are in tension or compression.



Resolve vert.

$$BC \sin 30^\circ = 20 + 20 \cos 25^\circ$$

$$BC = \frac{20(1 + 0.906)}{0.5}$$

$$= \underline{76.24 \text{ kN (C)}}$$

Resolve horiz

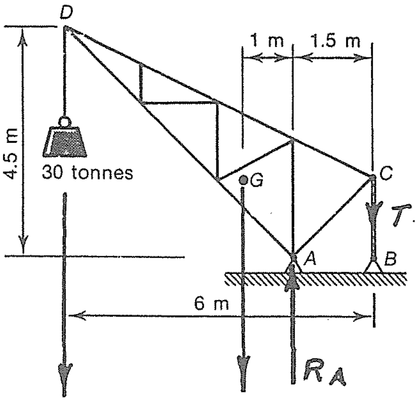
$$AB + BC \cos 30^\circ = 20 \sin 25^\circ$$

$$AB = 8.45 - 66.02$$

$$= -57.57 \text{ kN}$$

∴ Force in AB must be reversed to give $AB = \underline{57.57 \text{ kN (T)}}$

7/9
The crane has a mass of 8 tonnes with centre of gravity at G and is held in place by the pin connection at A and the vertical cable BC. Determine the force supported by the pin at A, and the tension in the cable BC.



300 kN   80 kN

Take moments about A

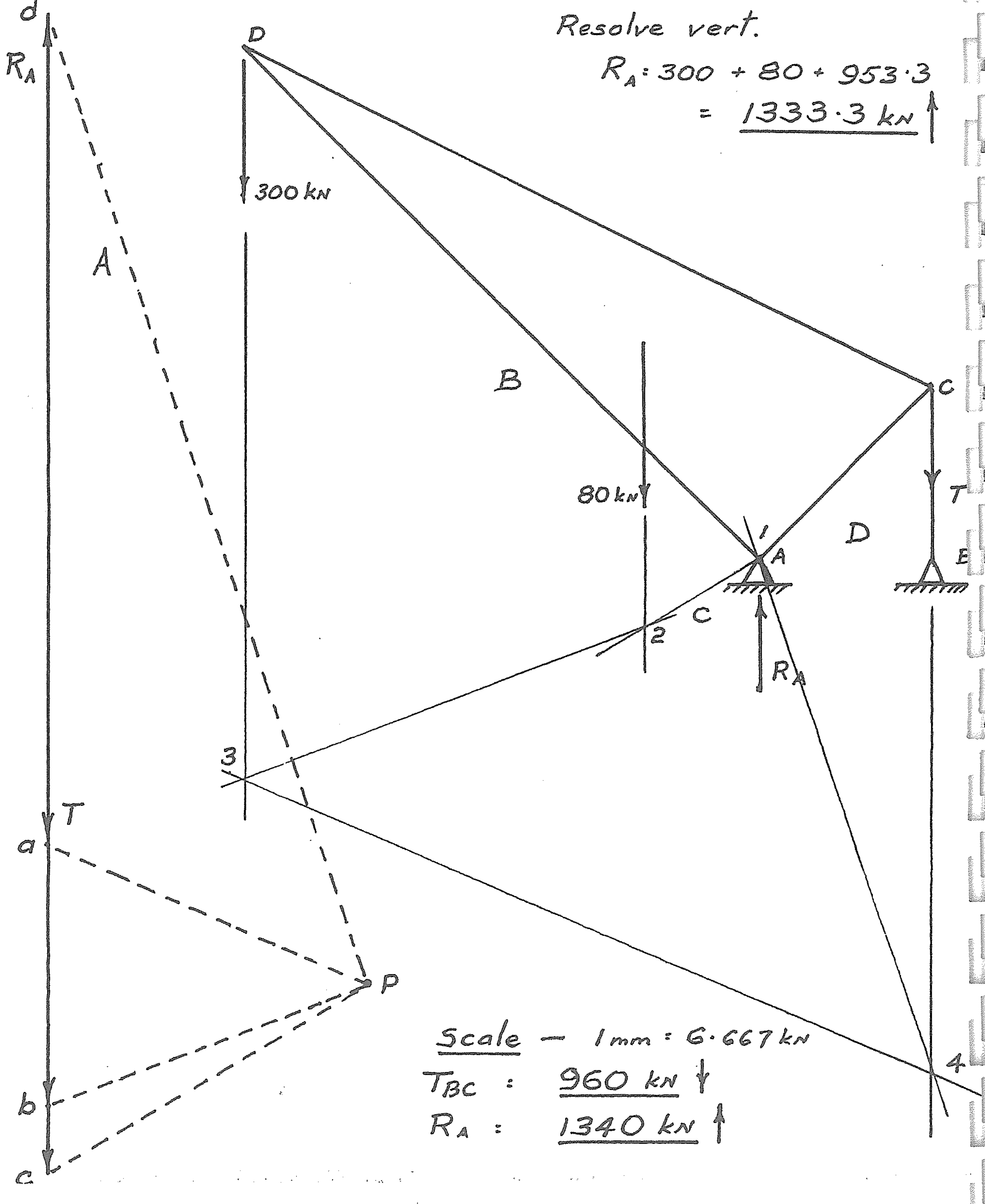
$BC \times 1{\cdot}5 = 80 \times 1 + 300 \times 4{\cdot}5$

$BC = \underline{953{\cdot}3\ kN} \downarrow$

Resolve vert.

$R_A = 300 + 80 + 953{\cdot}3$

$= \underline{1333{\cdot}3\ kN} \uparrow$



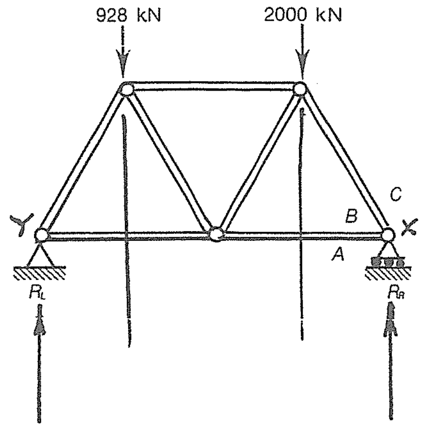
Scale — 1 mm = 6·667 kN

T_{BC} : $\underline{960\ kN} \downarrow$

R_A : $\underline{1340\ kN} \uparrow$

7/10
The structure shown in the diagram is in the form of equilateral triangles. For the loading shown determine:
(i) (a) The reaction R_L.
(b) The reaction R_R.
(ii) (a) The force in member AB.
(b) The force in member BC.

Let each member be 2 units long



(i)

(a) Take moments about X

$$R_L \times 4 = 928 \times 3 + 2000 \times 1$$

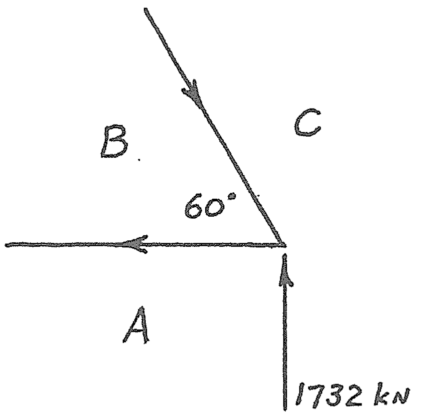
$$R_L = \underline{1196 \text{ kN}} \uparrow$$

(b) Resolve vert.

$$R_R = 2000 + 928 - 1196$$

$$= \underline{1732 \text{ kN}} \uparrow$$

(ii) Joint ABC



Resolve vert.

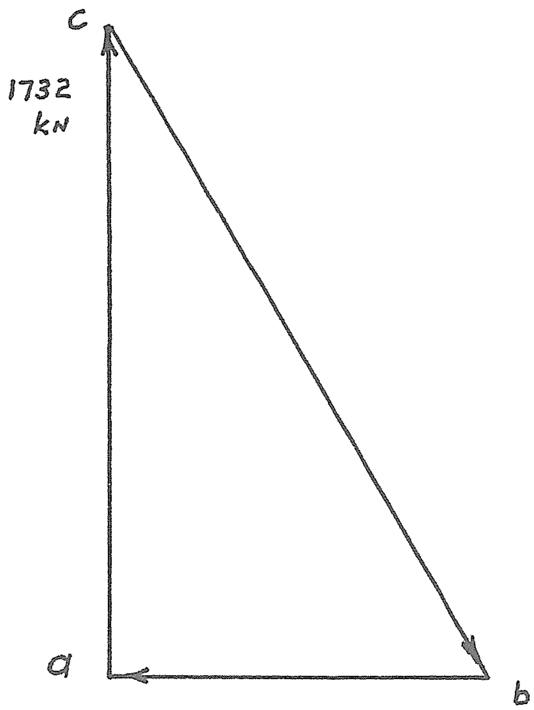
$$BC \sin 60^\circ = 1732$$

$$BC = \underline{2000 \text{ kN } (C)}$$

Resolve horiz.

$$AB = BC \cos 60^\circ$$

$$= \underline{1000 \text{ kN } (T)}$$



Scale – 5 mm = 100 kN

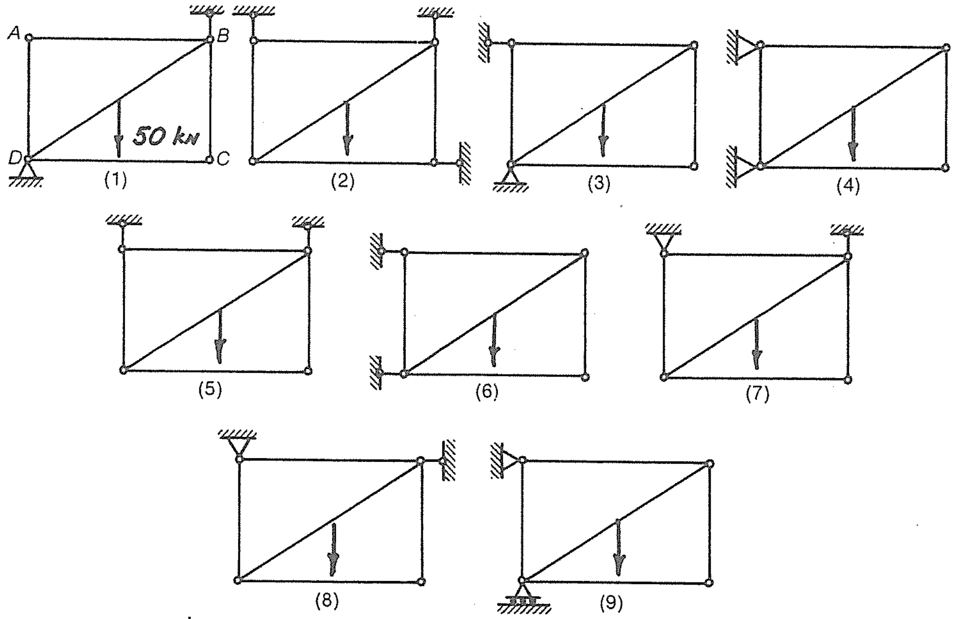
$BC = \underline{2000 \text{ kN } (C)}$

$AB = \underline{1000 \text{ kN } (T)}$

7/11

Simple 3 m × 4 m frameworks of mass 5 tonnes are supported vertically as shown. All connections consist of smooth pins, rollers, or short links. In each case, determine whether the reactions are statically determinate and wherever possible, calculate these reactions.

Assume the corresponding pin joints in each frame are lettered, A B C & D as in frame 1.



1. Take moments about D

$F_B \times 4 = 50 \times 2 \quad \therefore F_B = \underline{25\ kN} \uparrow$

Resolve vert. $\quad R_D = \underline{25\ kN} \uparrow$

2. Take moments about A

$F_B \times 4 = 50 \times 2 \quad \therefore F_B = \underline{25\ kN} \uparrow$

Take moments about C

$F_A \times 4 = 50 \times 2 \quad \therefore F_A = \underline{25\ kN} \uparrow$

No horiz. forces $\therefore F_C = \underline{0}$

3. Take moments about D

$F_A \times 3 = 50 \times 2 \quad \therefore F_A = \underline{33{\cdot}33\ kN} \leftarrow$

Resolve $\quad HR_D = \underline{33{\cdot}33\ kN} \rightarrow$

$VR_D = \underline{50\ kN} \uparrow$

4. Take moments about D & A

$HR_A \times 3 = 50 \times 2 \therefore HR_A = \underline{33{\cdot}33\ kN} \leftarrow$

& $HR_D \times 3 = 50 \times 2 \therefore HR_D = \underline{33{\cdot}33\ kN} \rightarrow$

Resolve vert. $VR_A + VR_D = 50$

These are <u>INDETERMINATE</u>

5. Resolve Vert.

$F_A = F_B = \underline{25\ kN} \uparrow$

6. No restraint at D.
Frame <u>will collapse</u>.

7. Resolve vert.

$R_A = F_B = \underline{25\ kN} \uparrow$

8. Vert. components at A & B are $25\,N \uparrow$, but horiz. components can't be found

<u>INDETERMINATE</u>

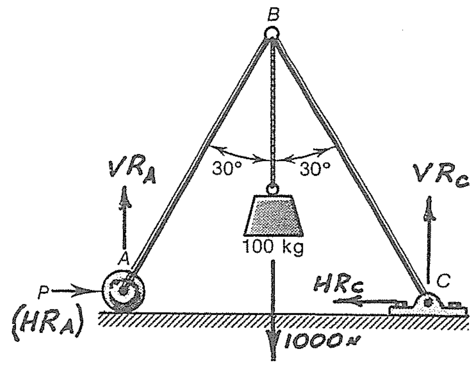
9. D is free to roll sideways so the frame <u>will collapse.</u>

7/12

Two bars, AB and BC, are pivoted at B. Both bars lie in a vertical plane and are of negligible weight. The end C is pin-jointed to a fixed bearing and end A is pinned to a roller which is free to move along a smooth horizontal surface. If a load of 100 kg is hung from B, find for the position shown:

(a) the horizontal force P required at A to prevent the roller moving outwards;
(b) the force in the link AB; and
(c) the vertical reaction on the roller.

Let AC be 2 units.



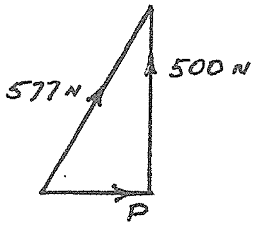
Take moments about C

$$VR_A \times 2 = 1000 \times 1$$

$$VR_A = \underline{500\ N}$$

The force in AB in order to produce a vert. component of 500 N will be: $500/\cos 30°$

$$= \underline{577\ N}$$

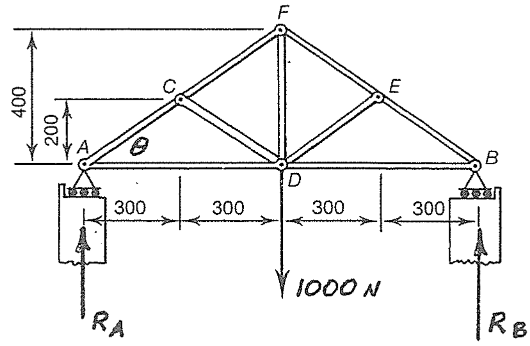


$$P = \sqrt{577^2 - 500^2}$$

$$= \underline{288.5\ N}$$

7/13

The model truss shown is under test. Determine the force in the members AD, AC and CD when a mass of 100 kg is hung from joint D.



Take moments about B

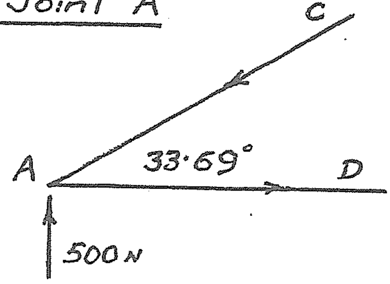
$$R_A \times 1200 = 1000 \times 600$$

$$\therefore R_A = 500\ N \uparrow$$

$\tan\theta = 400/600$

$\theta = 33.69°$

<u>At Joint A</u>



Resolve vert.

$$AC \sin 33.69° = 500$$

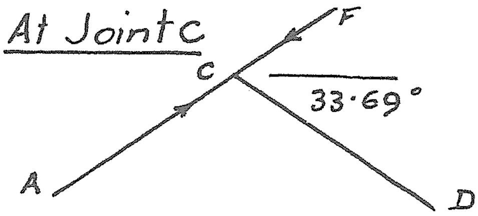
$$AC = \underline{901.4\ N\ (c)}$$

Resolve horiz.

$$AD = AC \cos 33.69°$$

$$= \underline{750\ N\ (T)}$$

<u>At Joint C</u>



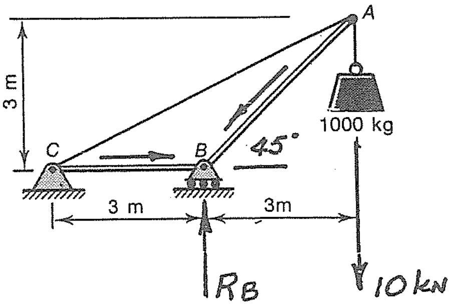
For equilibrium at C the force in CF $= \underline{901.4\ N\ (c)}$

Resolve horiz.

$$901.4 \cos 33.69° = CD \cos 33.69° + 901.4 \cos 33.69°$$

$$\therefore CD = \underline{0}$$

7/14
The simple derrick shown in the diagram must lift a 1000-kg vertical load by means of a rope suspended at point A.
(a) What is the magnitude of the reaction at B?
(b) Determine the forces in the members AC, AB, and BC.



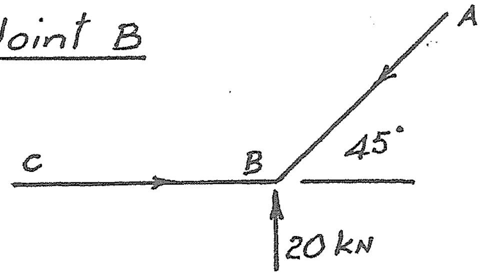
By geometry $\hat{ACB} = 26.57°$; $\hat{CAB} = 18.43°$

(a) Take moments about C

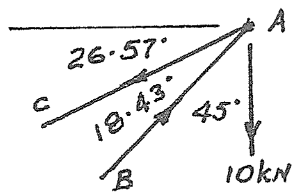
$$R_B \times 3 = 10 \times 6$$

$$R_B = \underline{20\ kN} \uparrow$$

(b) <u>At Joint B</u>



<u>At Joint A</u>



Resolve horiz.

$$AC \cos 26.57° = 28.28 \cos 45°$$

$$AC = \underline{22.36\ kN\ (T)}$$

Resolve vert.

$$AB \sin 45° = 20$$

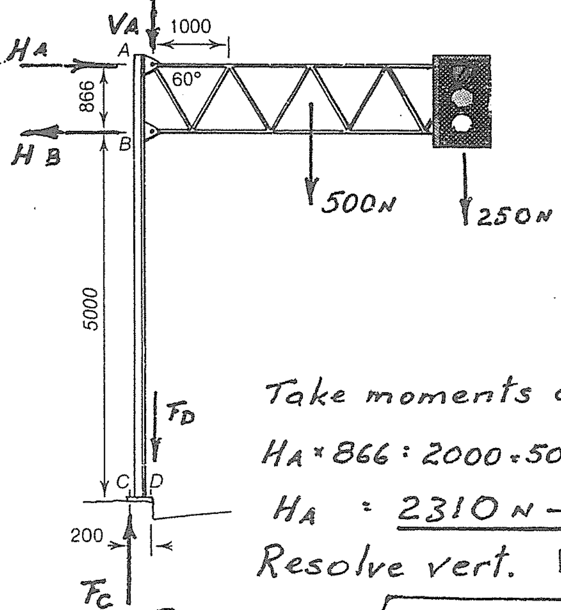
$$AB = \underline{28.28\ kN\ (C)}$$

Resolve horiz.

$$BC = AB \cos 45$$

$$= \underline{20\ kN\ (C)}$$

7/15
A set of traffic lights is cantilevered over the roadway on a light tubular steel framework. The framework has a mass of 50 kg and the signal lamp unit has a mass of 25 kg. Determine the maximum shear force in the bolts at A and B and the tensile force in the bolts at C and D.



Take moments about A

$$H_B \times 866 = 2000 \times 500 + 4000 \times 250$$

(S.F in B) $H_B = \underline{2310\ N} \leftarrow$

Take moments about B

$$H_A \times 866 = 2000 \times 500 + 4000 \times 250$$

$$H_A = \underline{2310\ N} \rightarrow$$

Resolve vert. $V_A = -750\ N \downarrow$

$$R\ (S.F.\ A) = \sqrt{2310^2 + 750^2}$$

$$= \underline{2429\ N}$$

$$\tan\theta = 750/2310$$

$$\theta = \underline{18°}$$

The couple at AB = 2 kNm

∴ Reaction couple by pins at AB = 2 kNm will be balanced by couple at CD

$$F_C \times 0.2 = 2$$

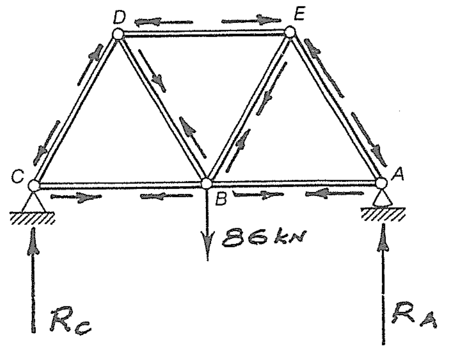
$$F_C = 10\ kN \uparrow$$

$$F_D = 10\ kN \downarrow$$

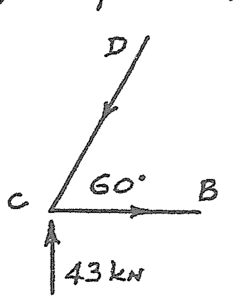
∴ Tension in bolt C = $\underline{10\ kN}$

& Tension in bolt D = $\underline{0}$

7/16
Calculate the forces in all members of the truss shown.



By inspection $R_C = R_A = 43$ kN ↑



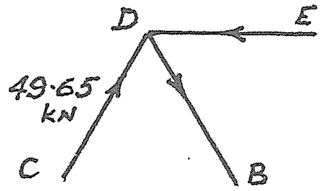
At Joint C

Resolve vert.

$CD \sin 60° = 43$

$CD = \underline{49 \cdot 65 \text{ kN (C)}}$

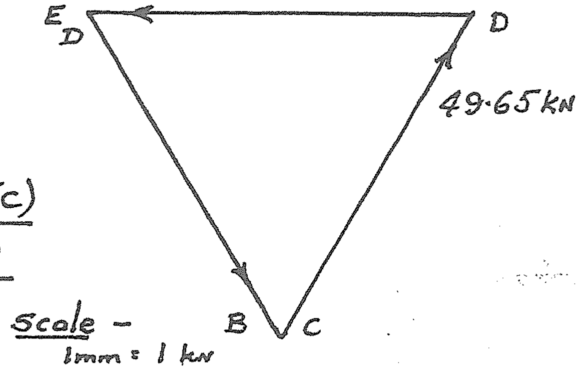
Resolve horiz. $BC = CD \cos 60° = \underline{24 \cdot 83 \text{ kN (T)}}$

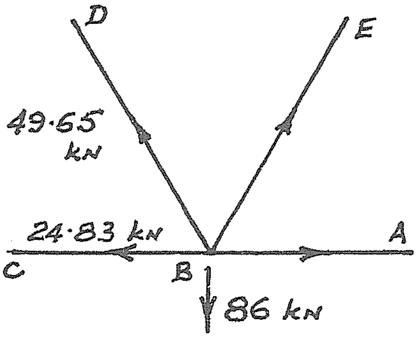


At Joint D

$DE = \underline{49 \cdot 65 \text{ kN (C)}}$

$DB = \underline{49 \cdot 65 \text{ kN (T)}}$





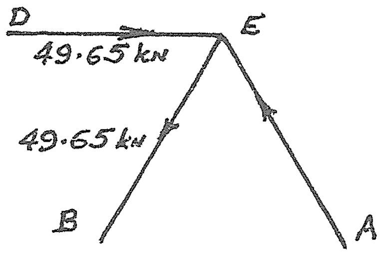
At Joint B

Resolve vert. $BE \sin 60° + 49{\cdot}65 \sin 60 = 86$

$BE = \underline{49 \cdot 65 \text{ kN (T)}}$

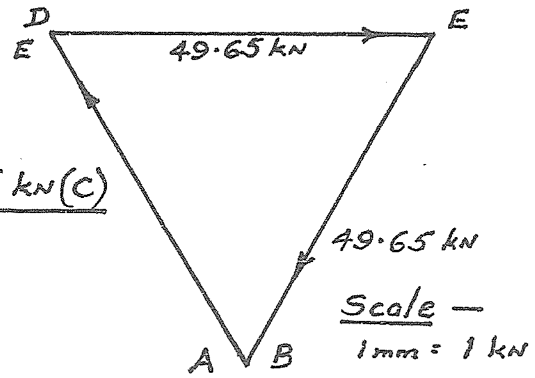
Resolve horiz. $AB + 49{\cdot}65 \cos 60° = 24{\cdot}83 + 49{\cdot}65 \cos 60°$

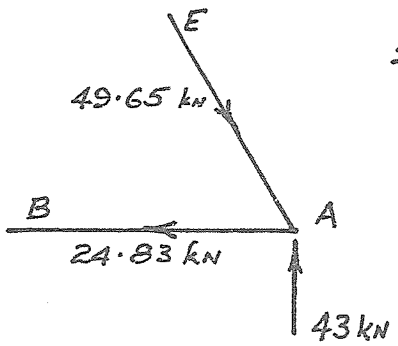
$AB = \underline{24 \cdot 83 \text{ kN (T)}}$



At Joint E

$AE = \underline{49 \cdot 65 \text{ kN(C)}}$





At Joint A

Resolve vert. $49{\cdot}65 \sin 60° = 43$

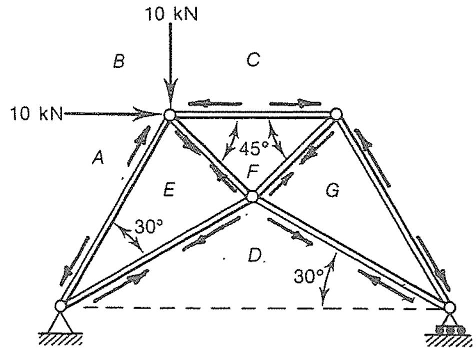
$\underline{42 \cdot 996 \doteqdot 43}$

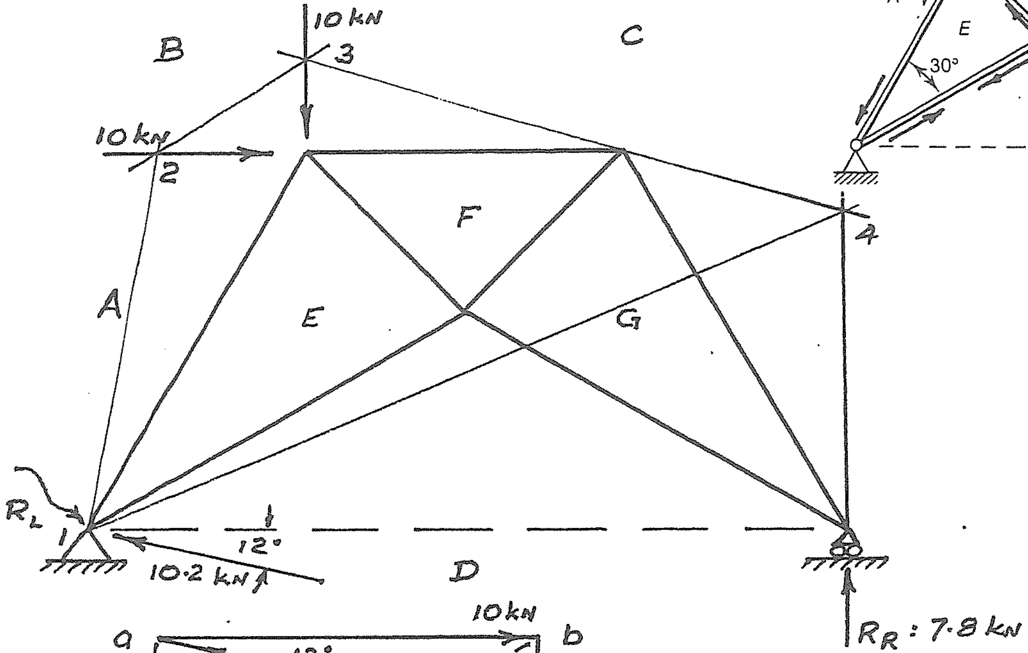
Resolve horiz. $49{\cdot}65 \cos 60° = 24 \cdot 83$

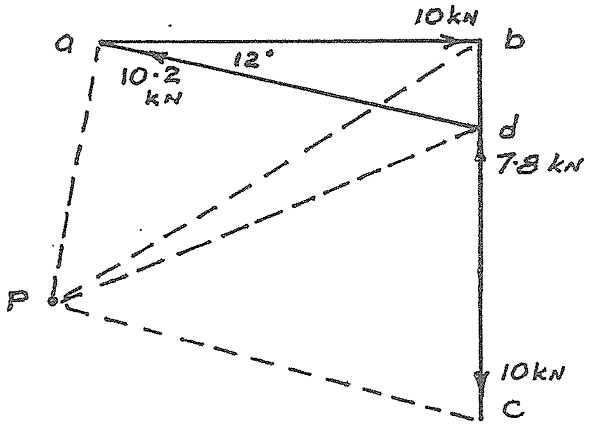
$\underline{24 \cdot 83 = 24 \cdot 83}$

The solution of Joint A provides a check for correctness

7/17
The geometry of a framework, loaded as shown, is given in the diagram. Find graphically the reaction at each support and the nature and magnitude of the force in each bar.



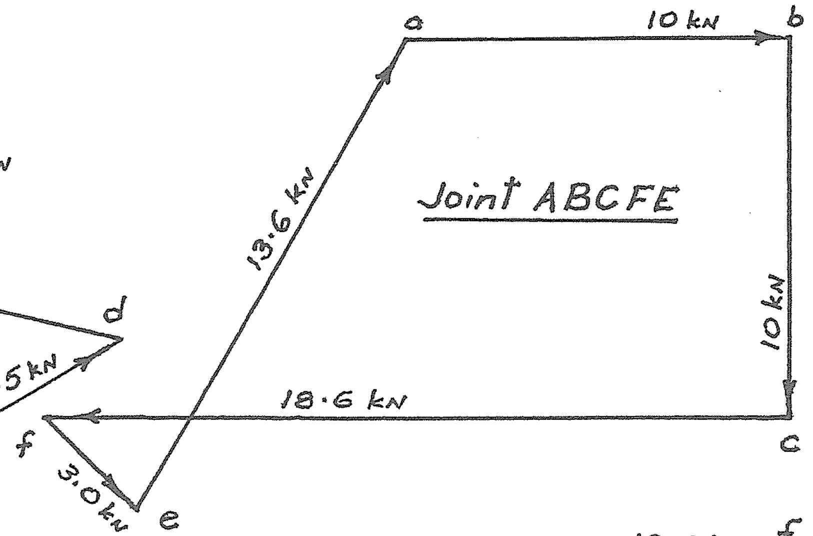




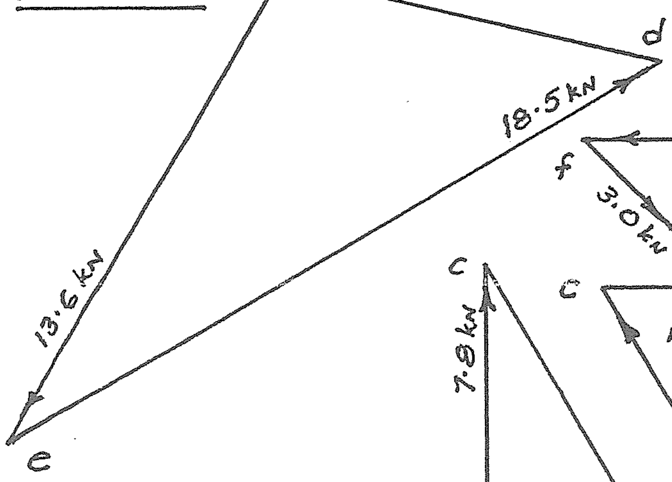
Scale – 10 mm : 2 kN

	kN
AE	= 13·6 (C)
ED	= 18·5 (T)
EF	= 3·0 (T)
FC	= 18·6 (C)
FG	= 16·4 (T)
GC	= 13·0 (C)
GD	= 7·6 (T)

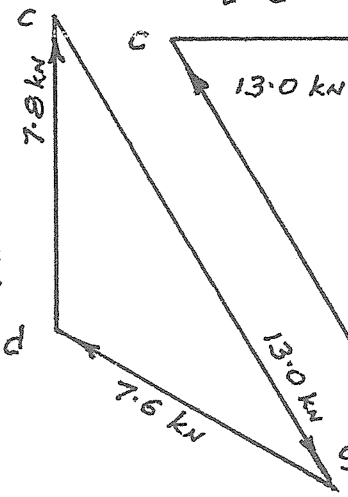
Joint ABCFE



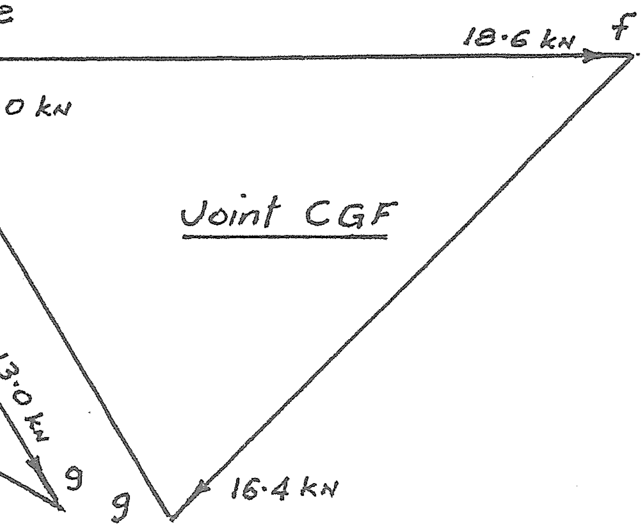
Joint DAE



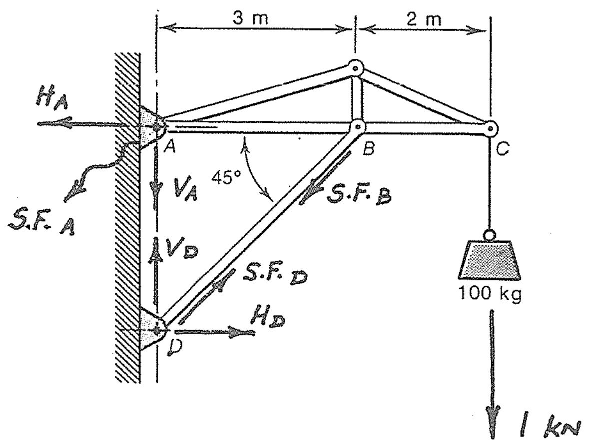
Joint DCG



Joint CGF



7/18
A simple pin-jointed frame is shown in the diagram. Determine
(a) the shear force in the pins at A, B, and D;
(b) the horizontal and vertical components of the reactive force at A;
(c) the horizontal and vertical components of the reactive force at D.



Take moments about D

$$H_A \times 3 = 1 \times 5$$
$$H_A = \underline{1 \cdot 67 \text{ kN}} \leftarrow$$

Take moments about A

$$H_D \times 3 = 1 \times 5$$
$$H_D = \underline{1 \cdot 67 \text{ kN}} \rightarrow$$

Take moments about B

$$1 \times 2 + V_D \times 3 = V_A \times 3 + 1 \cdot 67 \times 3$$
$$3(V_D - V_A) = 3$$
$$V_D - V_A = 1$$

Since reaction at D is at 45° $H_D = V_D = 1 \cdot 67$ kN

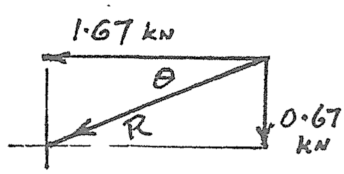
Subst. $1 \cdot 67 - V_A = 1$

$$V_A = \underline{0 \cdot 67 \text{ kN}} \downarrow$$
$$\& \quad V_D = \underline{1 \cdot 67 \text{ kN}} \uparrow$$

Reaction ($\therefore$ S.F.) at D $= \sqrt{1 \cdot 67^2 + 1 \cdot 67^2}$
$= \underline{2 \cdot 36 \text{ kN at } 45°}$ θ

Reaction ($\therefore$ S.F.) at B $= \underline{2 \cdot 36 \text{ kN at } 45°}$ θ

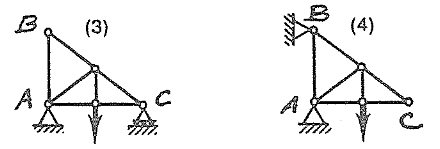
Reaction ($\therefore$ S.F.) at A $= \sqrt{1 \cdot 67^2 + 0 \cdot 67^2}$
$= \underline{1 \cdot 799 \text{ kN}}$

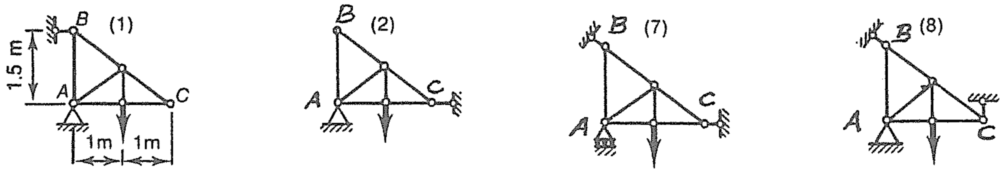


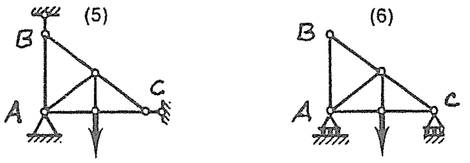
$$\tan\theta = 0 \cdot 67 / 1 \cdot 67$$

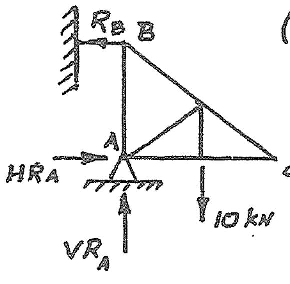
$$\theta = \underline{21 \cdot 86°} \; \theta$$

7/19
A small truss is supported in different ways as shown. All connections consist of smooth pins, rollers, or short links. For each structure, wherever possible, calculate the reactions, assuming that the magnitude of the force is 10 kN.









(1) Take moments about A

$10 \times 1 = R_B \times 1.5$

$R_B = \underline{6.67\ kN} \leftarrow$

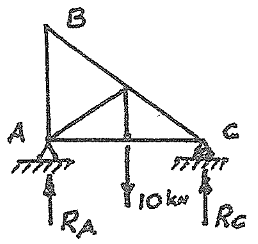
Resolve horiz.

$HR_A = \underline{6.67\ kN} \rightarrow$

Resolve vert.

$VR_A = \underline{10\ kN} \uparrow$

(2) INDETERMINATE: The mag. and direction of tension at C can't be found.



(3) No horiz. forces

$\therefore R_C$ and R_A are vert. From symmetry

$R_C = R_A = \underline{5\ kN}$

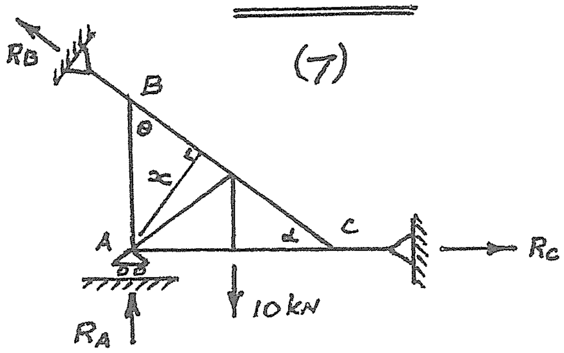
(4) INDETERMINATE : The mag. and direction of neither reaction can be found.

(5)

Same as for No. 4

(6) By symmetry and inspection R_A and R_C should be vertically up and equal to 5 kN, but the frame is unstable. It can move sideways.

(7)



(8) INDETERMINATE: There is no need for links at both B & C. Reactions can't be found.

Tan $\alpha = 1.5/2 \quad \therefore \alpha = 36.9°$

and $\theta = 53.1° \quad \therefore x = 1.2\ m$

Take moments about A

$R_B \times 1.2 = 10 \times 1$

$R_B = \underline{8.33\ kN}$ 36.9°

Take moments about B

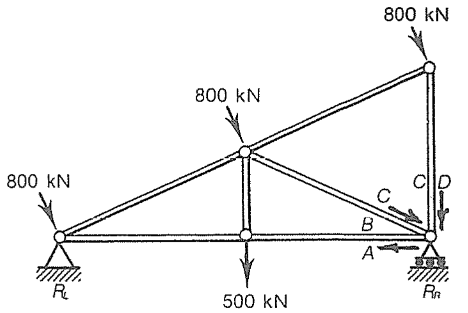
$R_C \times 1.5 = 10 \times 1$

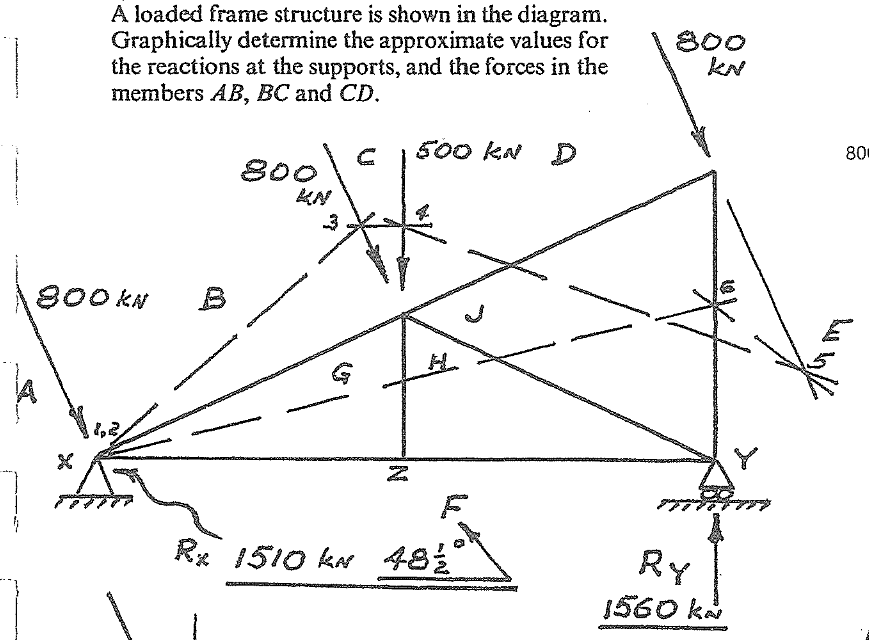
$R_C = \underline{6.67\ kN} \rightarrow$

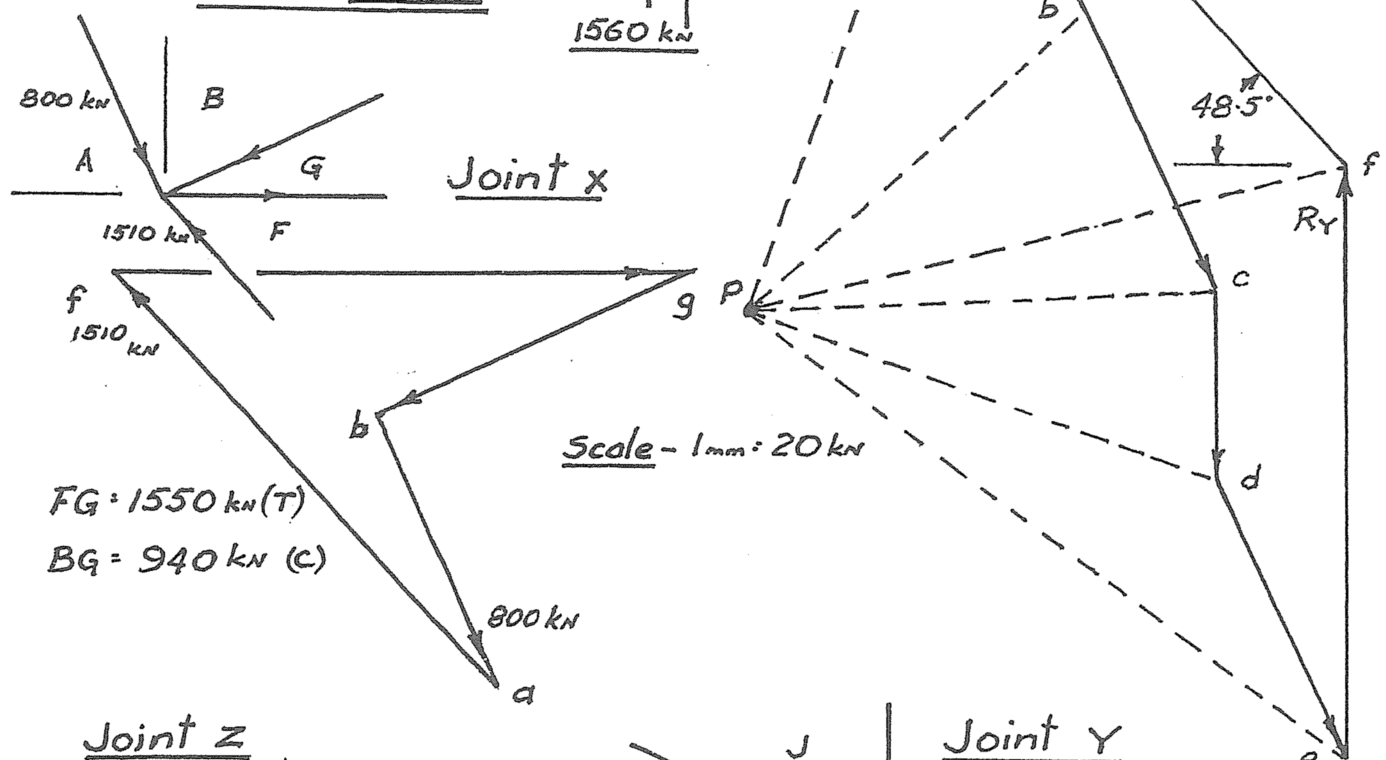
Resolve vert. $R_A + R_B \sin 36.9° = 10$

$R_A = \underline{5\ kN} \uparrow$

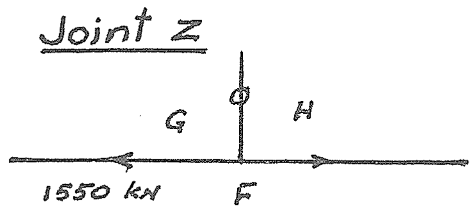
7/20
A loaded frame structure is shown in the diagram. Graphically determine the approximate values for the reactions at the supports, and the forces in the members *AB*, *BC* and *CD*.



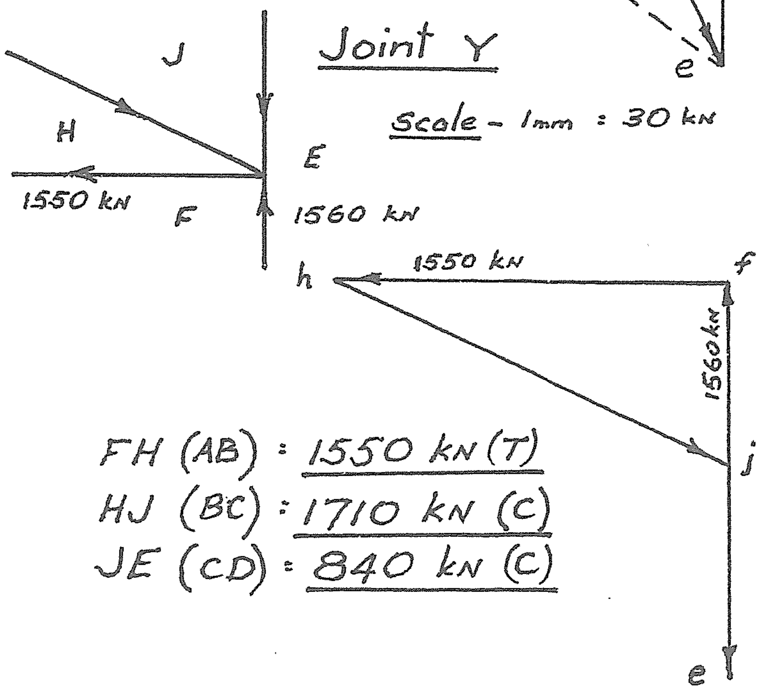




FG : 1550 kN (T)
BG = 940 kN (C)

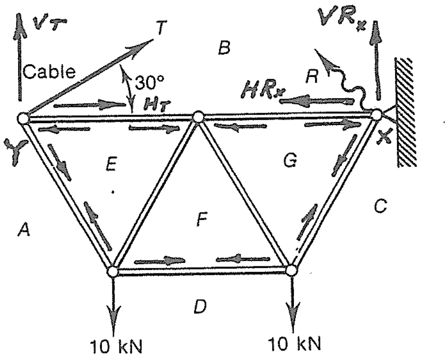


By inspection the force in HF = 1550 kN →
and in GH zero.



FH (AB) : 1550 kN (T)
HJ (BC) : 1710 kN (C)
JE (CD) : 840 kN (C)

7/21
The truss shown is made up of three equilateral triangles loaded as indicated. It is supported by a pin joint at the wall on the right-hand side and by a cable inclined at 30° to the horizontal on the left. Determine
(a) the tension in the cable;
(b) the reaction at the wall;
(c) the nature and magnitude of the force in each bar.



Let each member be 2 units long

(a) Take moments about x

$$T \sin 30 \times 4 = 10 \times 1 + 10 \times 3$$

$$\underline{T = 20 \text{ kN } \angle 30}$$

(b) Resolve Vert.

$$VR_x + 20 \sin 30^\circ = 10 + 10$$

$$VR_x = 10 \text{ kN} \uparrow$$

Resolve horiz.

$$HR_x = 20 \cos 30^\circ$$

$$= 17{\cdot}32 \text{ kN} \leftarrow$$

$$R = \sqrt{17{\cdot}32^2 + 10^2}$$

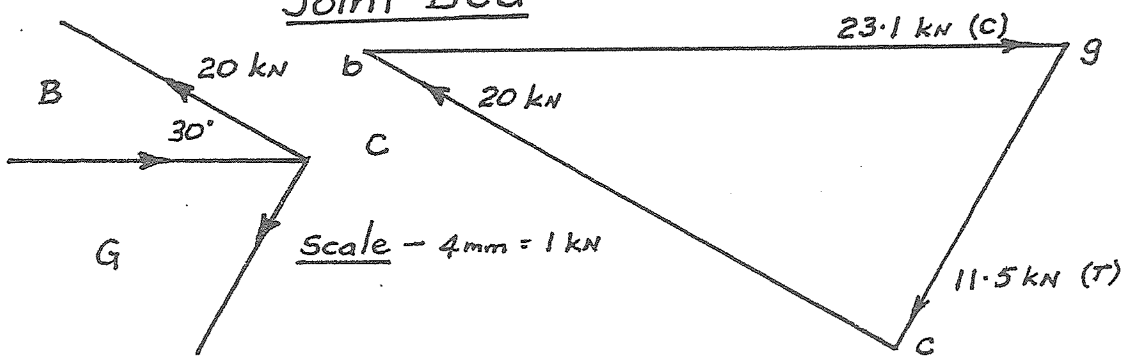
$$= \underline{20 \text{ kN}}$$

$$\tan \theta = 10/17{\cdot}32$$

$$\theta = \underline{30^\circ}$$

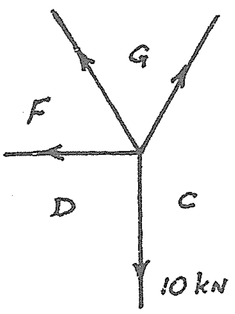
(c)

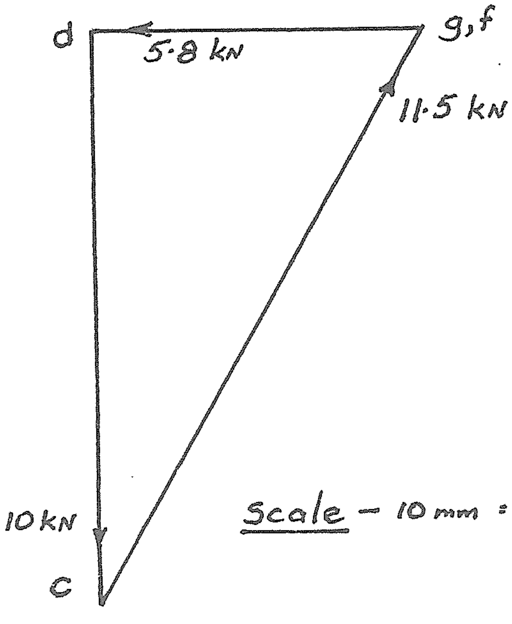
Joint BCG



Scale – 4mm = 1 kN

Joint DCGF



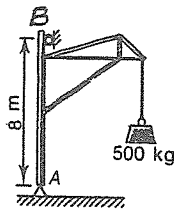


Using the symmetry of the frame:

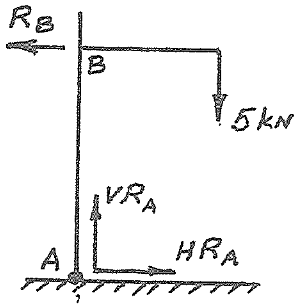
BG = BE = 23·1 kN (C)
GC = AE = 11·5 kN (T)
GF = EF = 0
FD = 5·8 kN (T)

Scale – 10 mm = 1·33 kN

7/22
The crane supports a 500-kg load. Find the reactions for each of the three types of supports shown.



(a)



Take moments about A

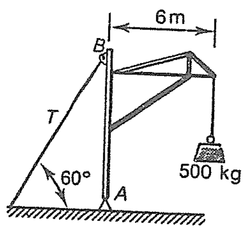
$R_B \times 8 = 5 \times 6$

$R_B = \underline{3 \cdot 75 \text{ kN}} \leftarrow$

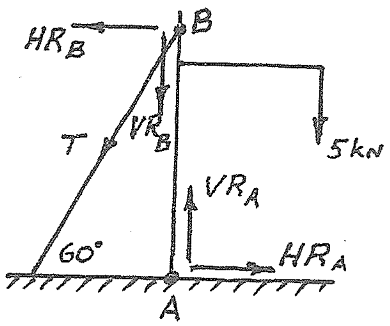
Resolve Vert. & horiz.

$VR_A = 5 \text{ kN} \uparrow$; $HR_A = 3 \cdot 75 \text{ kN} \rightarrow$

$\therefore R_A = \underline{6 \cdot 25 \text{ kN at } 53 \cdot 1^\circ \angle\theta}$



(b)



Take moments about B

$HR_A \times 8 = 5 \times 6$

$HR_A = 3 \cdot 75 \text{ kN} \rightarrow$

Resolve horiz.

$HR_B = 3 \cdot 75 \text{ kN} \leftarrow$

$\text{Cos.} 60^\circ = 3 \cdot 75 / T$ & $\text{Tan.} 60^\circ = VR_B / 3 \cdot 75$

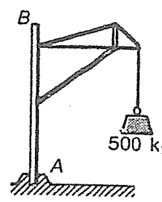
$\therefore \underline{T = 7 \cdot 5 \text{ kN}}$ $60°$ & $\underline{VR_B = 6 \cdot 5 \text{ kN}} \downarrow$

Resolve vert.

$VR_A = 6 \cdot 5 + 5$

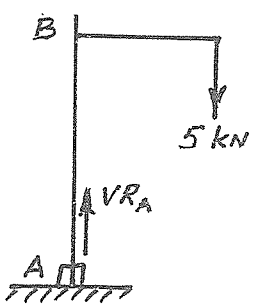
$= \underline{11 \cdot 5 \text{ kN}} \uparrow$

$\therefore R_A = \underline{12 \cdot 1 \text{ kN at } 71 \cdot 9^\circ \angle\theta}$



(c)

Since there is no support at B, there will be no reaction at B.



Resolve vert.

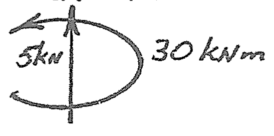
$VR_A = 5 \text{ kN}$

Take moments about A

5×6 or 30 kNm

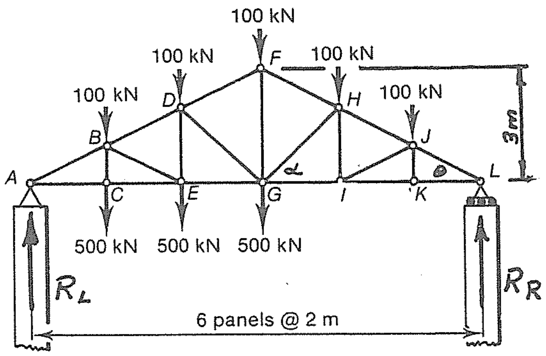
This is balanced by an anti-clockwise moment set up at A

i.e. $\underline{R_A} =$



7/23

Find the reactions in the supports and determine the forces in members *FH*, *GH*, and *GI* of the roof truss shown in the diagram.

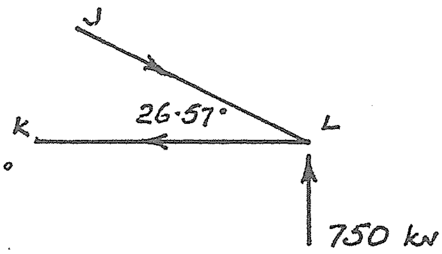


$\tan\theta = 1/2 \qquad \theta = 26{\cdot}57°$

$\tan\alpha = 2/2 \qquad \alpha = 45°$

Take moments about A to find $R_R = \underline{750\ \text{kN}} \uparrow$. Resolve vert. $R_L = \underline{1250\ \text{kN}} \uparrow$

Joint L



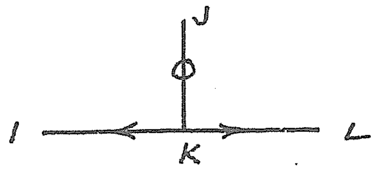
Resolve vert. $JL \sin 26{\cdot}57° = 750$

$JL = 1677\ \text{kN}\ (C)$

Resolve horiz. $KL = 1677 \cos 26{\cdot}57°$

$= 1500\ \text{kN}\ (T)$

Joint K



Resolve horiz. $IK = 1500\ \text{kN}\ (T)$

Resolve vert. $JK = 0$

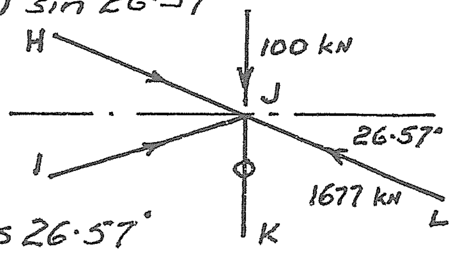
Joint J

Resolve vert.

$100 + HJ \sin 26{\cdot}57° = 1677 \sin 26{\cdot}57° + IJ \sin 26{\cdot}57°$

$\sin 26{\cdot}57° (HJ - IJ) = 650$

$HJ - IJ = 1453 \cdots (1)$



Resolve horiz.

$HJ \cos 26{\cdot}57° + IJ \cos 26{\cdot}57° = 1677 \cos 26{\cdot}57°$

$HJ + IJ = 1677 \cdots (2)$

Add (1) & (2)

$2HJ = 3130$

$HJ = 1565\ \text{kN}\ (C)$

Subst. in (2)

$1565 + IJ = 1677$

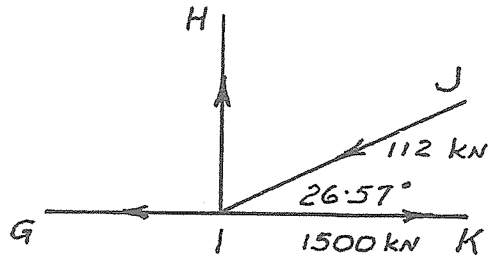
$IJ = 112\ \text{kN}\ (C)$

7/23 (cont.)

Joint I

Resolve horiz.

$$IG + 112 \cos 26.57° = 1500$$
$$IG = 1500 - 100$$
$$= 1400 \text{ kN (T)}$$



Resolve vert.

$$IH = 112 \sin 26.57°$$
$$= 50 \text{ kN (T)}$$

Joint H

Resolve horiz.

$$FH \cos 26.57° + GH \cos 45° = 1565 \cos 26.57°$$
$$0.8944\, FH + 0.7071\, GH = 1400 \text{ ---- (1)}$$

Resolve vert.

$$100 + FH \sin 26.57° + 50 = 1565 \sin 26.57° + GH \sin 45$$
$$0.4473\, FH - 0.7071\, GH = 550 \text{ ---- (2)}$$

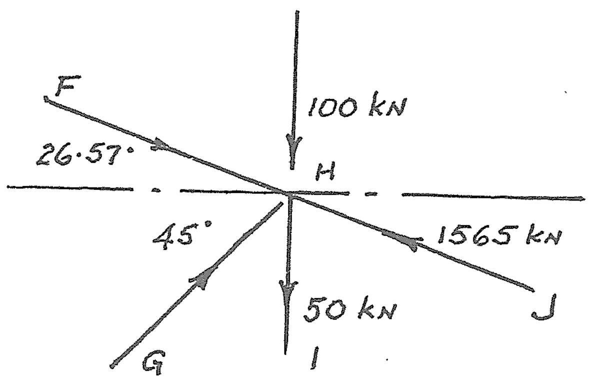
Add (1) & (2)

$$1.3417\, FH = 1950$$
$$FH = 1453 \text{ kN (C)}$$

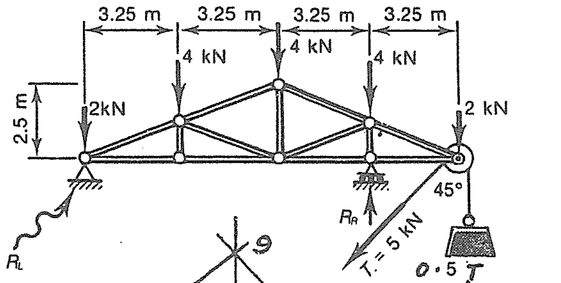
Subst. in (2)

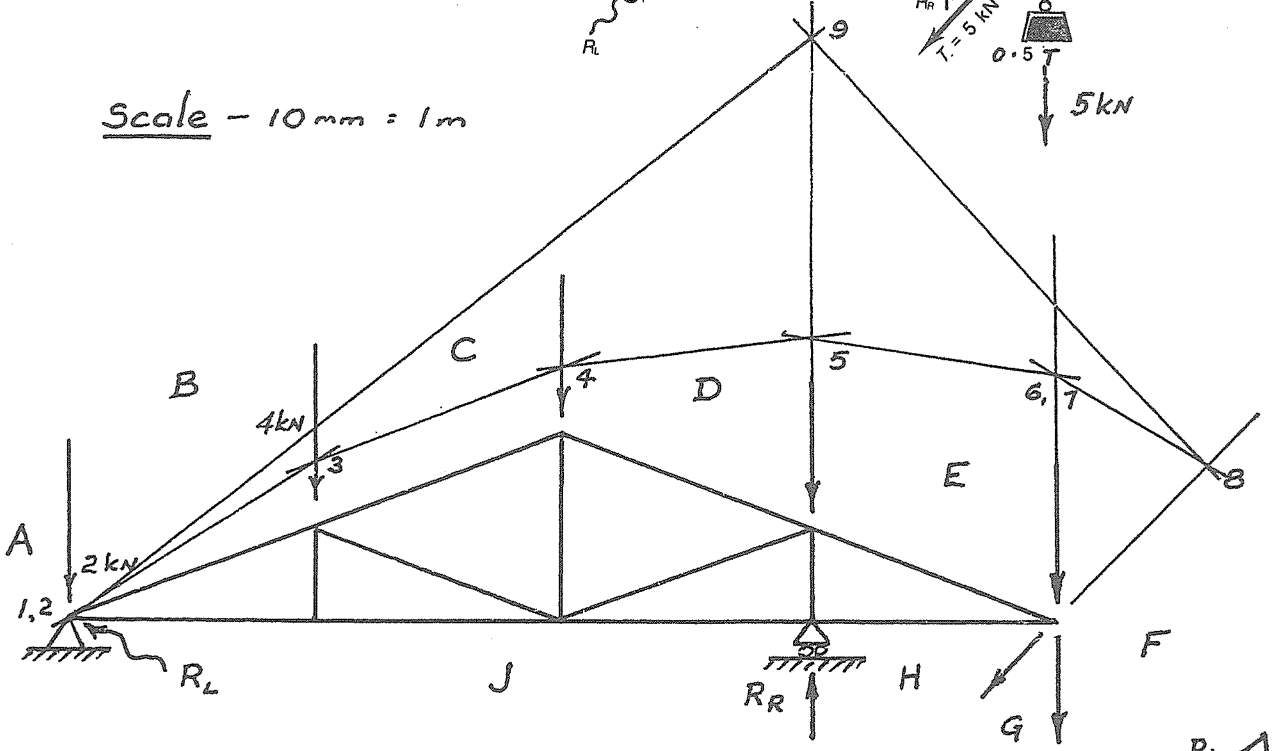
$$0.4473 \times 1453 - 0.7071\, GH = 550$$
$$GH = 141 \text{ kN (C)}$$

$FH = \underline{1453 \text{ kN (C)}}$
$GH = \underline{141 \text{ kN (C)}}$
$GI = \underline{1400 \text{ kN (T)}}$



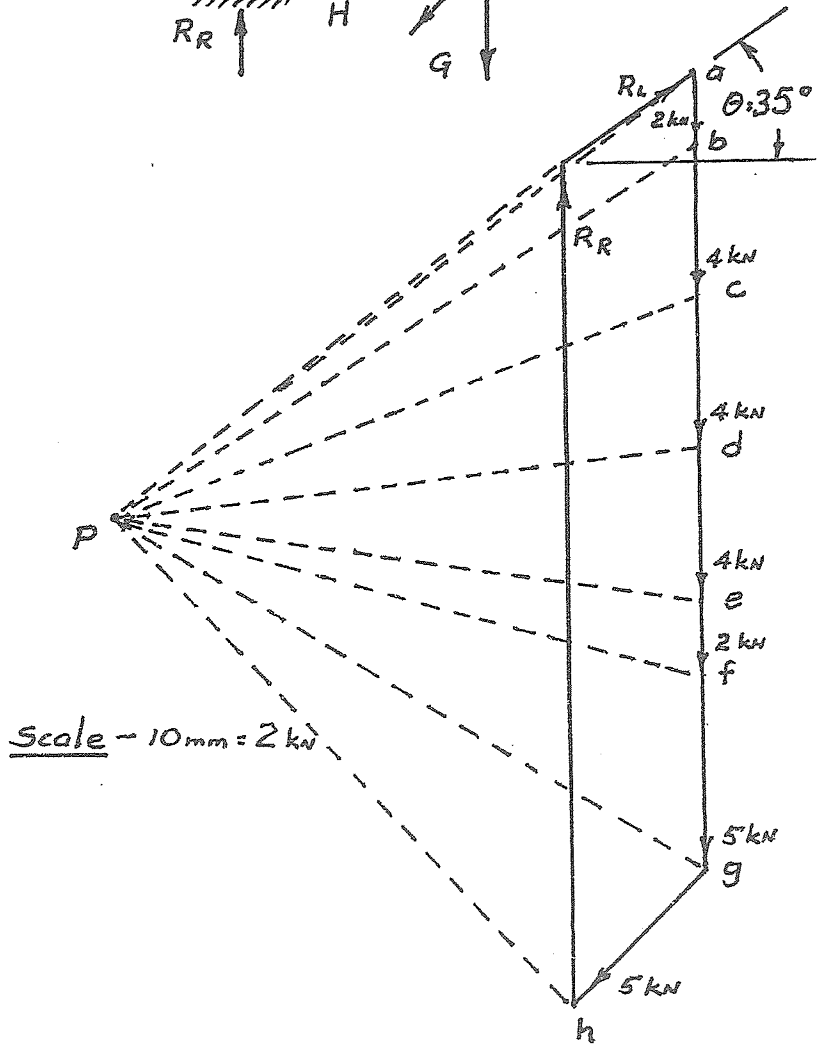
7/24
The truss shown in the diagram is subjected to the roof loads and the hoist load shown. Determine the *reactions* R_L and R_R using the funicular polygon.



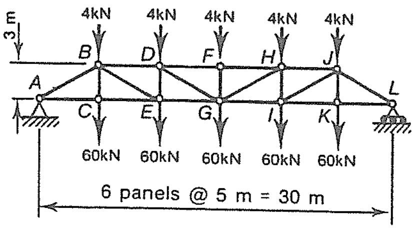


R_R = 22 kN

R_L = 4·3 kN θ: 35°



7/25
Determine the forces in members *DE* and *HJ* of the truss shown.

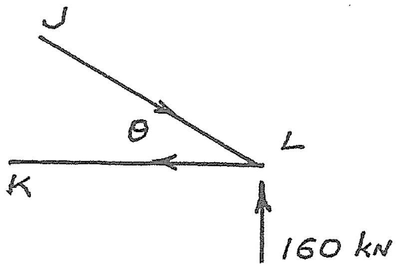


NOTE :

The truss is symmetrical and its loads are symmetrically arranged.

$\therefore$ Reaction at L will be $\underline{160\ \text{kN}}$ ↑

Joint L



Resolve vert.

$\tan\theta : 3/5$

$JL \sin\theta : 160$

$\theta : 30.96^\circ$

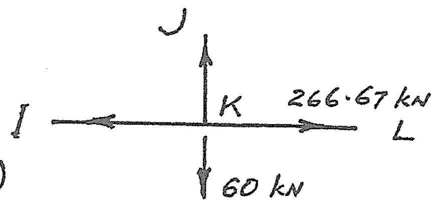
$\therefore JL : 310.98$ kN (C)

Resolve horiz.

$KL : JL \cos\theta$

$: 266.67$ kN (T)

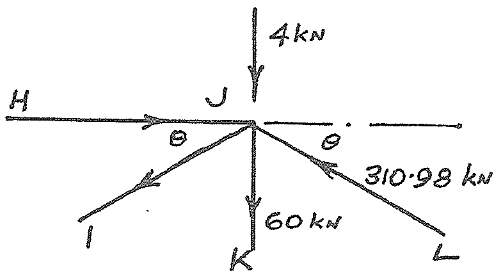
Joint K



Resolve vert. $JK : 60$ kN (T)

Resolve horiz. $IK : 266.67$ kN (T)

Joint J



Resolve vert.

$IJ \sin\theta + 4 + 60 : 310.98 \sin\theta$

$IJ : 186.6$ kN (T)

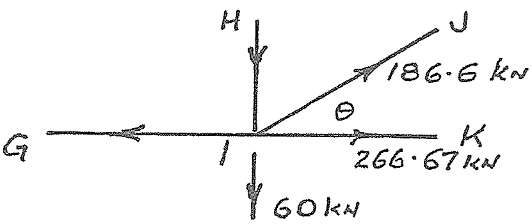
Resolve horiz.

$HJ : 186.6 \cos\theta + 310.98 \cos\theta$

$: 426$ kN (C)

Since the system is symmetrical the force in member DE = force in member HI

Joint I



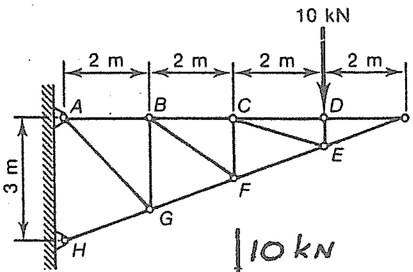
Resolve vert.

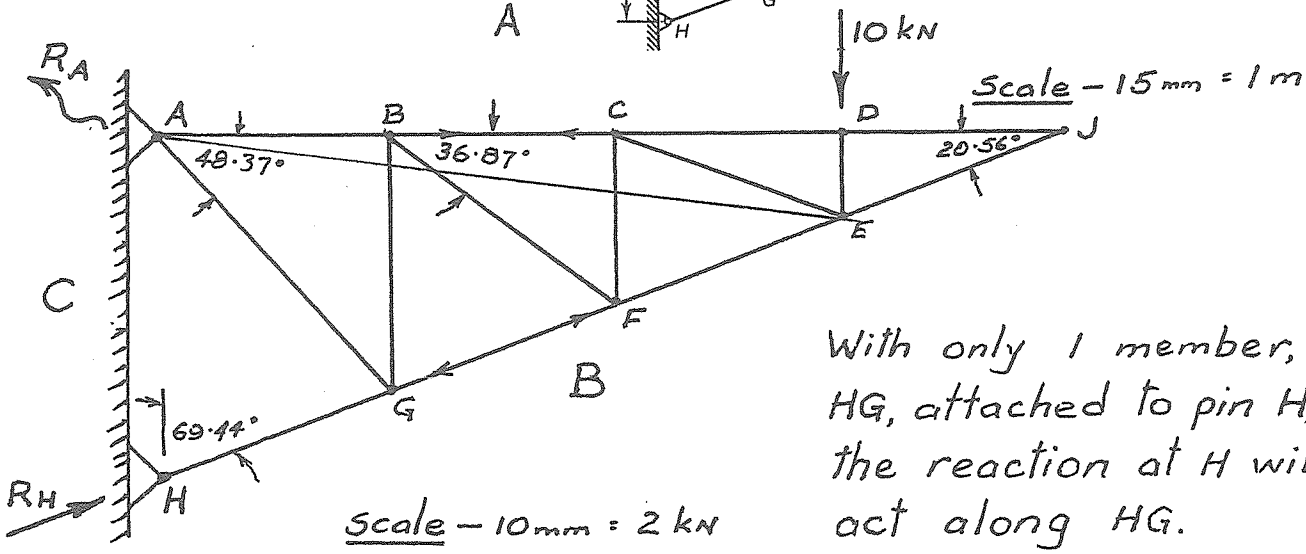
$HI + 60 = 186.6 \sin\theta$

$HI : 36$ kN (C)

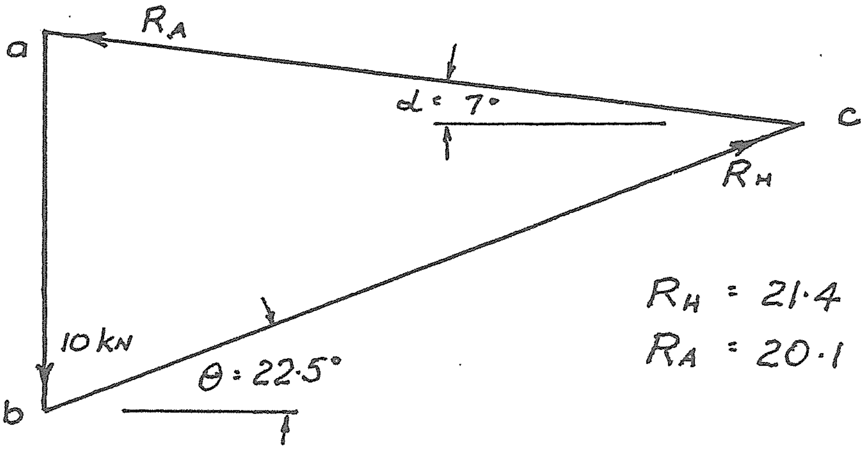
$DE : \underline{36\ \text{kN (C)}}$, $HJ : \underline{426\ \text{kN (C)}}$

7/26
Calculate the forces in members BC and GF of the cantilever truss shown in the diagram.





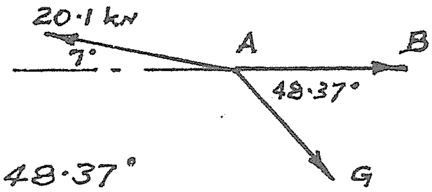
With only 1 member, HG, attached to pin H, the reaction at H will act along HG.



$R_H = 21{\cdot}4$ kN
$R_A = 20{\cdot}1$ kN

$HG = GF = FE = EJ = 2{\cdot}136$ m, $BG = 2{\cdot}25$ m, $CF = 1{\cdot}5$ m, $DE = 0{\cdot}75$ m

<u>Joint A</u>



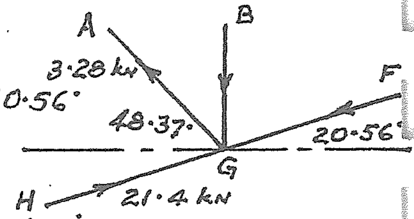
Resolve vert. $AG \sin 48{\cdot}37° = 20{\cdot}1 \sin 7°$

$AG = 3{\cdot}28$ kN

Resolve horiz. $AB = 20{\cdot}1 \cos 7° - 3{\cdot}28 \cos 48{\cdot}37°$

$= 17{\cdot}77$ kN

<u>Joint G</u>



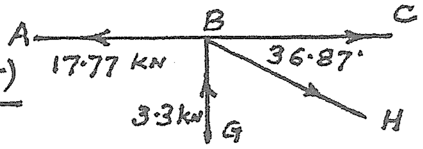
Resolve horiz. $3{\cdot}28 \cos 48{\cdot}37° + GF \cos 20{\cdot}56° = 21{\cdot}4 \cos 20{\cdot}56°$

<u>$GF = 19$ kN (C)</u>

Resolve vert. $BG + 19 \sin 20{\cdot}56° = 3{\cdot}28 \sin 48{\cdot}37° + 21{\cdot}4 \sin 20{\cdot}56°$

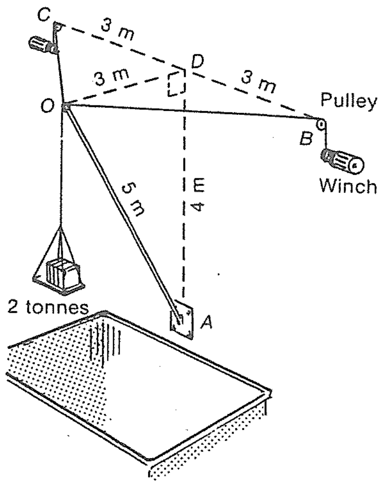
$BG = 3{\cdot}3$ kN

<u>Joint B</u> Resolve to find <u>$BC = 13{\cdot}3$ kN (T)</u>

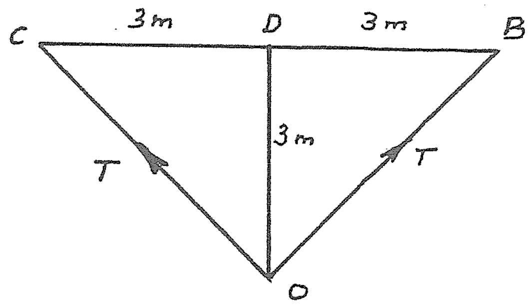


7/27

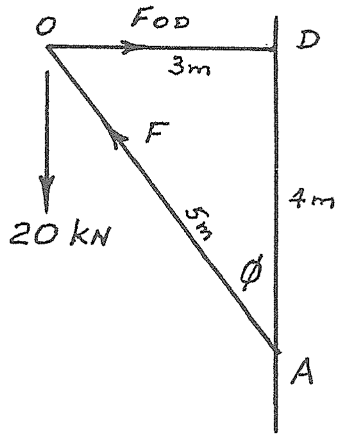
A ship's jib crane is shown diagrammatically. Determine the approximate compressive force in the boom OA and the tension in the cables OB and OC when they have positioned the boom as shown. The mass of the boom is negligible compared with that of the load and may be ignored in the calculations.



Resolve T_{OB} and T_{OC} in the direction OD



∴ Resultant force in direction OD, $F_{OD} = 2T\cos 45°$



$\tan \phi = 0.75$

$\phi = 36.87°$

Resolve vert.

$F \cos 36.87° = 20$

$F = \underline{25 \text{ kN}}$

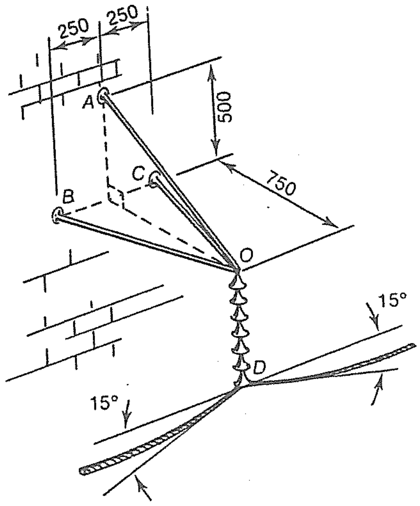
Resolve horiz.

$2T \cos 45 = F \sin 36.87°$

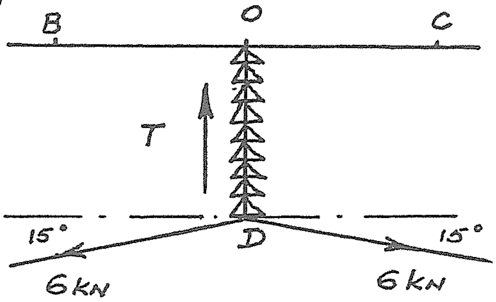
$$T = \frac{25 \times 0.6}{2 \times 0.7071}$$

$= \underline{10.6 \text{ kN}}$

7/28
A power line is suspended from the side of a building by the projecting framework shown. The tension in the power line is 6 kN at the insulator. Determine
(a) the tension in the insulator OD;
(b) the tension in the member OA;
(c) the compressive load in members OB and OC.



(a)



Resolve vert.

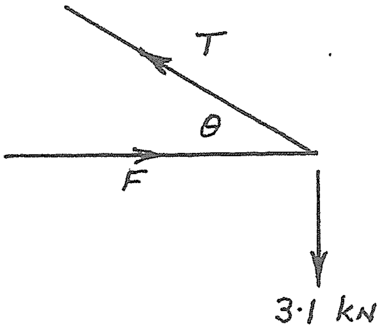
$$T_{OD} = 2 \times 6 \sin 15^\circ$$
$$= \underline{3.1 \text{ kN}}$$

(b)

$$\tan\theta = 0.6667$$
$$\theta = 33.69^\circ$$

Resolve vert.

$$T \sin 33.69^\circ = 3.1$$
$$T_{OA} = \underline{5.6 \text{ kN}}$$



Resolve horiz.

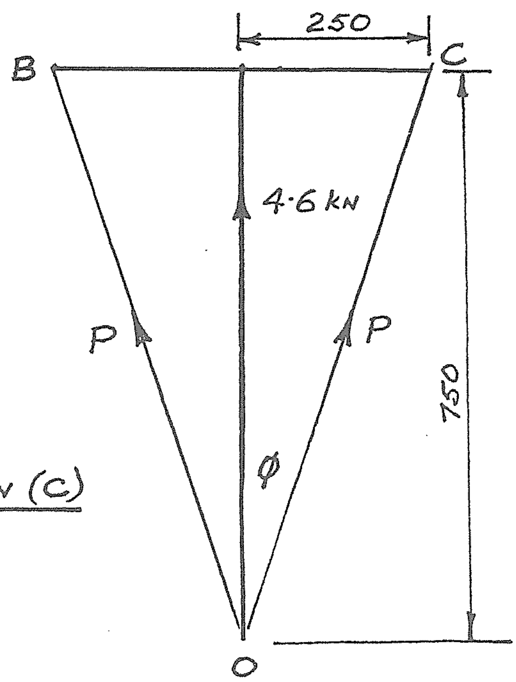
$$F = T\cos 33.69$$
$$= \underline{4.6 \text{ kN}}$$

(c)

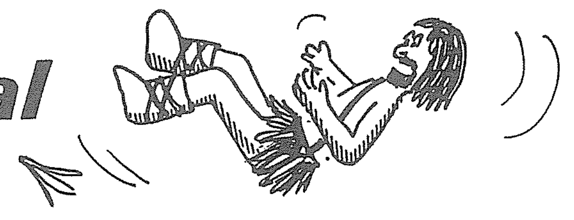
$$\tan\phi = 0.3333$$
$$\phi = 18.43^\circ$$

$$\cos\phi = 4.6/P$$
$$P = \underline{4.85 \text{ kN}}$$

$\therefore$ Load in OB and OC = $\underline{4.85 \text{ kN (C)}}$



8 Frictional Forces

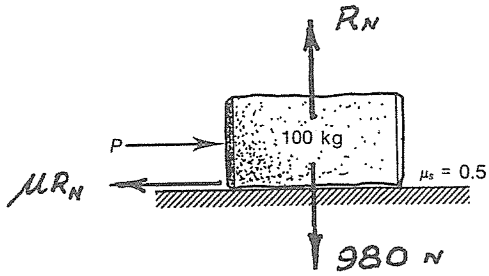


$$\mu = \frac{F}{N} = \tan\phi$$

$g = 9.8\ m/s^2$

8 FRICTIONAL FORCES
Frictional Forces: *Important Points About Frictional Forces.* Frictional Forces on Inclined Planes. *Angle of Friction.* Frictional Forces and Wedges.

8/11
A block of stone of mass 100 kg is resting on a horizontal concrete path. Determine the horizontal force P necessary to just cause the block to slide if the coefficient of static friction is 0.5.



Resolve vert.

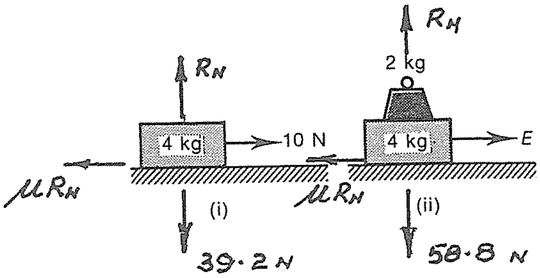
$R_N = 980\ N$

Resolve horiz.

$P = \mu R_N$

$= \underline{490\ N}$

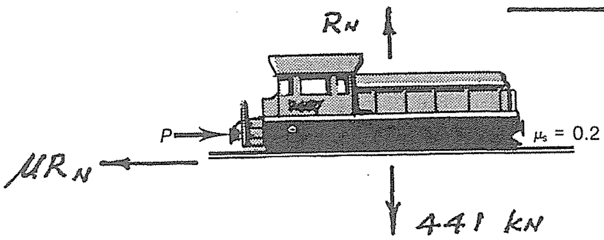
8/12
An effort of 10 N is required to cause the 4-kg block to be on the point of sliding.
(i) What is the coefficient of static friction present between the block and the plane?
(ii) Determine the effort E to just move the block if a 2-kg mass is placed on top of the 4-kg mass as shown in the diagram (b).



(i) Resolve vert. $R_N = 39.2$
Resolve horiz. $\mu R_N = 10$
$\mu = \underline{0.26}$

(ii) Resolve vert $R_N = 58.8$
Resolve horiz $E = \mu R_N$
$= \underline{15.3\ N}$

8/13
A diesel locomotive is stationary on the track. Given that the mass of the locomotive is 45 tonnes, find the greatest drawbar pull that the locomotive can exert if the coefficient of friction between the wheels and rails is 0.2.



Resolve vert.

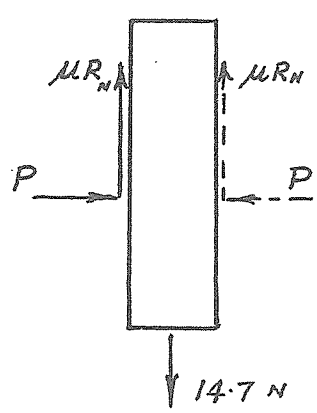
$R_N = 441$

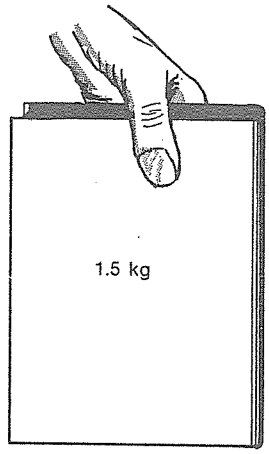
Resolve horiz.

$P = \mu R_N$

$= \underline{88.2\ kN}$

8/14
A book of mass 1.5 kg is held vertically and prevented from falling by being grasped by the fingers of one hand. What minimum clamping force must be exerted by the fingers if the coefficient of friction is 0.25?





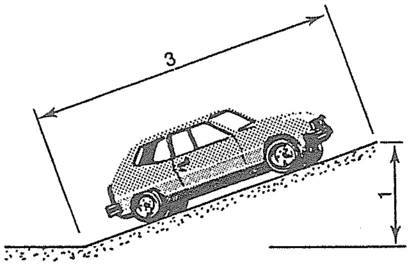
Resolve vert.

$$2 \times \mu R_N = 14{\cdot}7$$

$$2 \times 0{\cdot}25 \times P = 14{\cdot}7$$

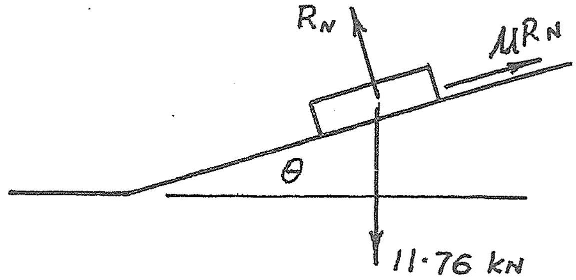
$$P = \underline{29{\cdot}4\ N}$$

8/15
A car of mass 1.2 tonnes is left stationary as shown on a concrete ramp of slope 1 in 3. The bonnet is bumped as the owner passes across in front of the car which then slides a short distance down the ramp. Determine the coefficient of static friction present between the wheels and the concrete.



$$\sin\theta = 0{\cdot}3333$$

$$\theta = 19{\cdot}47^\circ$$



Resolve $\perp$ to plane

$$R_N = 11{\cdot}76 \cos 19{\cdot}47^\circ$$

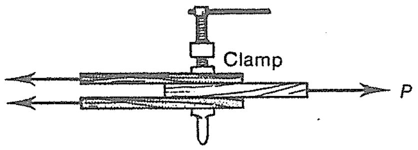
$$= 11{\cdot}08\ kN$$

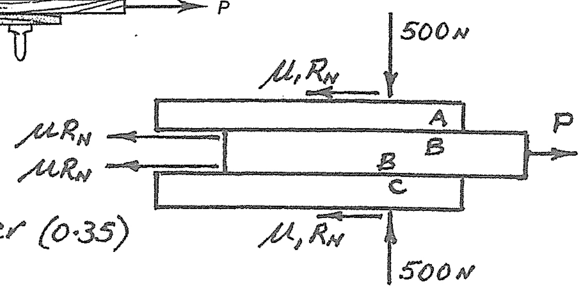
Resolve // to plane

$$\mu R_N = 11{\cdot}76 \sin 19{\cdot}47^\circ$$

$$\mu = \underline{0{\cdot}35}$$

8/16
A clamp exerts a normal force of 500 N on three pieces of wood A, B and C, held together as shown in the diagram. The coefficient of friction between the wooden pieces is 0.2 and between the wood and the clamp surfaces is 0.35. What is the maximum tensile force P that can be applied before sliding occurs?





Since μ for timber and timber (0·2) is less than μ for clamp and timber (0·35) piece B will slide before A or C

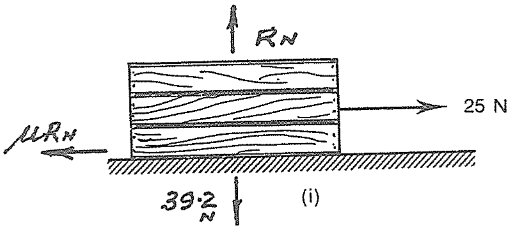
For limiting friction Resolve horiz.

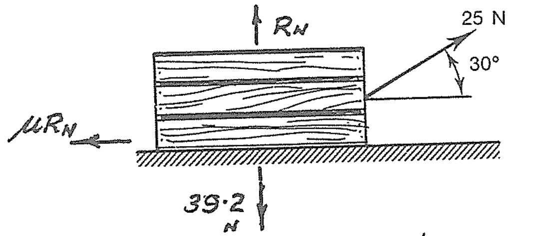
$$P = 2 \times \mu R_N$$

$$= 2 \times 0{\cdot}2 \times 500$$

$$= \underline{200\ N}$$

8/17
A small box of mass 4 kg is moved with constant velocity across a horizontal surface by a force of 25 N. Determine the coefficient of friction present and the frictional force operating, given that the 25-N force is applied
(i) horizontally;
(ii) at 30° to the horizontal.





i) Resolve vert.

$R_N = 39{\cdot}2$

Resolve horiz.

$\mu R_N = \underline{25\,N}$

$\mu = {}^{25}/_{39\cdot2}$

$= \underline{0{\cdot}64}$

(ii) Resolve vert.

$R_N + 25\sin 30^\circ = 39{\cdot}2$

$R_N = 26{\cdot}7$

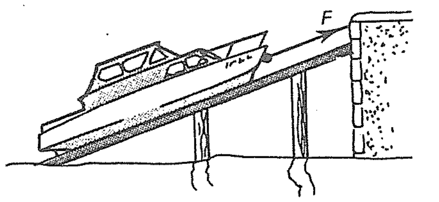
Resolve horiz.

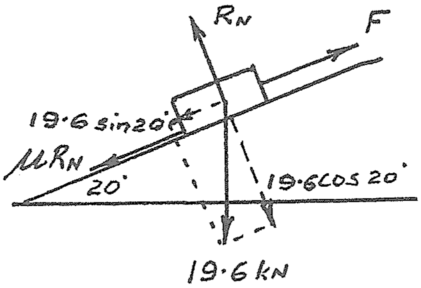
$\mu R_N = 25\cos 30^\circ = \underline{21{\cdot}65\,N}$

$\mu = 21{\cdot}65/26{\cdot}7$

$= \underline{0{\cdot}81}$

8/18
A boat of mass 2 tonnes rests on a slip which is inclined at 20° to the horizontal, the coefficient of static friction between the slip-rails and the boat being 0·3
Determine the least force, F, needed in the winch cable to move the boat
(i) up the slip, and
(ii) down the slip with constant velocity, given that the cable is parallel to the rails.





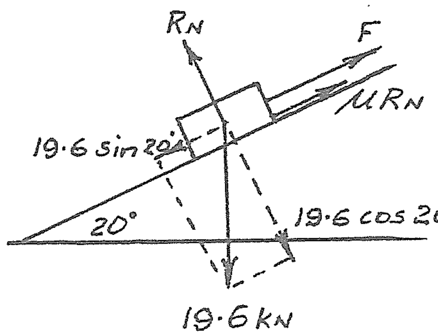
(i) Resolve ⊥ to plane

$R_N = 19{\cdot}6\cos 20^\circ$

Resolve // to plane

$F = \mu R_N + 19{\cdot}6\sin 20^\circ$

$= \underline{12{\cdot}23\ kN}$



(ii) Resolve ⊥ to plane

$R_N = 19{\cdot}6\cos 20^\circ$

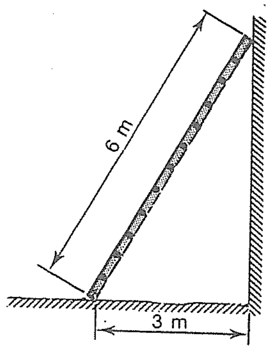
Resolve // to plane

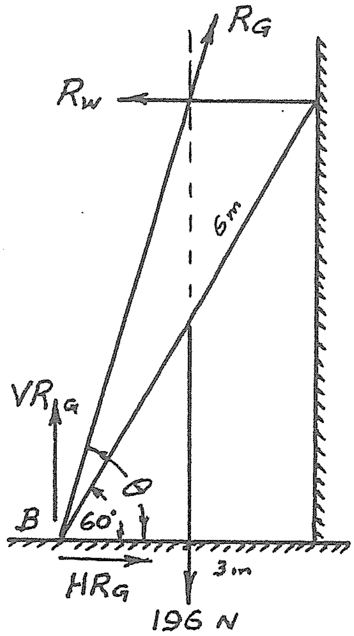
$F + \mu R_N = 19{\cdot}6\sin 20^\circ$

$F = \underline{1{\cdot}17\ kN}$

8/19

A ladder of mass 20 kg rests against a smooth vertical wall and on a rough concrete path as shown. Draw a free-body diagram of the ladder in this situation and determine the reactions at the wall and the path.





Resolve vert.

$VR_G = 196$

Resolve horiz.

$HR_G = R_W$

Take moments about B

$R_W \times 6 \sin 60° = 196 \times 1{\cdot}5$

$R_W = \underline{56{\cdot}58\ N} \leftarrow$

$R_G = \sqrt{196^2 + 56{\cdot}58^2}$

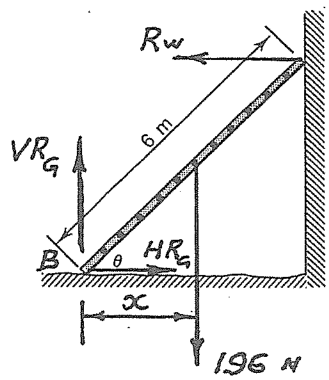
$= \underline{204\ N \text{ at } 73{\cdot}89°}$

$\tan\theta = 196/56{\cdot}5$

$\theta = \underline{73{\cdot}89°}$

8/20

The ladder in Problem 8/19 is now repositioned by moving its end further away from the vertical wall. What is the minimum angle θ possible between the concrete path and the ladder before the ladder slips under its own weight? The coefficient of static friction between ladder and path is 0.6.



<u>Note</u>: $HR_G = \mu R_N$

$x = 3\cos\theta$

Resolve vert.

$VR_G = 196$

Resolve horiz.

$HR_G\ (\mu R_N) = R_W$

Take moments about B

$R_W \times 6 \sin\theta = 196 \times 3\cos\theta \cdots\cdots$ (1)

But $R_W = \mu R_{N_G} = 0{\cdot}6 \times VR_G = 0{\cdot}6 \times 196$

Subst. in (1) $\quad 0{\cdot}6 \times 196 \times 6\sin\theta = 196 \times 3\cos\theta$

Divide by $\cos\theta$

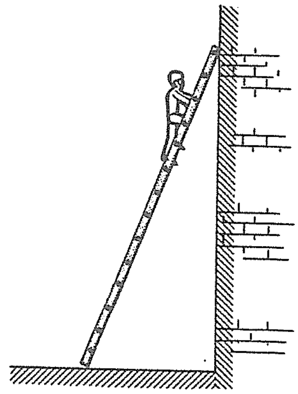
$0{\cdot}6 \times 196 \times 6\tan\theta = 196 \times 3$

$\therefore \tan\theta = \dfrac{196 \times 3}{0{\cdot}6 \times 196 \times 6}$

$\theta = \underline{39{\cdot}8°}$

8/21

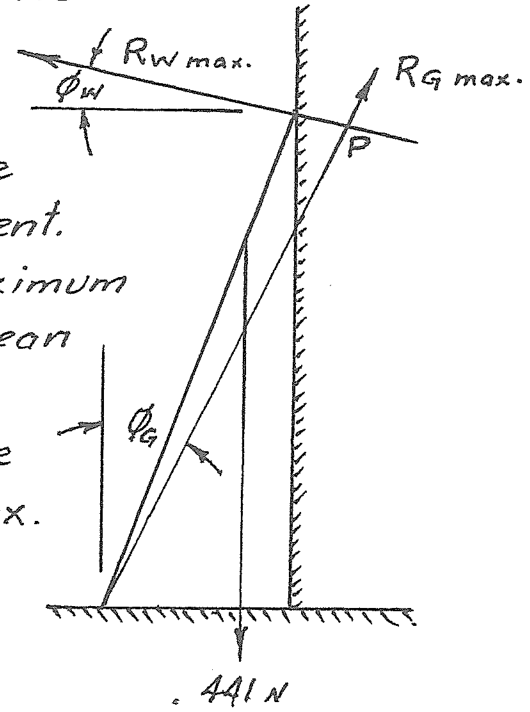
A boy of mass 45 kg places a 7-metre long ladder against a house with the bottom of the ladder 2.5 metres away from the wall. Assume a coefficient of friction at the bottom to be 0.50 and 0.2 between the top of the ladder and the wall. Is the ladder safe against slipping when the boy is three-quarters of the way to the top? Suppose the ladder is at 45° angle; is it safe against slipping? Solve this problem graphically.



Graphically

Scale : 10 mm = 1 m

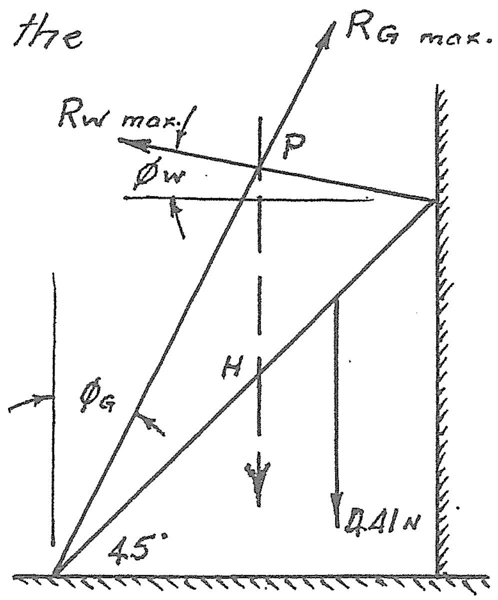
The angle of friction for the wall and ladder, $\phi_W = 11 \cdot 3^\circ$, and for the ladder and ground, ϕ_G $26 \cdot 6^\circ$



For 3 force equilibrium, the force lines of action must be concurrent. With the given values of μ, maximum values for the friction forces mean R_W and R_G will meet at P. This allows the boy to climb to the top without exceeding the max. friction force values.

For the new angle position of the ladder, the mass force line of action cannot pass through P, so the ladder will slip.

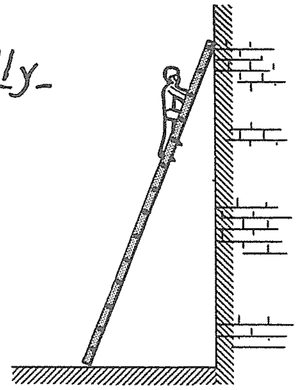
The maximum distance to which the boy can safely climb is height H.

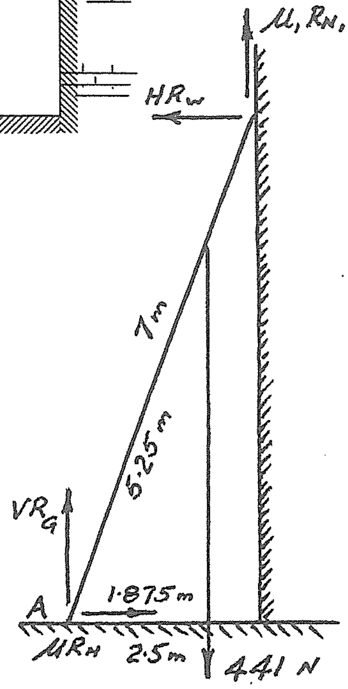


8/21

A boy of mass 45 kg places a 7-metre long ladder against a house with the bottom of the ladder 2.5 metres away from the wall. Assume a coefficient of friction at the bottom to be 0.50 and 0.2 between the top of the ladder and the wall. Is the ladder safe against slipping when the boy is three-quarters of the way to the top? Suppose the ladder is at 45° angle; is it safe against slipping? Solve this problem graphically.

Analytically





Resolve horiz.

$HR_W = \mu R_N$

$= 0{\cdot}5 \times VR_G$

$\therefore VR_G = 2HR_W$

Resolve vert.

$\mu_1 R_{N_1} + VR_G = 441 \cdots\cdots (1)$

$0{\cdot}2 \times HR_W + 2HR_W = 441$

$HR_W = 200\ N$

From (1) $\mu_1 R_{N_1} + 2HR_W = 441$

$\mu_1 R_{N_1} = 41\ N$

Take moments about A

$441 \times 1{\cdot}875 = 826{\cdot}87$ Nm clockwise

and $41 \times 2{\cdot}5 + 200 \times 6{\cdot}5 = 1402{\cdot}5$ Nm anti-clockwise.

In fact these are for max. values, and the anti-clockwise moment will not exceed the clockwise moment. $\therefore$ The ladder is safe.

Resolve horiz.

$HR_W = \mu R_N = 0{\cdot}5 \times VR_G$

Resolve vert.

$\mu_1 R_{N_1} + VR_G = 441$

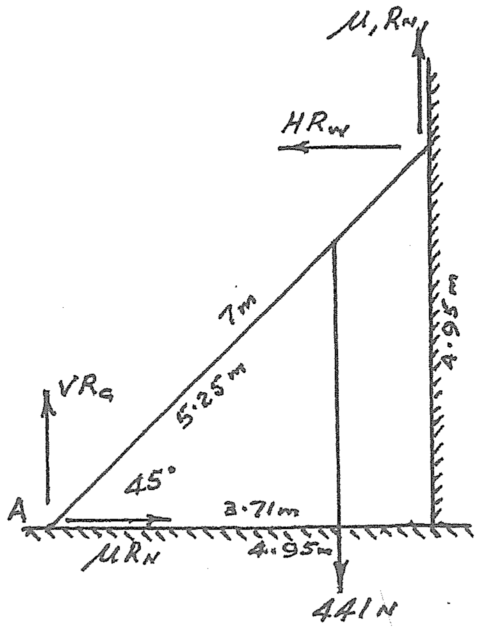
$0{\cdot}2\ HR_W + 2\ HR_W = 441$

$HR_W = 200\ N$

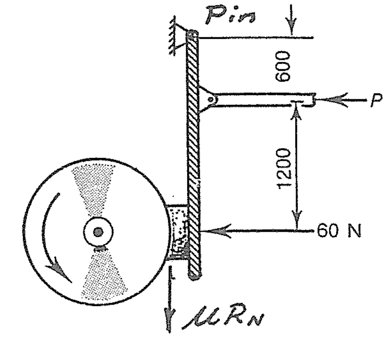
and $\mu_1 R_{N_1} = 41\ N$

Take moments about A

The anti-clockwise moment cannot match the clockwise moment $\therefore$ Ladder will slip.



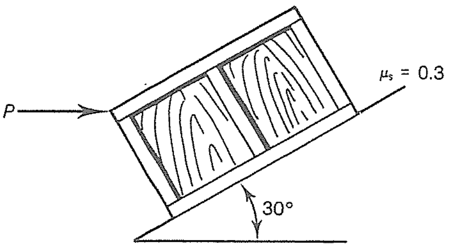
8/22
A brake pad operated by the lever system shown is pressed against a wheel with a force of 60 N. If the coefficient of friction between the brake pad and the wheel is 0.6,
(i) determine the tangential frictional force tending to stop the wheel, and
(ii) determine the force P necessary to produce this braking force.



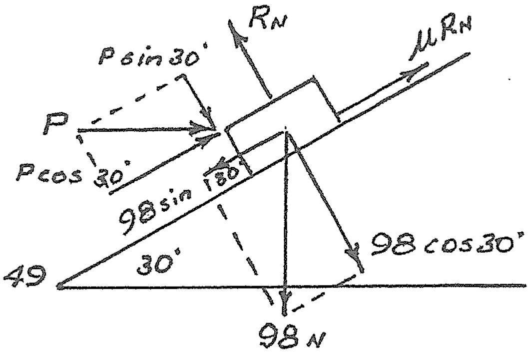
(i) $\mu R_N = 0.6 \times 60$
$= \underline{36\text{ N}}$

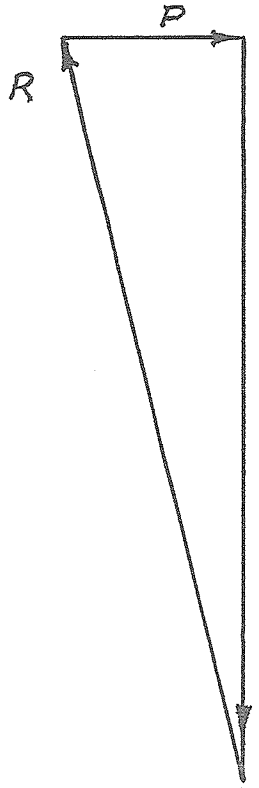
(ii) Take moments about Pin
$P \times 600 = 60 \times 1800$
$P = \underline{180\text{ N}}$

8/23
A box of mass 10 kg rests on a plane inclined at 30° to the horizontal as shown. If the coefficient of static friction between box and plane is 0.3, determine the magnitude of the horizontal force P that prevents the box sliding down the plane. [*Hint*: Solve graphically.]



Resolve ⊥ to plane
$R_N = P\sin 30° + 98\cos 30°$ ---①
Resolve ∥ to plane
$P\cos 30° + \mu R_N = 98\sin 30°$ ---②
From ① $R_N = (0.5P + 84.87)$
Subst. in ② $0.3(0.5P + 84.87) + 0.866P = 49$
$P = \underline{23.2\text{ N}}$

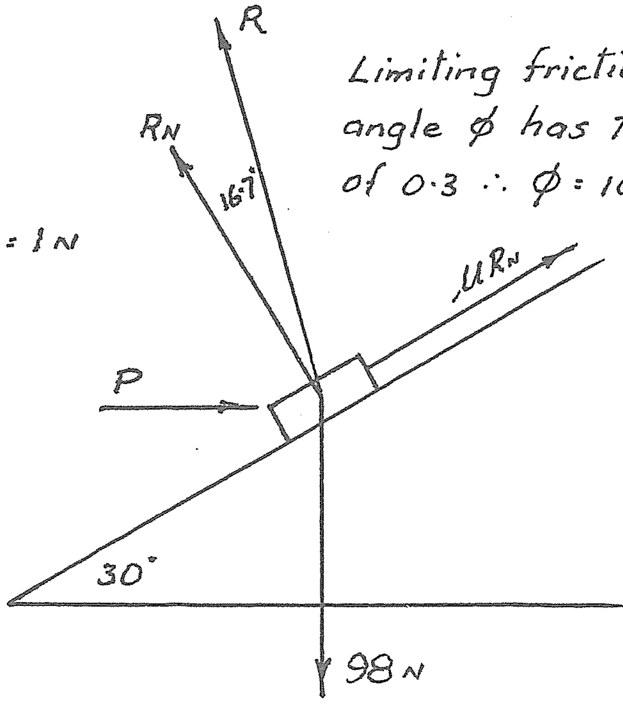




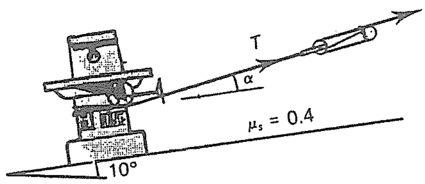
Scale – 1mm = 1 N

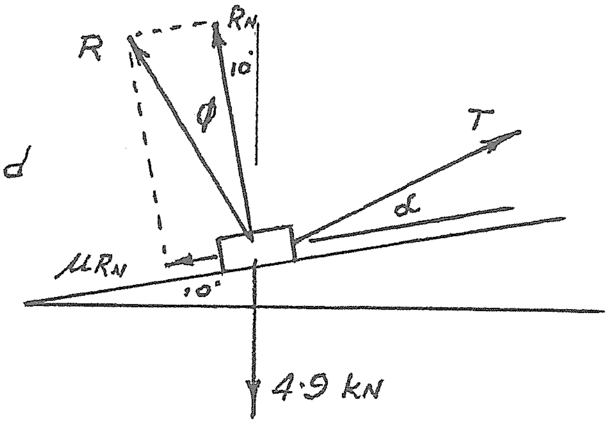
$P = \underline{23.5\text{ N}}$

Limiting friction angle ϕ has tan of 0.3 ∴ $\phi = 16.7°$



8/24
A machine of mass 500 kg is being dragged up a ramp by a block and tackle as shown. The coefficient of friction between the machine and ramp is 0.4 and the ramp is inclined at 10° to the horizontal. Determine, when motion is about to occur up the plane,
(i) the angle α between rope and ramp necessary to produce the least possible tension in the rope;
(ii) the rope tension at this instant.

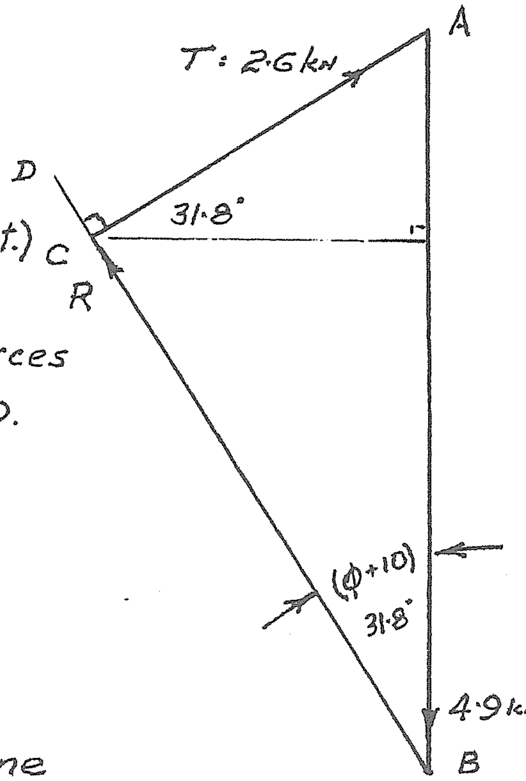




R is the resultant of R_N and μR_N and the angle ϕ is the angle of limiting friction with a tan of 0·4

i.e. $\phi = \underline{21 \cdot 8^\circ}$

<u>Scale</u> – 20 mm = 1 kN



In $\triangle$ ABC :

AB = 4·9

BD is parallel to R (31·8° to vert.)

The least value for T to complete the triangle of forces is AC perpendicular to BD.

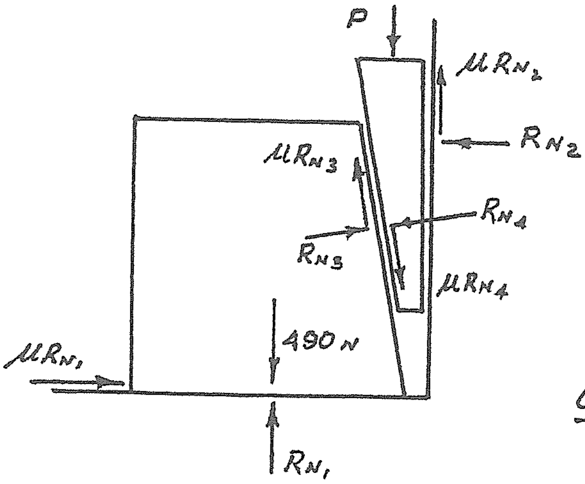
In $\triangle$ ABC

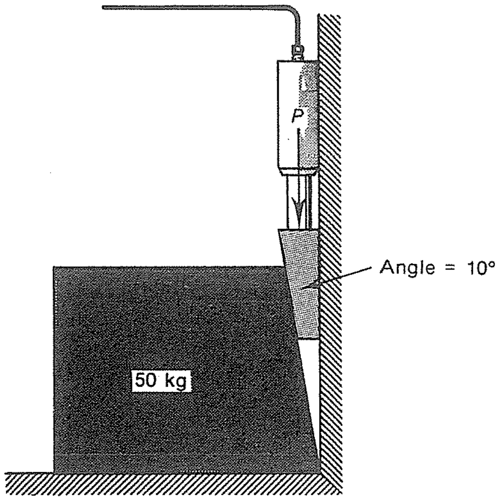
$\sin 31 \cdot 8^\circ = T/4 \cdot 9$

$T = \underline{2 \cdot 58 \text{ kN}}$

at $\underline{21 \cdot 8^\circ}$ (ϕ) <u>to plane</u>

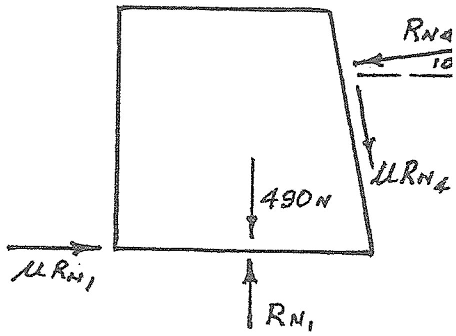
8/25
In a certain mechanism a hydraulic ram is used to apply a force P to the end of a wedge-shaped slider as shown. This slider then moves a block of mass 50 kg horizontally across a surface as shown. Determine the value of the force P necessary to put the system on the point of motion, given the coefficient of static friction is 0.23 for all surfaces in contact. (Take $\tan^{-1} 0.23 = 13°$.)





$R_{N_3} = R_{N_4}$

<u>Consider the forces on the Block</u>



Resolve horiz

$\mu R_{N_1} + \mu R_{N_4} \sin 10^\circ = R_{N_4} \cos 10^\circ \cdots (1)$

Resolve vert.

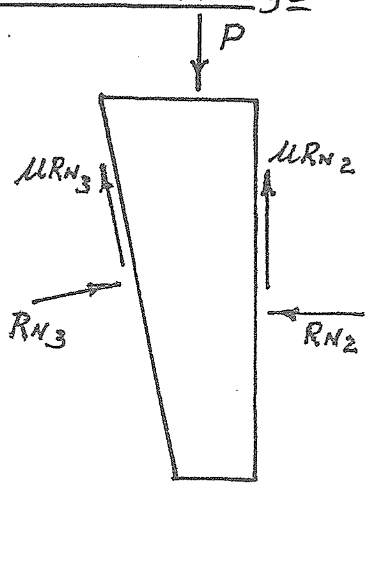
$R_{N_1} = \mu R_{N_4} \cos 10^\circ + R_{N_4} \sin 10^\circ + 490$

$= 0.4 R_{N_4} + 490$

Subst. in (1) $\quad 0.23(0.4 R_{N_4} + 490) + R_{N_4}(0.0399 - 0.9848) = 0$

$R_{N_4} = \underline{132.137 \text{ N}}$

<u>Consider the forces on the Wedge</u>



Resolve horiz.

$R_{N_2} = R_{N_3} \cos 10^\circ - \mu R_{N_3} \sin 10^\circ$

$= (132.137 \times 0.9848) - (0.23 \times 132.137 \times 0.1736)$

$= \underline{124.853 \text{ N}}$

Resolve vert.

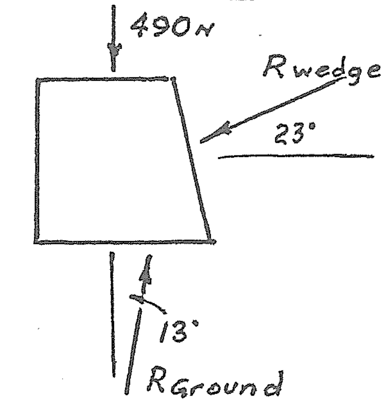
$P = \mu R_{N_2} + \mu R_{N_3} \cos 10^\circ + R_{N_3} \sin 10^\circ$

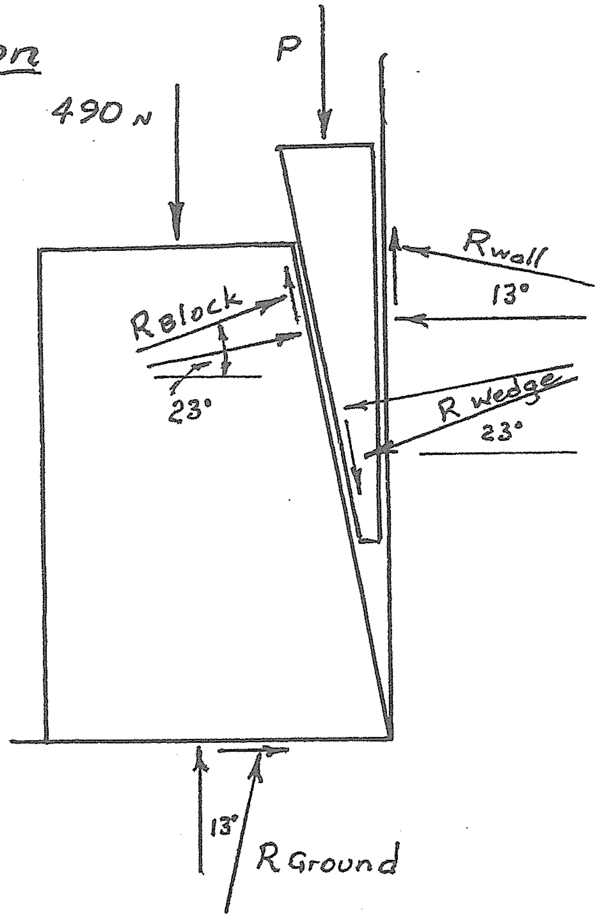
$= 28.716 + 29.929 + 22.939$

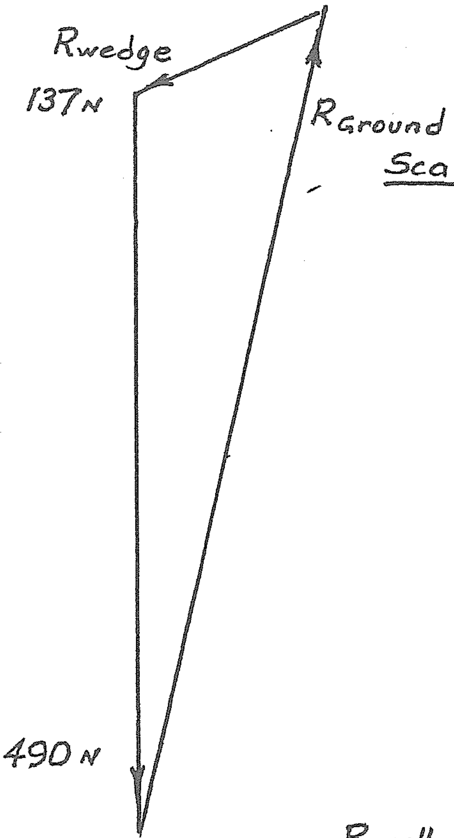
$= \underline{81.58 \text{ N}}$

8/25 **Graphical Solution**

Forces on the Block

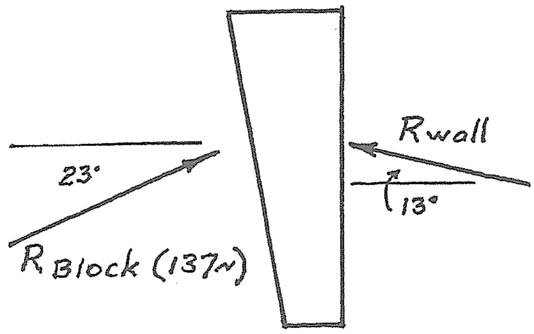


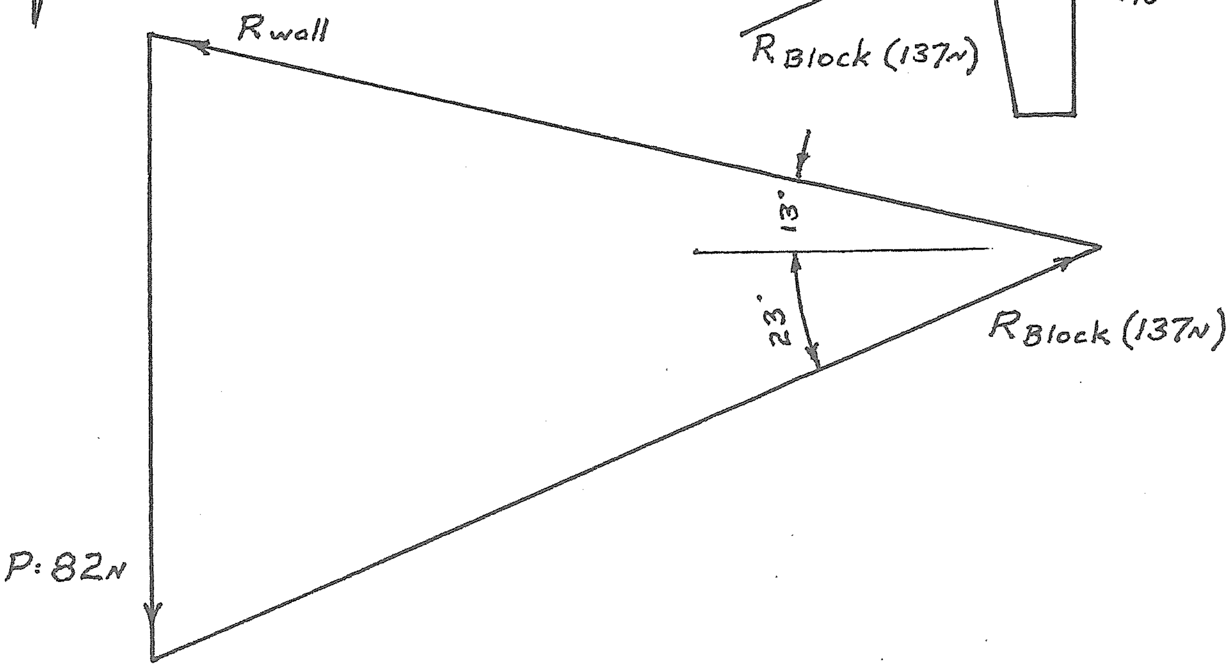




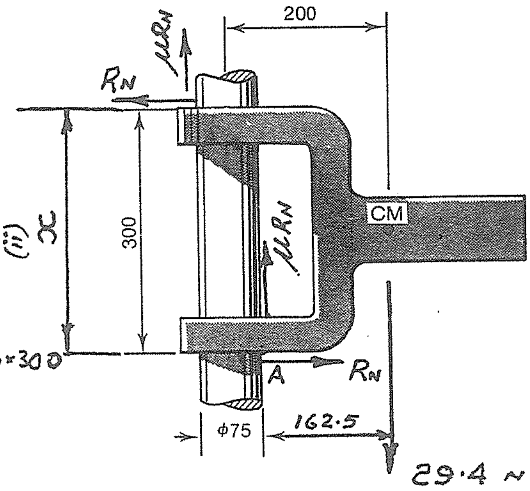
Scale - 1mm = 5N

Forces on the Wedge





8/26
A metal bracket is free to slide on a pipe as shown.
(i) If the mass of the bracket is 3 kg will it slide down the pipe under its own weight if the coefficient of static friction is 0.45?
(ii) What maximum length of bearing will just prevent the bracket sliding under its own weight?



(i) Take moments about A

$29{\cdot}4 \times 162{\cdot}5 + 0{\cdot}45 R_N \times 75 = R_N \times 300$

$266{\cdot}25 R_N = 4777{\cdot}5$

$R_N = 17{\cdot}94$

$\therefore \mu R_N = 8{\cdot}07\ N$

Resolve vert. $2\mu R_N = 16{\cdot}14\ N \uparrow$ and $29{\cdot}4\ N \downarrow$

$\therefore$ It will slide

(ii) If slip is not to occur

$2\mu R_N > 29{\cdot}4$

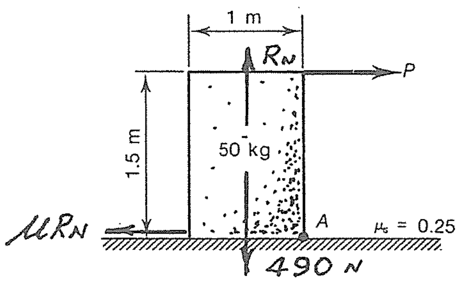
thus $R_N \geq 32{\cdot}67\ N$ and $\mu R_N \geq 14{\cdot}7\ N$

Take moments about A

$29{\cdot}4 \times 162{\cdot}5 + 14{\cdot}7 \times 75 = 32{\cdot}67 x$

$x = 180\ mm$

8/27
Determine the value of the force *P* which will just cause the 50-kg block to move. The coefficient of static friction between the block and the plane is 0.25. [*Hint*: Motion may occur by either *sliding*, or by *tipping* about the corner *A*.]



Resolve vert.

$R_N = 490$

Resolve horiz.

$\mu R_N = P$

$\therefore P = 122{\cdot}5\ N$ for sliding

Take moments about A

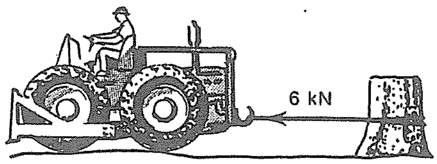
$490 \times 0{\cdot}5 = P \times 1{\cdot}5$

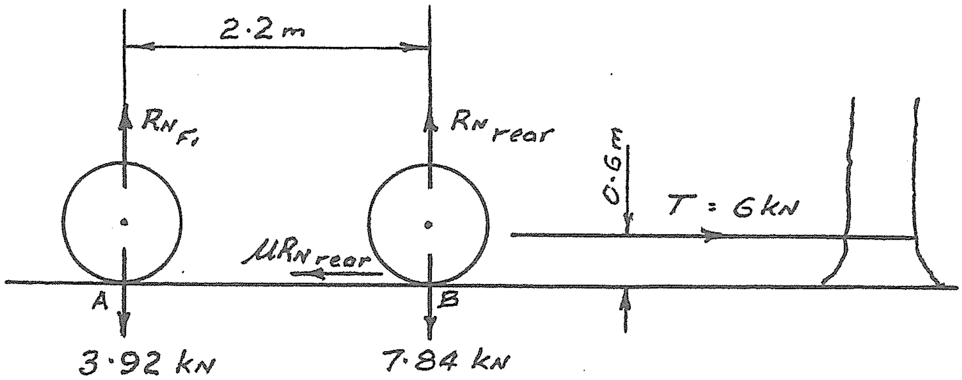
$\therefore P = 163{\cdot}3\ N$ to tip

$\therefore$ The block will slide

8/28

A tractor is used to pull out stumps. The pulling chain is horizontal and 0.6 metres above the ground. The tractor has a mass of 1200 kg of which 400 kg is on the front axle and 800 kg on the rear axle, when it is not pulling. The two axles are 2.2 metres apart. If the chain is pulled with a 6-kN force, then what is the vertical load on each axle? What coefficient of friction is needed between the rear driving tyres and the ground if no slipping is to occur?





Take moments about A

$$R_{N_R} \times 2.2 = 7.84 \times 2.2 + 6 \times 0.6$$

$$R_{N_R} = \underline{9.47 \text{ kN}}$$

Take moments about B

$$R_{N_F} \times 2.2 + 6 \times 0.6 = 3.92 \times 2.2$$

$$R_{N_F} = \underline{2.28 \text{ kN}}$$

Resolve horiz.

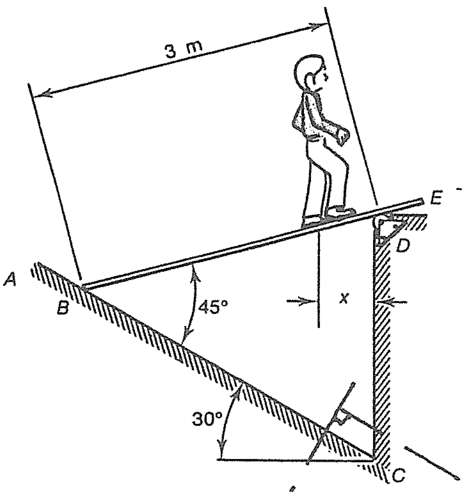
$$\mu R_{N_R} = 6$$

$$\mu = 6/9.47$$

$$= \underline{0.63}$$

8/29
A man of mass 81.6 kg walks up a plank *BE* of negligible weight, which rests with the end *B* on a ramp sloping at 30° to the horizontal. It rests on a frictionless roller at *D*, 3 m along the plank from *B*. What must be the minimum value of the coefficient of static friction, if the plank does not slide *down* the slope when the man steps on to the plank at *B*?

Supposing that the man can reach a position *x* from the wall *DC* before the plank will slide *up* the slope, and the coefficient of friction is still the same, *show by a sketch* the forces acting on the plank, and determine the distance *x*. Is it ever possible for the man to reach *D*?



As man steps onto plank at B

Take moments about B

$R_D \times 3 = 0 \quad \therefore R_D = 0$

Resolve $\perp$ to BC

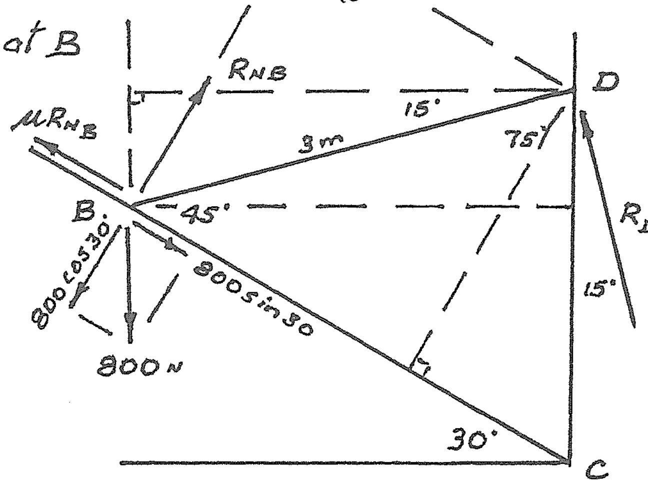
$R_{NB} = 800 \cos 30^\circ$

$= 692.8 \text{ N}$

Resolve $/\!/$ to BC

$\mu R_{NB} = 800 \sin 30$

$\mu = \underline{0.58}$



Take moments about B

$800 \cdot y = 3 R_D \cdots\cdots (1)$

Resolve vert.

$R_D \cos 15^\circ + R_B \cos 60^\circ = 800 \cdots (2)$

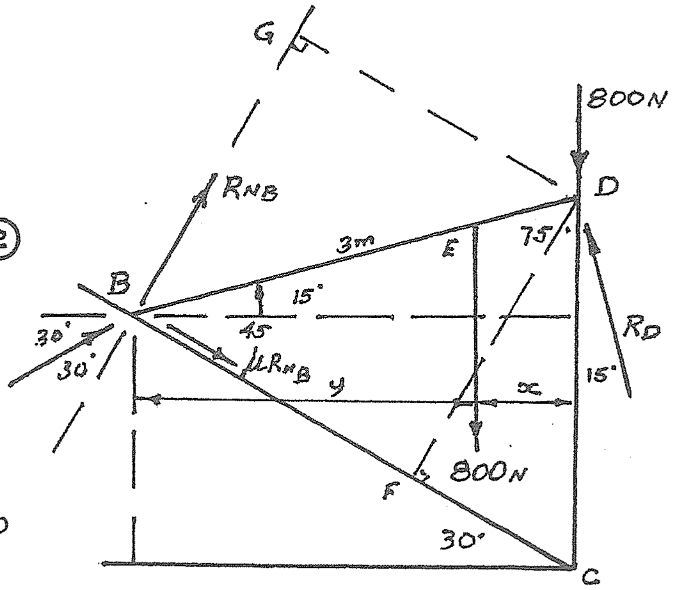
Resolve horiz.

$R_B \cos 30^\circ = R_D \sin 15^\circ$

$R_B = \dfrac{R_D \sin 15^\circ}{\cos 30^\circ}$

Subst. in (2)

$R_D \cos 15^\circ + \dfrac{R_D \sin 15^\circ}{\cos 30^\circ} \cos 60^\circ = 800$



$R_D = 717.3 \text{ N}$

From (1) $\quad y = 2.69 \text{ m}$

$\therefore x = 3\cos 15 - 2.69$

$= 0.208 \text{ m}$

or $\underline{208 \text{ mm}}$.

Take moments about D

$R_{NB} \times GD = \mu R_{NB} \times FD$

$GD = FD$

$\therefore \mu = 1 \qquad \underline{\text{No}}$

With $\mu = 0.58$ the end of the plank at B will slide up

9 Linear Motion

9 LINEAR MOTION

The Motion of Bodies. Displacement (s). Velocity (v). Acceleration (a). *Acceleration due to Gravity* (g). Equations Describing Straight-line (Rectilinear) Motion. *Sign Convention.* Problem Solving. Graphical Solutions in Kinematics: *Displacement–Time Graphs, Velocity–Time Graphs. Examples of More Complex Problems.*

$$v \text{ (average)} = \frac{u+v}{2}$$
$$v = u + at$$
$$v^2 = u^2 + 2as$$
$$s = ut + \tfrac{1}{2}at^2$$

$g = 9.8 \text{ m/s}^2$

9/16
A man walks for 35 minutes at 6.5 km/h and then 45 minutes at 4.5 km/h. Determine his average speed (in m/s).

6.5 km/hr = 108.33 m/min.
Distance walked = 108.33 × 35
= 3791.55 m

4.5 km/hr = 75 m/min.
Distance walked = 75 × 45 = 3375 m
Total distance = 7166.55 m
Av. vel. = 7166.55/4800 = 1.49 m/s

9/17
A motorcycle travelling along a straight road accelerates from 18 km/h to 108 km/h in 10 seconds. Calculate its average acceleration in m/s^2.

18 km/hr = 5 m/s
108 km/hr = 30 m/s

u = 5 m/s
v = 108 m/s
a = ?
s = —
t = 10s

$v = u + at$
$30 = 5 + 10a$
$a = 2.5 \text{ m/s}^2$

9/18
A car's speed increases uniformly from 36 km/h to 108 km/h in 30 seconds. Calculate
(i) the average speed (in m/s) for the 30-second period;
(ii) the acceleration operating (in m/s^2);
(iii) the distance travelled during the 30-second period.

36 km/hr = 10 m/s
108 km/hr = 30 m/s

(i) Average speed = $\frac{10+30}{2}$ = 20 m/s
(ii) u = 10 m/s
v = 30 m/s
a = ?
s = —
t = 30s

$v = u + at$
$30 = 10 + 30a$
$a = 0.67 \text{ m/s}^2$

(iii) Dist. = Av. speed × time = 20 × 30 = 600 m

9/19
A car travelling at 50 km/h has its brakes applied and slows to 40 km/h in 10 metres. Given the same rate of braking, how much further (in metres) will the car run before stopping?

50 km/hr : 13·89 m/s
40 km/hr : 11·11 m/s

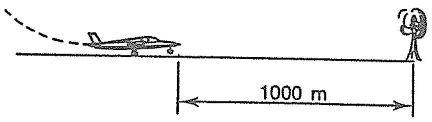
u : 13·89 m/s
v : 11·11 m/s
a : ?
s : 10 m
t : –

$v^2 = u^2 + 2as$
$123.43 = 192.89 + 20a$
$a = -3.47\ m/s^2$

u : 11·11 m/s
v : 0
a : –3·47 m/s
s : ?
t : —

$v^2 = u^2 + 2as$
$0 = 123.43 - 6.94s$
$s = \underline{17.8\ m}$

9/20
A jet fighter touches down on an airstrip and decelerates at 4 m/s² for 20 seconds before coming to rest. If the pilot touches down 1000 m from the trees at the end of the runway, will he crash?



u : ?
v : 0
a : $-4\ m/s^2$
s : –
t : 20s.

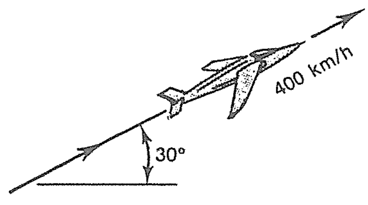
$v = u + at$
$0 = u - 4 \times 20$
$u = 80\ m/s$

u : 80 m/s
v : 0
a : $-4\ m/s^2$
s : ?
t : —

$v^2 = u^2 + 2as$
$0 = 6{,}400 - 8s$
$s = \underline{800\ m}$

<u>NO</u>

9/21
An aircraft travelling at 400 km/h is climbing at an angle of 30 degrees to the horizontal.
(i) What is its ground speed?
(ii) How high will it climb in 30 seconds assuming its rate of climb and speed remain constant?

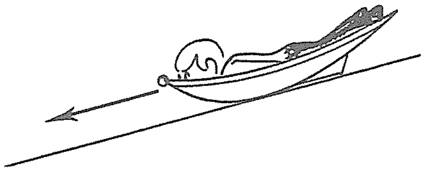


(i) Ground speed = 400 cos30° = <u>346·4 km/hr.</u>

(ii) Vert. component = 400 sin 30°
= 200 km/hr
or 55·55 m/s.

In 30s plane climbs 30 × 55·55 or <u>1·67 km</u>

9/22
A boy on a snow-sled starts from rest down a run which has a uniform gradient. If the sled travels 10 metres in 4 seconds how long will it take to reach a velocity of 25 m/s down the plane?



$u : 0$
$v : -$
$a = ?$
$s = 10\,m$
$t : 4\,s$

$s : ut + \frac{1}{2}at^2$
$10 : 0 + \frac{1}{2}a \times 16$
$8a = 10$
$a : \underline{1{\cdot}25\,m/s^2}$

$u : 0$
$v : 25\,m/s$
$a : 1{\cdot}25\,m/s^2$
$s = -$
$t = ?$

$v = u + at$
$25 : 0 + 1{\cdot}25t$
$t = \underline{20\,s}$

9/23
An astronaut making repairs to his space vehicle drops a wrench from a height of 15 metres. With what velocity does the wrench strike the surface if the space vehicle is standing on (i) the earth and (ii) the moon? Take $g = 1.7$ m/s^2 on the moon.

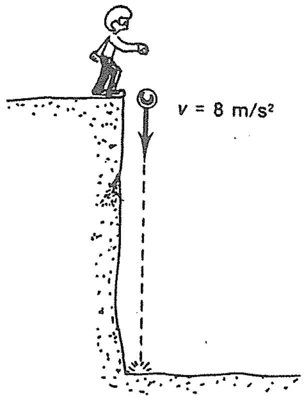
(i) $u : 0$
$v : ?$
$a : 9{\cdot}8\,m/s^2$
$s : 15\,m$
$t : -$

$v^2 : u^2 + 2as$
$: 0 + 294$
$v : \underline{17{\cdot}15\,m/s}$

(ii) $u : 0$
$v : ?$
$a : 1{\cdot}7\,m/s^2$
$s : 15\,m$
$t : -$

$v^2 : u^2 + 2as$
$: 0 + 51$
$v : \underline{7{\cdot}14\,m/s}$

9/24
A stone thrown vertically down from a cliff with an initial velocity of 8 m/s hits the ground in 2 seconds. What is the height of the cliff and what is the striking velocity of the stone?



$u : 8\,m/s$
$v = ?$
$a : 9{\cdot}8\,m/s^2$
$s : ?$
$t : 2\,s$

$s : ut + \frac{1}{2}at^2$
$: 8 \times 2 + \frac{1}{2} \times 9{\cdot}8 \times 4$
$: \underline{35{\cdot}6\,m}$

$v : u + at$
$: 8 + 9{\cdot}8 \times 2$
$: \underline{27{\cdot}6\,m/s}$

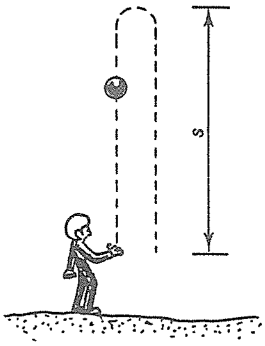
9/25
Fred's favourite football team, the Roosters, call for an "up and under". Fairfax obliges with a punt that gives the ball a velocity of 25 m/s. How much time have the players to get under the ball before it returns to the ground?

$u = 25\,m/s$
$v : 0$
$a : -9{\cdot}8\,m/s^2$
$s : -$
$t : ?$

$v : u + at$
$0 : 25 - 9{\cdot}8t$
$t : 2{\cdot}55\,s$ to reach max. height.

and <u>5·1 s to return to ground.</u>

9/26
A ball thrown vertically upwards returns to the thrower in three seconds. Find its initial upward velocity and the height to which it rises.

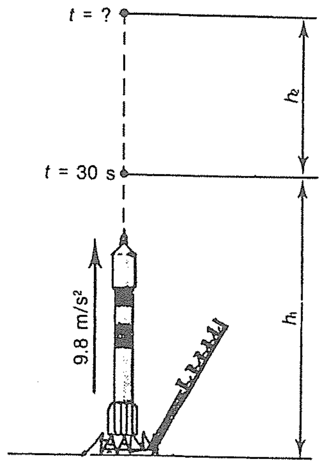


$u = ?$
$v = 0$
$a = -9.8\ m/s^2$
$s = ?$
$t = 1.5s$

$v = u + at$
$0 = u - 1.5 \times 9.8$
$u = \underline{14.7\ m/s}$

$s = ut + \frac{1}{2}at^2$
$= 14.7 \times 1.5 - \frac{1}{2} \times 9.8 \times 2.25$
$= \underline{11.025\ m}$

9/27
A rocket is fired vertically and ascends with a constant resultant vertical acceleration of 9.8 m/s^2 for 30 seconds, after which its fuel is exhausted. Assuming that the rocket continues to rise vertically, calculate the maximum height reached and the time taken for the flight.



$u = 0$
$v =$
$a = 9.8\ m/s^2$
$s = ?$
$t = 30s.$

$s = ut + \frac{1}{2}at^2$
$= 0 + \frac{1}{2} \times 9.8 \times 30^2$
$= 4410\ m$
$v = u + at$
$= 0 + 9.8 \times 30$
$= 294\ m/s$

$u = 294\ m/s$
$v = 0$
$a = -9.8\ m/s^2$
$s = ?$

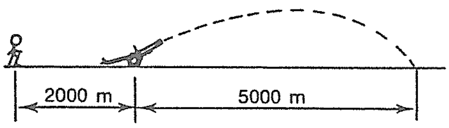
$v^2 = u^2 + 2as$
$0 = 86436 - 2 \times 9.8s$
$s = 4410\ m.$

$v = u + at$
$0 = 294 - 9.8t$
$t = 30s.$

$\therefore$ Total time $= \underline{60s.}$ Max. ht. $= \underline{8820\ m}$

9/28
A man standing behind a gun hears the report of the gun and sees the shell-burst at the same instant. The shell, gun and man are in the same straight line, the man being 2000 metres behind the gun and the shell-burst occurring 5000 metres in front of the gun.

Given that sound travels at 335 m/s, determine the horizontal velocity of the shell.

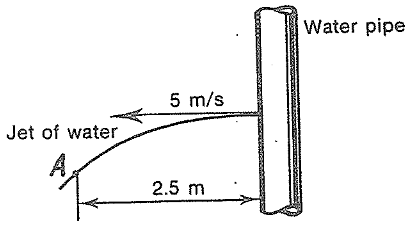


Time for sound to reach man
$= 2000/335$
$= 5.97$ secs.

$\therefore$ Time from firing to shell burst $= 5.97$ secs
The horizontal vel. of shell is constant
Vel. $= 5000/5.97$

$= \underline{837.5\ m/s}$

9/29
A jet of water issues from a small hole in the vertical pipe. If the initial velocity of the jet is 5 m/s horizontally, determine the vertical distance through which it will have fallen when its horizontal distance from the pipe is 2.5 m.



The horiz. vel. of the water jet is constant at 5 m/s

$\therefore$ time to reach A : $2{\cdot}5/5 = 0{\cdot}5\,s$

For vert. motion

$u : 0$

$v : -$

$a : 9{\cdot}8\ m/s^2$

$s : ?$

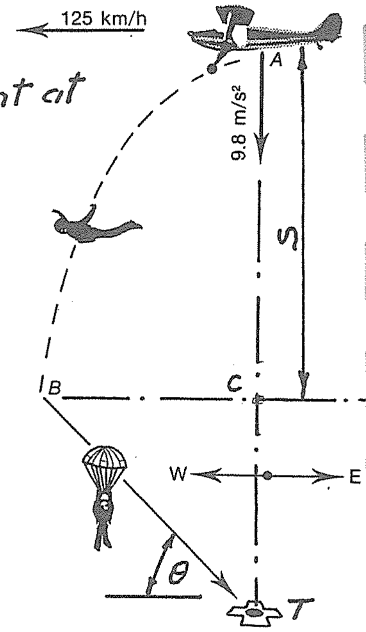
$t : 0{\cdot}5\,s$

$s = ut + \frac{1}{2}at^2$

$= 0 + 1/2 \times 9{\cdot}8 \times 0{\cdot}25$

$= \underline{1{\cdot}225\ m}$

9/30
A parachutist exits from an aircraft that is in level flight at an altitude of 1000 metres and free-falls in a flat stable attitude for 10 seconds. If the aircraft is travelling with a ground speed of 125 km/h and is directly over the target at the time of exit, determine the position of the parachutist relative to the target at the end of his free fall. Assume that it is a windless day.



Horiz. vel. is constant at 125 km/hr or 34·7 m/s

$\therefore$ BC : 34·7 m/s for 10 s. = 347 m

$u : 0$

$v : -$

$a : 9{\cdot}8\ m/s^2$

$s : ?$

$t : 10\,s$

$s = ut + \frac{1}{2}at^2$

$= 0 + 1/2 \times 9{\cdot}8 \times 100$

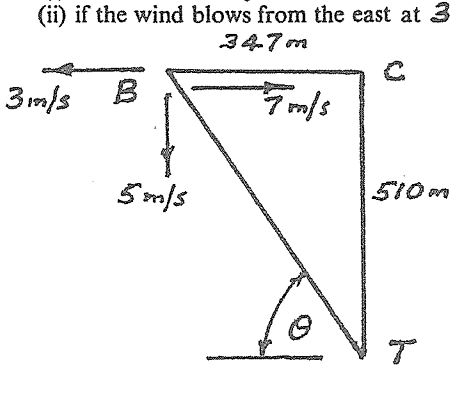
$= 490\ m.$

$\therefore$ Ht. above target = 510 m.

<u>Pos. is 510 m high & 347 m west</u>

9/31
After his 10-second free fall, the sport parachutist in Problem 9/30 pulls his ripcord and opens his parachute at point *B*. The canopy is designed to expel air from the rear, so providing the parachutist with a maximum horizontal air-speed of 7 m/s.

If his vertical rate of descent is a uniform 5 m/s, is it possible for him to land on the target
(i) on a windless day?
(ii) if the wind blows from the east at 3 m/s?



$\tan\theta = 510/347 \quad \theta = 55{\cdot}77°$

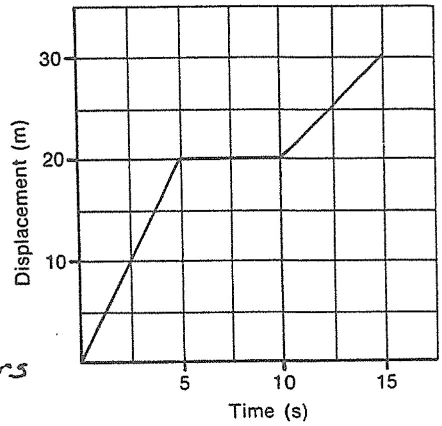
(i) The path he must follow is BT
With a constant vert. rate of 5 m/s
his horiz. speed must be:

$5/\tan 55{\cdot}77° = 3{\cdot}4\ m/s \rightarrow$

This is less than 7 m/s. <u>He will land on T.</u>

(ii) With a 3 m/s East wind,
max. horiz. air speed is 4 m/s $\rightarrow$
<u>By adjusting this to 3·4 m/s $\rightarrow$
he will land on T</u>

9/32
The graph shown illustrates the relationship between distance and time for a certain moving body. From the graph determine
(i) the velocity during the first 5 seconds;
(ii) the velocity for the 7th second;
(iii) the velocity during the 15th second.



(i) 20 m in 5 s

$= \underline{4 \text{ m/s}}$

(ii) 0 m in 5 s

$= \underline{0 \text{ m/s}}$

(iii) From 10^{th} to 15^{th} second, body covers $(30-20)$ m in $(15-10)$ s.

$\therefore$ vel. $= 10/5 = \underline{2 \text{ m/s}}$

9/33
Draw a velocity–time graph for a machine element which has the following motion:
(i) uniform acceleration from rest to a velocity of 1.5 m/s in 3 seconds;
(ii) travels at 1.5 m/s velocity for the next 5 seconds;
(iii) decelerates at a uniform rate for 6 seconds, the final velocity being 1 m/s.
From this graph, determine
(a) the displacement in the first 2 seconds;
(b) the displacement in the 12th second;
(c) the total displacement;
(d) the acceleration in the first 3 seconds;
(e) the acceleration in the last second.

(a) Av. vel. – 0·5 m/s for 2 s.

$\therefore$ Displ. $=$ 1 m (shaded A)

(b) Av. vel. in 12^{th} sec — 1·2 m/s

$\therefore$ Displ. $= \underline{1 \cdot 2 \text{ m}}$.

(c) Tot. Displ. $= B + C + D$

$= (1 \cdot 5 \times 3/2) + (1 \cdot 5 \times 5) + (0 \cdot 5 \times 6/2) + (1 \times 6)$

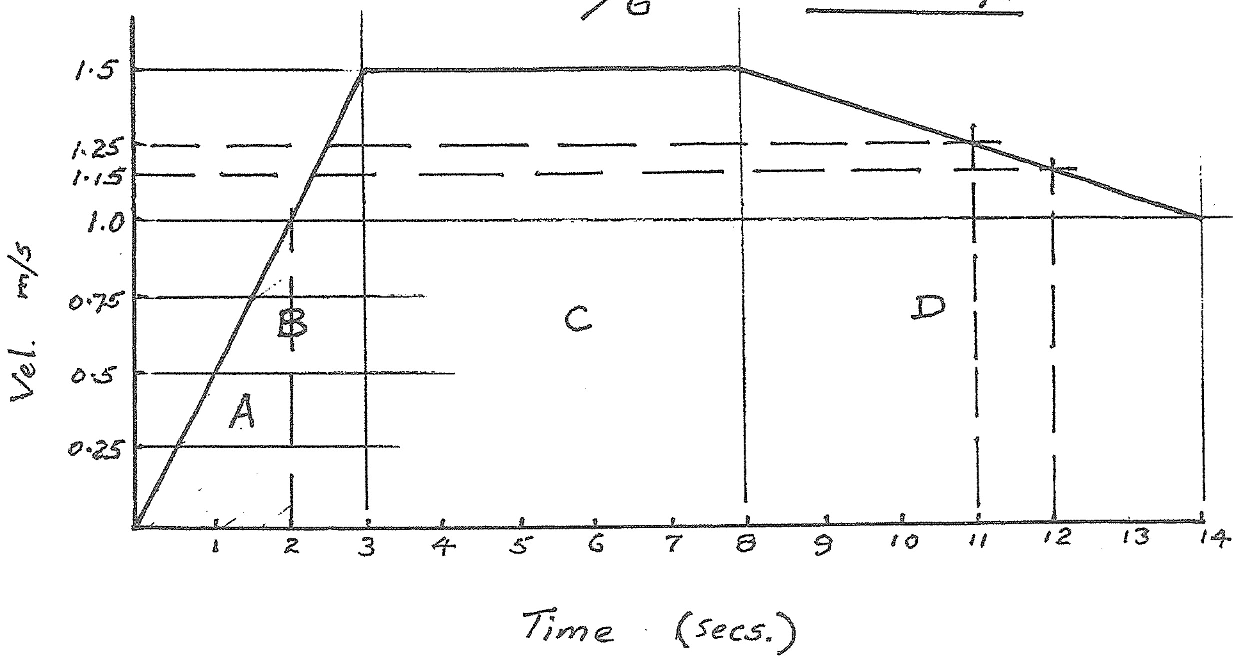
$= \underline{17 \cdot 25 \text{ m}}$

Accel. is slope of graph.

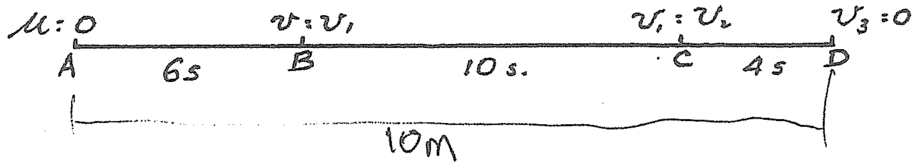
(d) $1 \cdot 5/3 = \underline{+0 \cdot 5 \text{ m/s}^2}$

(e) Accel. is constant and neg. over last 6 secs

Acc. $= \dfrac{1 \cdot 0 - 1 \cdot 5}{6} = \underline{-0 \cdot 08 \text{ m/s}^2}$



9/34
A conveyer belt in a factory carries partly completed sub-assemblies from one operator to the next in 20 seconds. The conveyer accelerates for the first 6 seconds, travels at uniform velocity for the next 10 seconds, then decelerates to rest for the next 4 seconds. If the operators are 10 metres apart, what is the uniform velocity during the 10-second period?



A–B

$u = 0$ $\quad v = u + at$ $\quad s = ut + \frac{1}{2}at^2$

$v = v$ $\quad v = 0 + 6a_1$ $\quad s_1 = 0 + \frac{1}{2} \times \frac{v}{6} \times 36$

$a = a_1$

$s = s_1$ $\quad a_1 = v/6\ m/s^2$ $\quad \underline{s_1 = 3v}$

$t = 6s.$

B–C uniform vel. $s_2 = s_2$, $v_1 = v$, $t = 10s.$, $a = 0$

$\therefore \underline{s_2 = 10v}$

C–D

$u = v\ (v_2, v_1)$ $\quad v = u + at$ $\quad s = ut + \frac{1}{2}at_2^2$

$v = 0$ $\quad 0 = v + 4a_2$ $\quad s_3 = 4v + \frac{1}{2} \times a_2 \times 16$

$a = a_2$

$s = s_3$ $\quad a_2 = -v/4$ $\quad = 4v + 8a_2$

$t = 4s.$ $\quad = 4v - 8/4\,v$

$s_1 + s_2 + s_3 = 10$ $\quad = \underline{2v}$

$3v + 10v + 2v = 10$

$v = \underline{0.67\ m/s}$

9/35
An electric car travelling between two stations 500 metres apart is uniformly accelerated for the first 10 seconds and covers 40 metres. It then runs with constant velocity for a certain time, after which it is uniformly decelerated and brought to a stop in the last 20 metres.

Draw a velocity–time graph for the motion of the electric car and determine
(i) the maximum velocity of the electric car;
(ii) the total time taken to cover the 500 metres.

(i) $u = 0$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v = u + at$

$v = ?$ $\quad 40 = 0 + \frac{1}{2}a \times 100$ $\quad = 0 + .8 \times 10$

$a = ?$ $\quad a = 0.8\ m/s^2$ $\quad = \underline{8\ m/s}$

$s = 40m$

$t = 10s$

(ii) $u = 8m/s$ $\quad v^2 = u^2 + 2as$

$v = 0$ $\quad 0 = 64 + 40a$

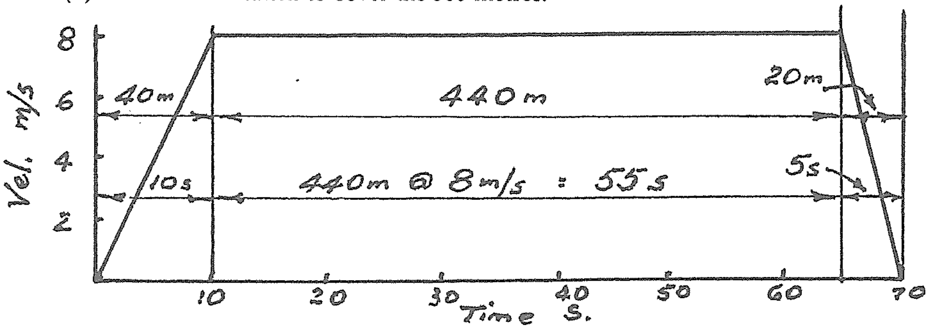
$a = ?$ $\quad a = -1.6\ m/s^2$

$s = 20m$

$t = ?$ $\quad v = u + at$

$t = \underline{5s.}$

Tot. time = $\underline{70s}$



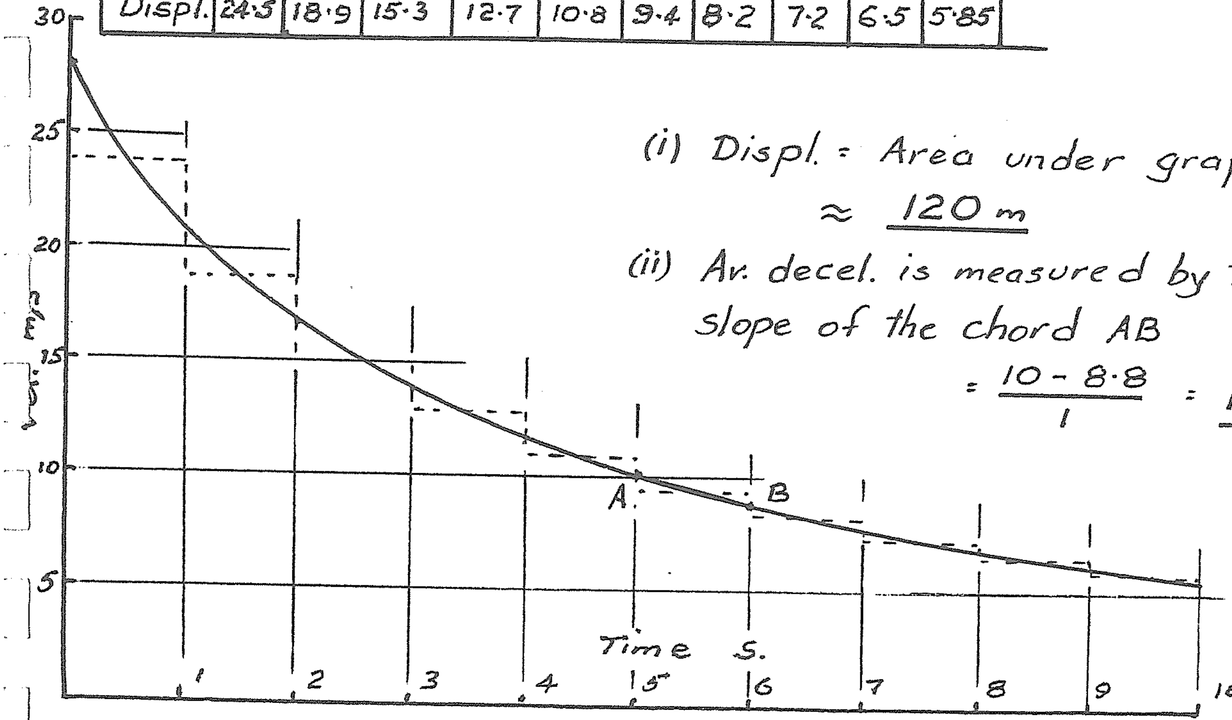
9/36

A car is travelling at 28 m/s when the brakes are applied. The relationship that occurs between velocity and time is set out below. Draw the velocity–time graph and determine

(i) the approximate displacement that occurred during braking;

(ii) the average deceleration during the 6th second.

Time (s)	0	1	2	3	4	5	6	7	8	9	10
Velocity (m/s)	28	21	16.8	13.8.	11.6	10	8.8	7.6	6.8	6.2	5.5
Displ.	24·5	18·9	15·3	12·7	10·8	9·4	8·2	7·2	6·5	5·85	



(i) Displ. = Area under graph
$\approx$ 120 m

(ii) Av. decel. is measured by the slope of the chord AB
$= \frac{10 - 8 \cdot 8}{1} = 1 \cdot 2 \, m/s^2$

9/37

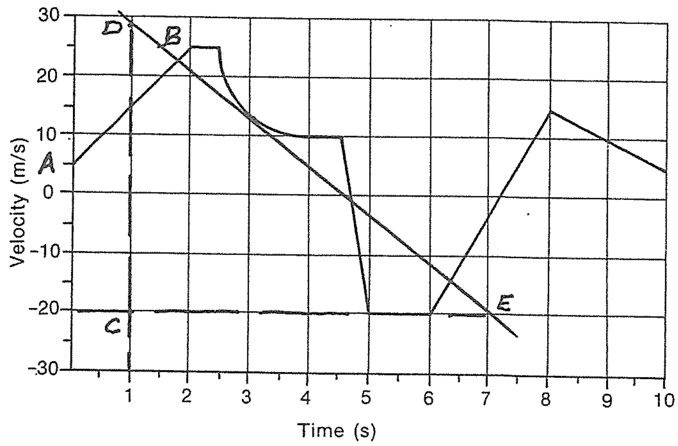
One cycle of the motion of a machine member is represented by the velocity–time graph shown. From the graph determine

(i) the acceleration during the first 2 seconds;

(ii) the maximum velocity;

(iii) the approximate acceleration after 3 seconds;

(iv) the approximate net displacement during the cycle.



(i) Accel. = slope of AB
$= \frac{25-5}{2} = +10 \, m/s^2$

(ii) Max. vel. occurs at B
= 25 m/s

(iii) Accel. is slope of tangent at 3 s.
= CD/CE = 48/6 = $-8 \, m/s^2$

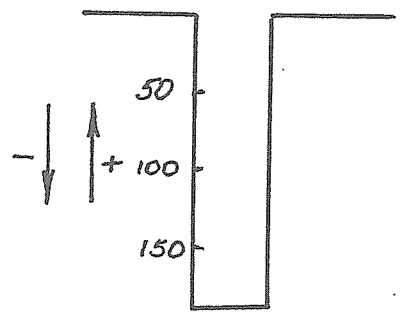
(iv) + Displ. = Area between zero and graph. $\approx$ +9 squares

− Displ. = Area between zero and graph $\approx$ -3·5 squares

Each sq. represents 10 m/s

∴ Total Displ. = (9·0 − 3·5) × 10
55 m.

9/38
A mine cage starts from the 50-m gallery level and descends to the 150-m gallery level. It then ascends to the surface. What was its displacement
(i) after descending?
(ii) after ascending?
(*Note:* A suitable sign convention is essential in this problem.)



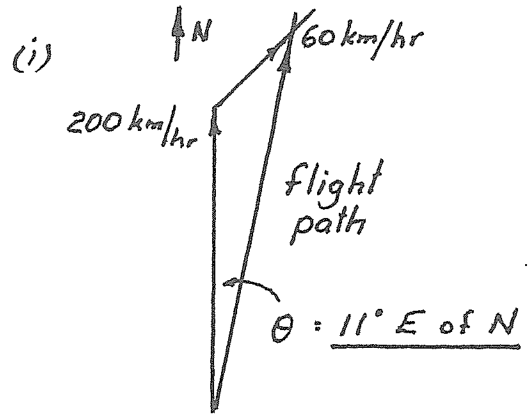
(i) $\underline{-100\text{ m}}$

(ii) $-100 + 150 = \underline{+50\text{m}}$

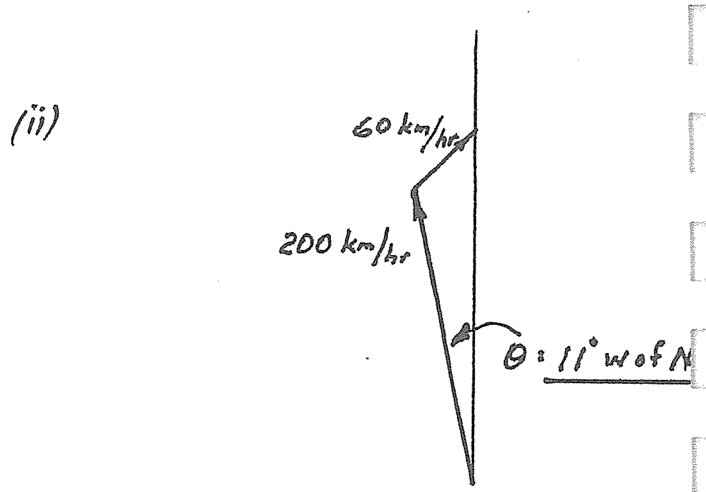
9/39
An aircraft flying at 200 km/h in level flight due north encounters a 60 km/h wind blowing from the SW direction.
(i) If no corrections are made to the aircraft heading, what will the actual flight path be?
(ii) What correction would the pilot need to make to his heading in order to maintain his original flight path?

Scale – 1mm = 5 km/hr

(i)



(ii)

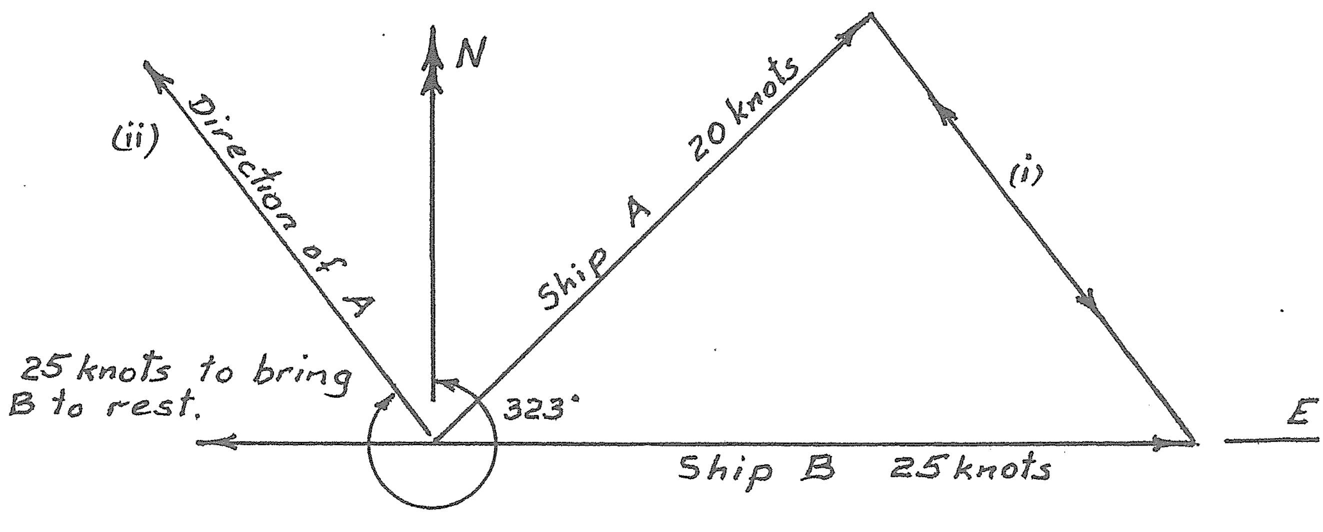


9/40
Two ships leave a port at the same time. Ship *A* is set on a northeasterly course at 20 knots. Ship *B* steams due east at 25 knots. (1 knot = 1 nautical mile/hour.)
(i) At what speed are the ships moving away from each other?
(ii) In what direction is ship *A* moving relative to ship *B*?

Scale – 4mm = 1 knot

(i) 18 knots

(ii) 323°



9/41

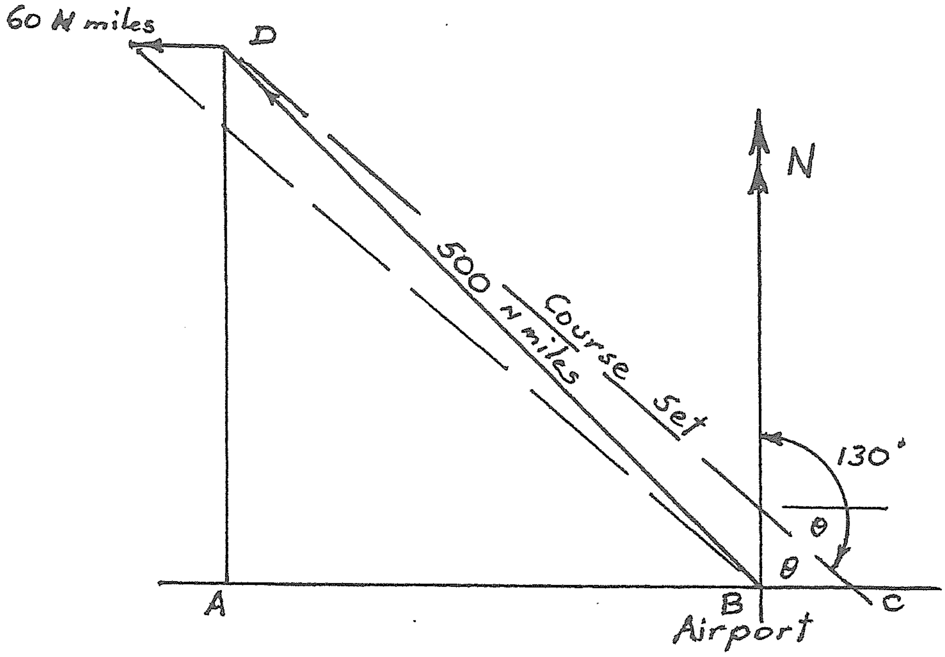
An aeroplane is 500 nautical miles northwest of the airport which it has to reach in one hour. If the pilot observes that the wind is blowing from the east at 60 nautical miles per hour

(i) on what course will he set the plane?

(ii) at what *ground speed* must he fly, to arrive at his destination on time?

(iii) at what *air speed* must he fly, to arrive at his destination on time?

Scale – 10mm : 50 N. miles



AB : 500 cos 45°

: 353·5

BC : 60

∴ AC : 413·5 N. Miles

AD : 500 sin 45°

: 353·5

Tan θ : 353·5/415·5

θ : 40·5°

(i) Course set would be N 130·5°E

(ii) Ground speed is 500 N. miles/hr.

(iv) Air speed is DC : Sin 40·5° : AD/DC

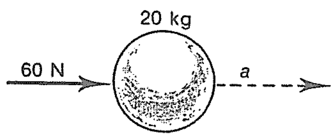
DC : 544 N miles/hr

10 Forces and Motion

$g : 9.8\ m/s^2$ $\quad$ $F = ma$

10 FORCES AND MOTION
Newton's First Law and Inertia. Newton's Second Law and Acceleration. *Further Worked Examples Not Involving Friction.* Dynamic Friction.

10/17
A body of mass 20 kg is acted upon by a resultant force of 60 newtons. Determine the acceleration that occurs in the direction of that force.

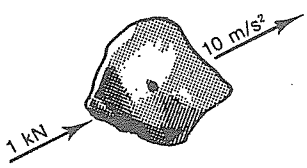


$F : ma$

$60 : 20a$

$a : \underline{3\ m/s^2}$

10/18
A horizontal force of 1 kN acts on a body and produces an acceleration of 10 m/s^2. What is the mass of the body?



$F : ma$

$1 \times 10^3 : m \times 10$

$m : \underline{100\ kg}$

10/19
An air-to-air missile of mass 500 kg is fired from a fighter in level flight. If the missile accelerates at 90 m/s^2 relative to the fighter, what average thrust is provided by the rocket motor?

$F : ma$

$: 500 \times 90$

$: \underline{45\ kN}$

10/20
A rocket of mass 300 tonnes travelling horizontally at 3600 km/h has its velocity increased to 5400 km/h by a 3-second "burn" of its motor. What average thrust was exerted by the engine?

$u : 3{,}600\ km/hr : 1000\ m/s$

$v : 5{,}400\ km/hr : 1500\ m/s$

$a : ?$

$s : -$

$t : 3\ s.$

$v : u + at$

$1500 : 1000 + 3a$

$a : 166 \cdot 67\ m/s^2$

$F : ma$

$: 300 \times 10^3 \times 166 \cdot 67$

$: 5 \times 10^7\ N$

Or $\underline{50\ MN}$

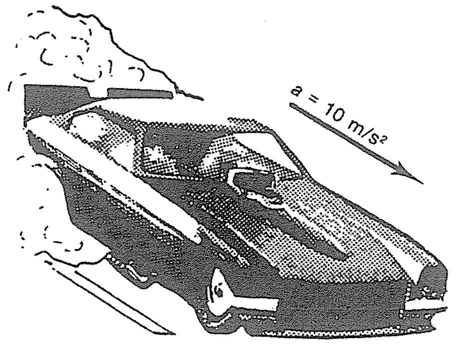
10/21

A car on a dragstrip accelerates at 10 m/s² from a standing start.

(i) How long does it take to cover the 400 m course?

(ii) What is the final velocity of the car?

(iii) What average tractive force was exerted by the rear wheels on the concrete dragstrip if the car had a mass of 0.8 tonnes and the total force opposing motion remained constant at 500 N during the run?



(i) $u = 0$ $\quad s = ut + \frac{1}{2}at^2$

$v = -$ $\quad 400 = 0 + \frac{1}{2} \times 10t^2$

$a = 10\ m/s^2$

$s = 400\ m$ $\quad t^2 = 80$

$t = ?$ $\quad t = \underline{8.94\ s.}$

(ii) $u = 0$ $\quad v^2 = u^2 + 2as$

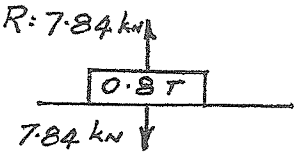
$v = ?$ $\quad = 0 + 2 \times 10 \times 400$

$a = 10\ m/s^2$ $\quad = 8000$

$s = 400$ $\quad v = \underline{89.4\ m/s}$

$t = -$ $\quad$ or $\underline{322\ km/hr}$

(iii)



$F = ma$

$= 800 \times 10$

$= 8000\ N$

8 kN needed to produce 10 m/s² without friction

∴ <u>8.5 kN needed with friction</u>

10/22

A man on a rocket sled mounted on a straight track is travelling at 360 km/h when the retro-rockets are fired and the velocity is reduced to 36 km/h in 165 metres.

If the man has a mass of 80 kg, calculate the total force exerted by the man on his seat belts during deceleration. Express your answer in kilonewtons and as "g-force".

$u = 360\ km/hr = 100\ m/s$ $\quad v^2 = u^2 + 2as$ $\quad F = ma$

$v = 36\ km/hr = 10\ m/s$ $\quad 100 = 10000 + 2 \times 165a$ $\quad = 80 \times -30$

$a = ?$ $\quad a = -30\ m/s^2$ $\quad = \underline{-2.4\ kN}$

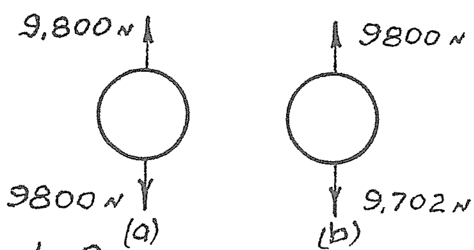
$s = 165\ m$

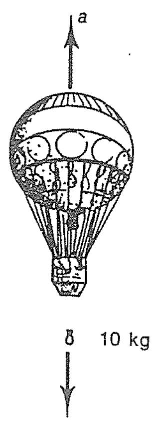
$t = -$

Force in terms of "g" $= a/g = -30/9.8 = \underline{-3g}$

10/23

A balloon is drifting horizontally in still air when 10 kg of ballast is released. Determine the resultant vertical acceleration of the balloon if its mass is 1 tonne.





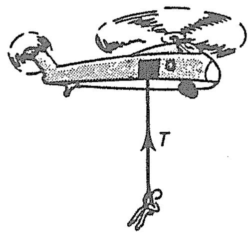
(a) Equilibrium (No nett force) <u>Accel = 0</u>

(b) Nett force 98 N ↑ $\quad$ ∴ $F = ma$

$98 = 990a$

$a = \underline{0.099\ m/s^2} \uparrow$

10/24
A 75-kg man is winched into a search-and-rescue helicopter which is hovering directly above the building in which the man was trapped. If the mass of the sling used is 10 kg, determine the tension in the cable when the man is ascending,
(i) with constant velocity, and
(ii) with a constant acceleration of 0.5 m/s^2. (Neglect the mass of the winch cable.)



(i) T, $\boxed{85\ kg}$, 833 N

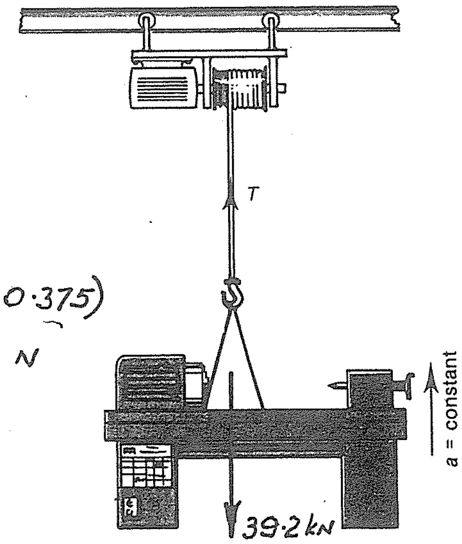
For const. vel (equil.)

$T = \underline{833\ N}$

(ii) T, $\boxed{85\ kg}$, $0.5\ m/s^2$, 833 N

$F(T) = ma$
$= 85(9.8 + 0.5)$
$= 85 \times 10.3$
$= \underline{875.5\ N}$

10/25
A machine of mass 4 tonnes is hauled vertically upwards by a cable and moves a distance of 3 metres in 4 seconds from rest. Assuming that the acceleration is constant, determine the tension in the cable.



$u = 0$
$v = -$
$a = ?$
$s = 3\ m$
$t = 4\ s.$

$s = ut + \frac{1}{2}at^2$
$3 = 0 + \frac{1}{2}a \times 16$
$a = 0.375\ m/s^2$

$F(t) = ma$
$= 4 \times 10^3(9.8 + 0.375)$
$= 40.7 \times 10^3\ N$
Or $\underline{40.7\ kN}$

10/26
A surface-to-air missile is fired vertically from its launcher and its motor provides an average thrust of 5 kN over a 10-second period. If the missile has a mass of 102 kg, determine its acceleration and the altitude it reaches.

$F = ma$
$5 \times 10^3 = 102a$
$a = 49\ m/s^2$
Effective $a = 49 - 9.8$
$= \underline{39.2\ m/s^2}$

$v = ?$
10s $a = ?$
$u = 0$

$u = 0$
$v = -$
$a = 39.2\ m/s^2$
$s = ?$
$t = 10\ s$

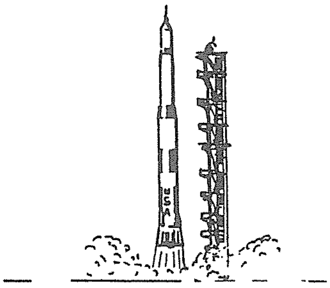
$s = ut + \frac{1}{2}at^2$
$= 0 + \frac{1}{2} \times 39.2 \times 100$
$= \underline{1960\ m}$

10/27
During a test, a thrust of 5 kN is developed for 15 seconds by a small rocket motor. If the minimum vertical acceleration of the rocket into which this motor is to be placed is 65 m/s^2, determine the maximum mass of the rocket. (Neglect the mass of the motor in your calculations.)

Effective minimum acceleration
$= (65 + 9.8)\ m/s^2$
$= 74.8\ m/s^2$

$F = ma$
$5 \times 10^3 = m \times 74.8$
$m = \frac{5 \times 10^3}{74.8}$
$= \underline{66.8\ kg.}$

10/28
The Saturn V rocket had a mass of approximately 2.7×10^6 kg, of which 2.5×10^6 kg was the liquid oxygen/kerosene propellant. This produced a lift-off thrust of 33.5 MN.
(i) What was the initial acceleration of the rocket?
(ii) How does this compare with that of an average family car?
(iii) How does the acceleration vary during the rocket launch and why?



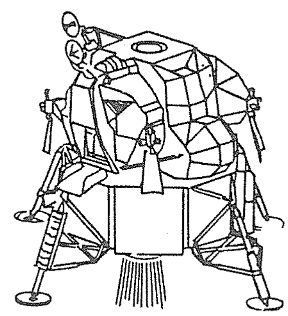
(i) $F = ma$
$33.5 \times 10^6 = 2.7 \times 10^6 a$
$a = 12.4$ m/s²
less 9.8 m/s² $\therefore$ Eff. $a = \underline{2.6 \text{ m/s}^2}$

(ii) If a car accel. from rest to 60 km/hr in 8 s., the accel. will be 2.1 m/s²
<u>About the same</u>

(iii) As the fuel is used up, the mass is dramatically reduced
eg With fuel almost gone mass is 0.2×10^6 kg
$F = ma$
$33.5 \times 10^6 = 0.2 \times 10^6 \times a$
$\therefore a = \underline{167.5 \text{ m/s}^2}$
Also the further away from earth, the less gravity

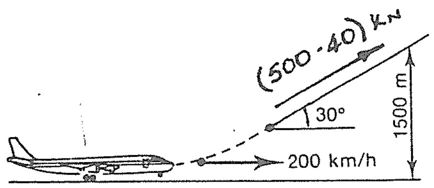
10/29
During the last stage of its descent on to the moon's surface, a lunar module decelerated vertically at 1 m/s². Given that the module had a mass of 13×10^3 kg and that the acceleration due to gravity is 1.67 m/s² on the moon, determine the thrust exerted by the descent engine during this stage.

Nett accel.
$= 1.67\downarrow - 1.00\uparrow$
$= 0.67 \text{ m/s}^2 \downarrow$



$F = ma$
$= 13 \times 10^3 \times 0.67$
$= \underline{8.7 \text{ kN}}$

10/30
An aircraft climbs to an altitude of 1500 m at a constant angle of 30 degrees to the horizontal. If the total thrust of the jet engines is 500 kN and the total mass of the plane and its contents is 55 tonnes, determine the approximate time taken to reach the 1500 m altitude. Assume a ground speed at take-off of 200 km/h and an average air resistance of 40 kN.

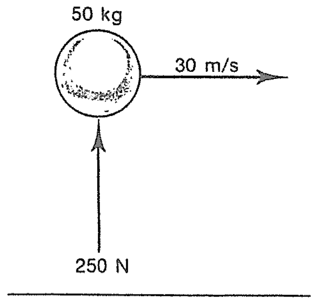


Vert. comp. of force
$= 460 \sin 30°$
$= 230$ kN
$F = ma$
$230 \times 10^3 = 55 \times 10^3 a$
$a = 4.18$ m/s²

Vert. comp. of $u = 0$
$u = 0$
$v = -$
$a = 4.18$ m/s²
$s = 1500$ m
$t = ?$

$s = ut + \frac{1}{2}at^2$
$1500 = 0 + \frac{1}{2} \times 4.18 t^2$
$t = \underline{26.7 \text{ s}}$

10/31
A body travelling at 30 m/s in a direction due east is acted upon by a force of 250 N acting due north for 2 seconds. If the body has a mass of 50 kg, determine its final velocity.



$F = ma$

$250 = 50a$

$a = 5\ m/s^2$

$u = 0$

$v = ?$

$a = 5\ m/s^2$

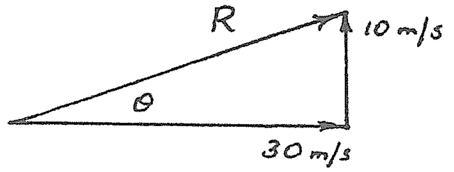
$s = -$

$t = 2\ s.$

$v = u + at$

$= 0 + 5 \times 2$

$= 10\ m/s$



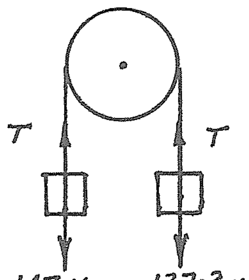
$R = \sqrt{30^2 + 10^2}$

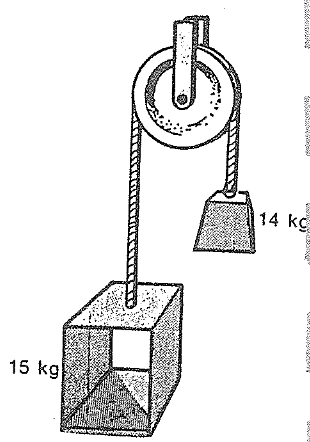
$= \underline{31 \cdot 62\ m/s}$

$\tan\theta = 10/30$

$\theta = \underline{18.4^\circ}$

10/32
A motorless service elevator consists of a platform plus cage and a counterweight connected by a rope which passes over a small, free-running pulley. If the platform and the cage have a mass of 15 kg and the counterweight a mass of 14 kg, calculate the tension in the rope and the acceleration of the system if it is allowed to move freely when unloaded. Neglect friction, inertia and the weight of the rope.





The rope has same tension throughout.

__L.H.S.__ $F\downarrow = (147 - T)\ N$

$F = ma$

$147 - T = 15a$

$T = 147 - 15a$

__R.H.S.__ $F\uparrow = (T - 137.2)\ N$

$F = ma$

$T - 137.2 = 14a$

$T = 14a - 137.2$

$29a = 147 - 137.2$

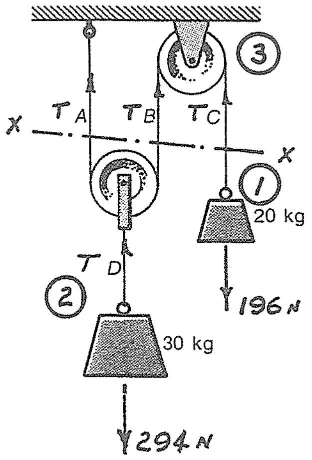
$a = \underline{0.33\ m/s^2}$

Subst. $147 - T = 15 \times 0.33$

$T = \underline{141.9\ N}$

10/33
In an experiment, the single-fixed, single-movable pulley system is loaded as shown. Neglecting friction, the mass of the cord, the mass of the pulley sheave blocks and the inertia of the pulleys, determine:
(i) the tensions in the cord at the points A, B and C;
(ii) the tension in the tie, D;
(iii) the accelerations of the two masses.

$T_A = T_B = T_C$
(single rope)



Note: If a system is unbalanced, any part of the system behaves according to $f = ma$
The M.A. and ∴ V.R. of this system is 2.

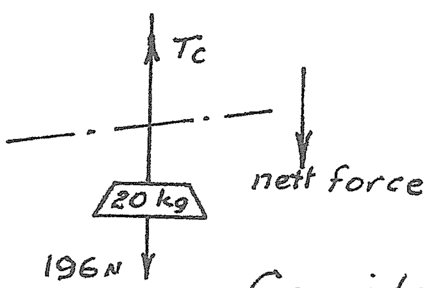
20 kg mass	30 kg mass
$u = 0$	$u = 0$
$v = v_1$	$v = v_2$
$a = a_1 \quad \therefore v_1 = a_1 t \ldots (1)$	$a = a_2 \quad \therefore v_2 = a_2 t \ldots (2)$
$t = t$	$t = t$

$v_1 = 2v_2$ (V.R. = 2) ∴ from (1) & (2) $a_1 = 2a_2 \ldots (3)$

Take a section XX through the single rope, dividing the system into a number of parts with 3 unknowns, T, a_1, a_2.

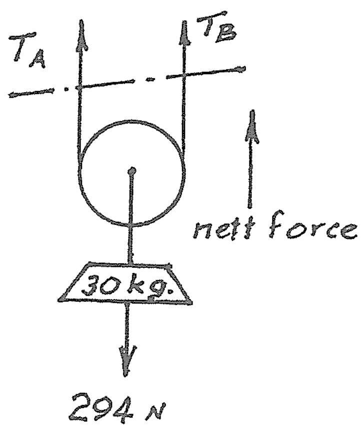
With a M.A. of 2, a 15 kg mass would maintain equilibrium. ∴ the 20 kg mass will accel. down and 30 kg. up.

Consider the 20kg part of the system



$f = ma$
$196 - T_C = 20a_1 \ldots (4)$

Consider the 30kg part of the system

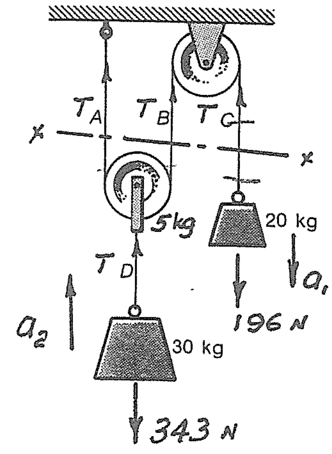


$f = ma$
$2T - 294 = 30a_2 \ldots (5)$

Solve eqns. (3), (4) & (5) simultaneously to give :-

a_1 (20kg.) = 1·78 m/s²
a_2 (30kg.) = 0·89 m/s²
T_A, T_B & T_C = 160·4 N
and T_D balances T_A & T_B & = 320·8 N

10/34
The fixed and movable pulleys and sheave blocks of the pulley system shown in Problem 10/33 are found to have a mass of 5 kg. For the same loads determine
(i) the tensions in the cord at the points A, B and C;
(ii) the tension in the tie, D;
(iii) the acceleration of the system.
(Neglect friction, the mass of the cord, and the inertia of the pulleys.)



$V.R. = 2 \quad , \quad T_A = T_B = T_C$

$a_1 = 2a_2 \cdots\cdots (1)$

20 kg. mass $\quad f = ma$

$196 - T = 20\,a_1 \cdots\cdots (2)$

30 kg. mass $\quad f = ma$

$2T - 343 = 35\,a_2 \cdots\cdots (3)$

Subst. $2a_2$ for a_1 in (2)

$196 - T = 40\,a_2$

Mult. by 2 $\quad 392 - 2T = 80\,a_2 \cdots\cdots (4)$

Add (3) & (4)

$49 = 115\,a_2$

$a_2 = \underline{0{\cdot}426\ m/s^2}$

$a_1 = \underline{0{\cdot}852\ m/s^2}$

$T_A,\ T_B\ \&\ T_C = \underline{178{\cdot}8\ N}$

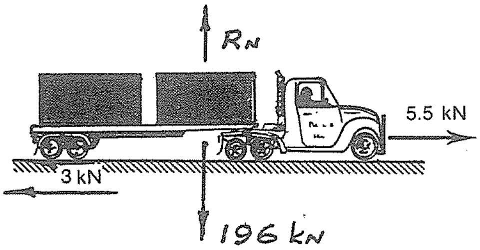
The 5kg mass has no direct effect on the tension in D, but since it is accelerating upwards at $0{\cdot}426\ m/s^2$, the value of "f" to produce this accel. is $2{\cdot}13$ N (from $f = ma$) This tends to reduce the tension in Tie D

$\therefore\ T_D = T_A + T_B - 49 - 2{\cdot}13$

$= \underline{306{\cdot}8\ N}$

10/35

A trailer together with its load has a mass of 20 tonnes and is attached to a prime mover which can provide a maximum pull of 5.5 kN. If the total frictional force resisting the forward motion of the trailer and load is 3 kN, determine the resultant acceleration of the trailer and prime mover.

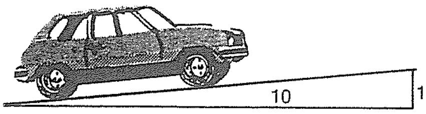


$$\text{Nett force} = 5.5 - 3$$
$$= 2.5 \text{ kN}$$
$$F = ma$$
$$2.5 \times 10^3 = 20 \times 10^3 a$$
$$a = \underline{0.125 \text{ m/s}^2}$$

10/36

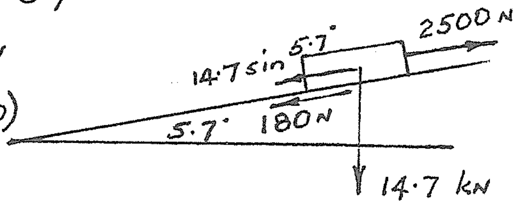
A motor vehicle of mass 1.5 tonnes starts from rest to climb a gradient of 1 in 10 against a frictional resistance of 180 N. If the tractive effort exerted by the rear wheels is 2.5 kN, determine

(i) the acceleration of the vehicle up the gradient;
(ii) the time taken to reach 60 km/h;
(iii) the total distance travelled.
(iv) What would be the acceleration on a horizontal stretch of road if the frictional resistance is unchanged?



$$\tan\theta = 0.1$$
$$\theta = 5.7^\circ$$

(i) Mass force down plane $= 14.7 \sin 5.7^\circ$
$= 1460 \text{ N}$



$\therefore$ Nett force $= 2{,}500 - (1460 + 180)$
$= 860 \text{ N}$

$$F = ma$$
$$860 = 1500a \qquad \therefore a = \underline{0.57 \text{ m/s}^2}$$

(ii) $u = 0$
$v = 16.67$ m/s
$a = 0.57$ m/s^2
$s = -$
$t = ?$

$v = u + at$
$16.67 = 0.57t$
$t = \underline{29.2 \text{ s.}}$

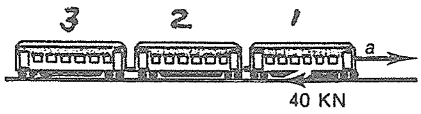
(iii) $u = 0$
$v = 16.67$ m/s
$a = 0.57$ m/s^2
$s = ?$
$t = -$

$v^2 = u^2 + 2as$
$277.89 = 1.14s$
$s = \underline{243 \text{ m}}$

(iv) Force resisting motion $= 180$ N
Nett force $= 2{,}320$ N
$$F = ma$$
$$2320 = 1500a$$
$$a = \underline{1.546 \text{ m/s}^2}$$

10/37

A suburban train consists of 3 carriages each having a mass of 15 tonnes. The first carriage acts as an engine and exerts a driving force of 40 kN on the rails. The frictional drag of each carriage on the rails is 1 kN. Determine the acceleration of the train and the tension in the couplings between the carriages.



Nett force $= 40 - 3\times 1 = 37\ kN$

$F = ma$

$37\times 10^3 = 45\times 10^3 a$

$a = \underline{0.82\ m/s^2}$

Coupling between 1 & 2

Nett force $= T - 2$

$F = ma$

$T - 2 = 2\times 15 \times 0.82$

$T = \underline{26.6\ kN}$

Coupling between 2 & 3

Nett force $= T - 1$

$F = ma$

$T - 1 = 1\times 15 \times 0.82$

$T = \underline{13.3\ kN}$

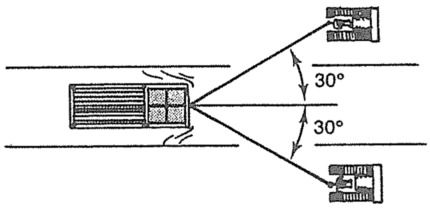
10/38

A barge of mass 10 tonnes is moved along a straight canal by two horizontal tow ropes attached to two prime movers on opposite sides of the canal. The prime movers operate so that the tow ropes always make angles of 30° with the direction of motion.

If the barge moves from rest a distance of 40 metres in 20 seconds, and the average force resisting the forward motion of the barge is 180 N per tonne, determine

(i) the average resultant force causing the forward motion of the barge;

(ii) the tension in either tow rope.



(i) $u = 0$

$v = -$

$a = ?$

$s = 40\ m$

$t = 20\ s.$

$s = ut + \frac{1}{2}at^2$

$40 = 0 + \frac{1}{2}\times a \times 400$

$a = 0.2\ m/s^2$

$F = ma$

$= 10\times 10^3 \times 0.2$

$= 2000\ N$

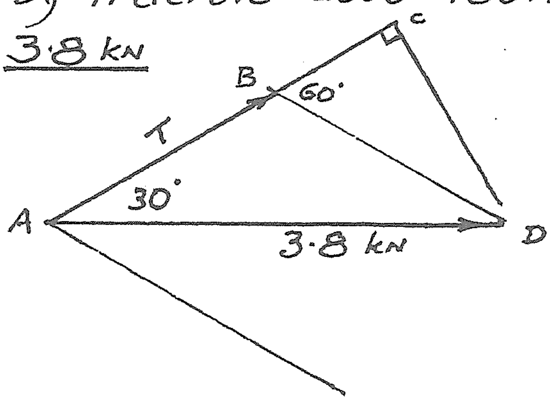
$\therefore$ Force supplied by tractors $= 2000 + 180\times 10$

$= \underline{3.8\ kN}$

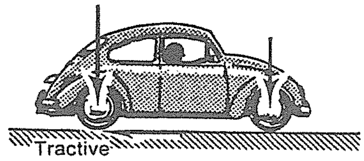
(ii) $AC = 3.8\cos 30^\circ = 3.29\ kN$

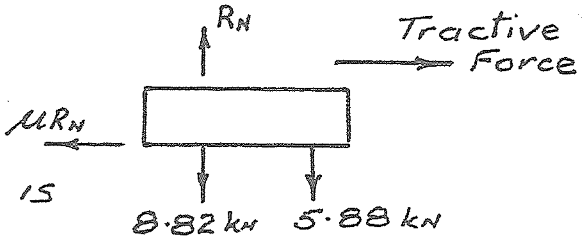
$BC = CD/\tan 60^\circ = 1.09\ kN$

$\therefore AB\ (T) = \underline{2.2\ kN}$



10/39
The engine of a motor car drives the rear wheels only. If the axle loads are 900 kg rear and 600 kg front, determine the maximum tractive force that the rear tyres can exert during acceleration. Take the coefficient of friction (skidding) between rubber and bitumen as 0.6.





Resolve Vert.

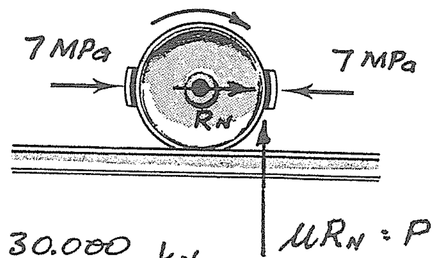
$R_N = 8{\cdot}82$ kN

(Disregard Front since there is no force on front with max. traction at rear.)

Resolve horiz.

Tractive Force $= \mu R_N = 0{\cdot}6 \times 8{\cdot}82 = \underline{5{\cdot}29 \text{ kN}}$

10/40
Determine the tangential braking force P acting on the rotating railway carriage wheel when a pressure of 7 MPa exists between the cast iron brake shoes and the rim of the wheel. The coefficient of friction μ_k present is 0.2, and each brake shoe has an area of 30 000 square millimetres.



$1 \text{ MPa} = 1 \text{ n/mm}^2$

$\therefore$ Force of shoe on wheel $= \dfrac{7 \times 30.000}{1000}$ kN

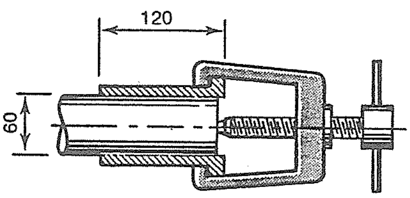
$= 210$ kN

$\therefore R_N = 210$ kN

and $\mu R_N = 0{\cdot}2 \times 210$

$= 42$ kN per shoe or $\underline{84 \text{ kN}}$

10/41
A bronze bush is drawn from the end of the shaft using the screw-operated puller as shown. If the radial pressure between the bush and shaft is 3.5 MPa and the coefficient of sliding friction is 0.2, determine the force required to remove the bush.



Surface area of contact $= \pi D \times 120 \text{ mm}^2$

$\therefore$ Pressure $= \dfrac{3{\cdot}5 \times \pi D \times 120}{10^3}$ kN

This also equals R_N between bush & shaft

$\therefore$ Friction to be overcome

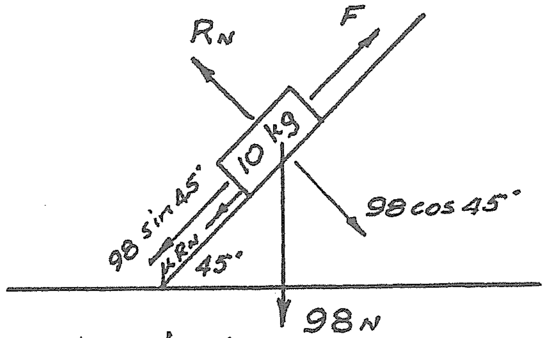
$= \dfrac{0{\cdot}2 \times 3{\cdot}5 \times \pi D \times 120}{10^3}$

$= \underline{15{\cdot}8 \text{ kN}}$

10/42
The start of a sand sled run has a constant slope of 45° to the horizontal. One competitor of mass 60 kg has a sled of mass 10 kg.

(i) What tension will he need to exert on the tow rope when pulling his sled up the sled run with constant velocity?
(ii) With what rate of acceleration will he come down this section of the sled run?
(iii) If this section of the sled run is 20 metres long, what will be his velocity after he covers that distance, assuming that his initial velocity was zero?

The coefficient of friction between sand and sled is 0.35.



(i) Resolve ⊥ to plane.

$R_N = 98\cos 45°$

Resolve ∥ to plane

$F = 98\sin 45° + \mu R_N$

$= \underline{93.5\ N}$

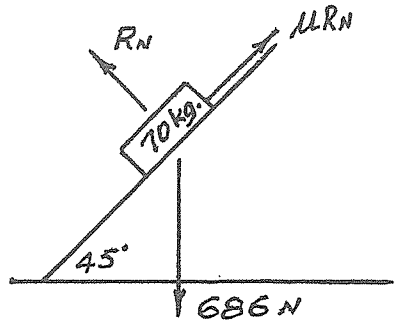
(ii) Resolve ⊥ to plane

$R_N = 686\cos 45°$

Resolve ∥ to plane

Nett force $= 686\sin 45° - \mu R_N$

$= 315.2\ N$



$F = ma$

$315.2 = 70a$

$a = \underline{4.5\ m/s^2}$

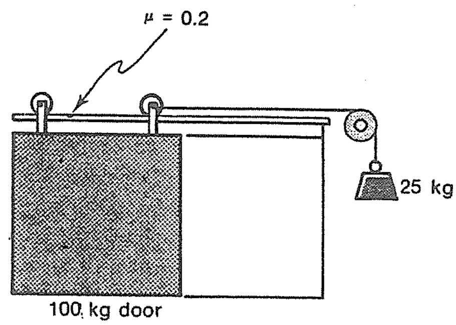
(iii) $u = 0$
$v = ?$
$a = 4.5\ m/s^2$
$s = 20\ m$

$v^2 = u^2 + 2as$

$= 0 + 2 \times 4.5 \times 20$

$v = \underline{13.4\ m/s}$

10/43
A 100-kg fire door has a self-closing mechanism operated by the 25-kg counterweight as shown. Assuming negligible friction in the pulley, determine the approximate acceleration of the door when the restraining latch is released, if the coefficient of friction present in the overhead door track system is 0.2.



Force causing horiz. motion

$= T - \mu R_N$

$= (T - 196)\ N$

$F = ma$

$T - 196 = 100a \ldots\ldots (1)$

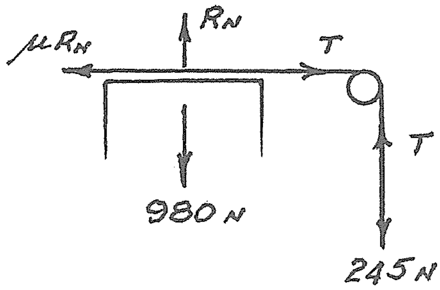
Force causing vert. motion

$= (245 - T)\ N$

$245 - T = 25a \ldots\ldots (2)$

Add (1) & (2) $\quad 125a = 49$

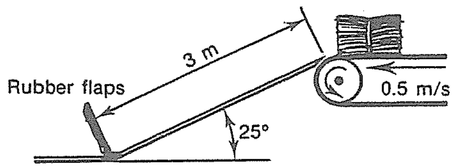
$a = \underline{0.392\ m/s^2}$

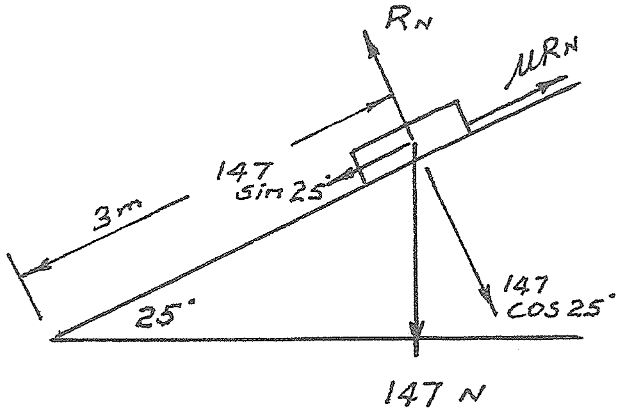


10/44

Part of a delivery system consists of a conveyer belt that carries 15-kg bundles of newspapers to the top of a metal shute 3 metres long and inclined at 25° to the horizontal.

At the bottom of the shute a pair of rubber flaps slow the bundles before they enter the storage area. If the bundles have an initial horizontal velocity of 0.5 m/s when they reach the slide and the coefficient of sliding friction present is 0.1, calculate the velocity with which the bundles hit these flaps.





Resolve ⊥ to plane

$R_N = 147 \cos 25°$

Nett force down plane

$= 147 \sin 25° - \mu R_N$

$= 48 \cdot 81$ N

$F = ma$

$48 \cdot 81 = 15a$

$a = 3 \cdot 25$ m/s²

$u = 0 \cdot 5$ m/s

$v = ?$

$a = 3 \cdot 25$ m/s²

$s = 3$ m

$t = -$

$v^2 = u^2 + 2as$

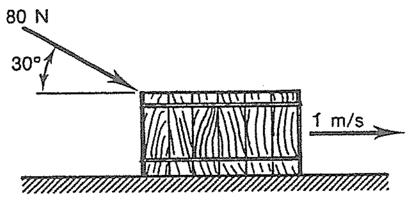
$= 0{\cdot}25 + 19{\cdot}5$

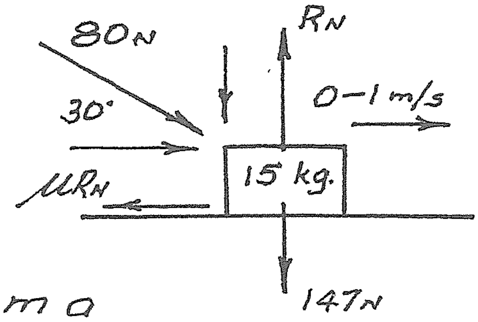
$= 19 \cdot 75$

$v = \underline{4 \cdot 44 \text{ m/s}}$

10/45

A constant push of 80 N applied at an angle of 30° to a crate of mass 15 kg gives the crate a horizontal velocity of 1 m/s in 3 seconds. Determine the coefficient of dynamic friction acting between the crate and the horizontal plane.





$u = 0$

$v = 1$ m/s

$a = ?$

$s = -$

$t = 3$ s.

$v = u + at$

$1 = 0 + 3a$

$a = 1/3$ m/s²

$F = ma$

$= 15 \times 1/3$

$= 5$ N.

Resolve vert.

$R_N = 147 + 80 \sin 30$

$= 187$ N

∴ 5N ← will balance the system

Resolve horiz.

$\mu R_N + 5 = 80 \cos 30°$

$187 \mu = 64 \cdot 28$

$\mu = \underline{0 \cdot 34}$

11 Impulse and Momentum

11 IMPULSE AND MOMENTUM
Impulse. Conservation of Momentum. *Rocket and Jet Engines. Elastic Collisions and the Coefficient of Restitution.*

11/10
Calculate the momentum (in kg m/s) of the following bodies:
(i) a motorcycle and rider, combined mass 400 kg moving at 72 km/h;
(ii) a truck, mass 8 tonnes, moving at 54 km/h;
(iii) a bullet, mass 15 g, moving at 280 m/s;
(iv) the Saturn V rocket of mass 2.7×10^6 kg when it is travelling at the "escape velocity" of 11 000 m/s.

(i) Momentum $= mv$
$= 400 \times 20$
$= 8000$
or $\underline{8 \times 10^3 \text{ kgm/s}}$

(ii) Mom. $= mv$
$= \frac{8000 \times 54 \times 1000}{3,600}$
$= \underline{1 \cdot 2 \times 10^5 \text{ kgm/s}}$

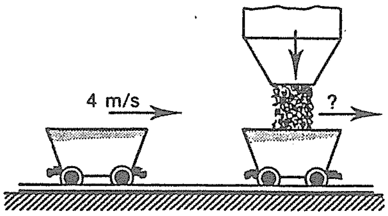
(iii) $M = mv$
$= \frac{15 \times 280}{1000}$
$= \underline{4 \cdot 2 \text{ kgm/s}}$

(iv) $M = mv$
$= 2 \cdot 7 \times 10^6 \times 11000$
$= \underline{2 \cdot 97 \times 10^{10} \text{ kgm/s}}$

11/11
A 6-gram bullet is fired horizontally into a 10-kg block of wood. The block and bullet move off with an initial velocity of 0.4 m/s. Determine the impact velocity of the bullet.

$m_1 v_1 = m_2 v_2$
$\frac{6}{1000} \times v_1 = \left(10 + \frac{6}{1000}\right) \times 0 \cdot 4$
$v_1 = \underline{667 \text{ m/s}}$

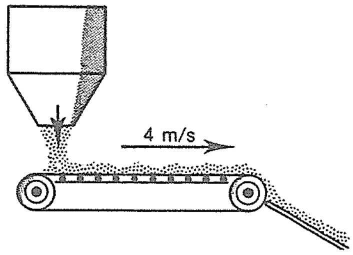
11/12
An empty 5-tonne skip loader is coasting at 4 m/s along a horizontal section of track. As it passes under a loader, 3 tonnes of sinter are suddenly dumped into it. Assuming that the sinter had no horizontal component to its velocity, determine the resultant velocity of the loader plus sinter.



Momentum of empty skip $= 5 \times 10^3 \times 4$ kgm/s
" " loaded " $= 8 \times 10^3 \times v$ kgm/s
$8 \times 10^3 v = 5 \times 10^3 \times 4$
(sinter had no horiz. vel. comp.)

$v = \underline{2 \cdot 5 \text{ m/s}}$

11/13
Sand drops at the rate of 50 kg per second from an overhead hopper on to a horizontal conveyer belt which moves at 4 m/s. Determine the additional force required to drive the conveyer belt, due to its load of sand.



$I : F \times t$

$I :$ change in momentum

$F \times 1 : mv - mu$

$: 50 \times 4 - 50 \times 0$

$F : \underline{200\ N}$

11/14
An agitator truck full of concrete has a total mass of 19 tonnes. It is travelling at 60 km/h when it collides head-on with a small sedan of mass 1 tonne travelling at 100 km/h. If the wreckage moves off as one mass, determine its velocity immediately after impact.

60 km/hr → $\boxed{19t}$ ← −100 km/hr $\boxed{1t}$

Total mom. before impact

$: m_1 u_1 + m_2 u_2$

$: (19 \times 60) + (1 \times -100)\ t\,km/hr$

Total mom. after impact

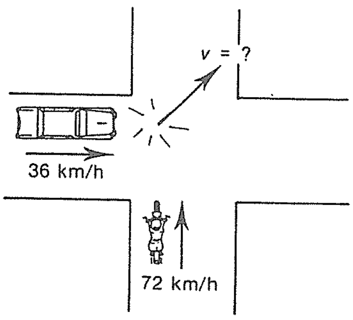
$: m_3 v$

$: (19 + 1) v$

$20v : 19 \times 60 - 100$

$v : \underline{52\ km/hr}$

11/15
A car and driver of combined mass 1 tonne are moving due east through an intersection at 36 km/h when struck centrally by a motorcycle plus rider moving due north at 72 km/h. If the combined mass of the motorcycle and rider is 300 kg, determine the velocity after impact if the car driver, motorcycle and rider become entangled and move off as one mass.

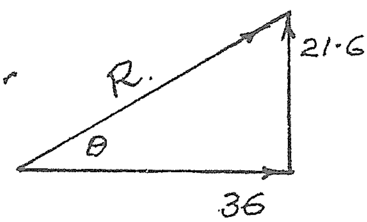


Mom. of Truck before collision : $1 \times 36 : 36\ t\,km/hr$ →

Mom. of Bike before collision : $0.3 \times 72 : 21.6\ t\,km/hr$ ↑

Total mom.(R) : $\sqrt{36^2 + 21.6^2}$

$: 41.98\ t\,km/hr$



$M : mv$

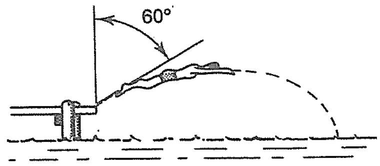
$41.98 : 1.3v$

$v : \underline{32.2\ km/hr}$

$\tan\theta : 21.6/36$

$\theta : \underline{31^\circ\ N\ of\ E}$

11/16
An 80-kg man dives off the end of a pier with an initial velocity of 2.5 m/s in the direction shown. Determine the horizontal and vertical components of the force exerted on the pier by the diver if he takes 0.75 seconds to leave the pier.



Mom. before leaving = 0
Mom. on leaving = $80 \times 2.5 = 200$ kgm/s
Horiz. comp. = 173.2 kgm/s; vert. comp. 100 kgm/s

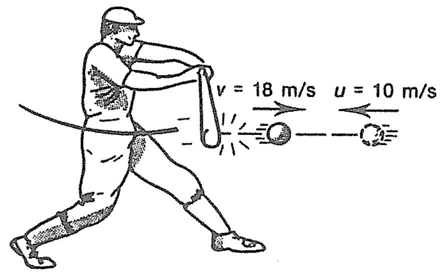
Horiz. comp.
I = change in mom.
$= 173.2 - 0$
$Ft = 173.2$
$F = 173.2/0.75$
$= \underline{231 \text{ N}} \rightarrow$

Vert. comp.
I = change in mom.
$= 100 - 0$
$Ft = 100$
$F = 100/0.75$
$= 133.3 + mg$
$= \underline{917 \text{ N}} \uparrow$

11/17
A sledge hammer of mass 6 kg has a velocity of 5 m/s when it hits squarely on to the end of a wedge. It rebounds with an initial velocity of 0.5 m/s. If the blow occurs in 0.01 seconds, what average force is exerted on the wedge?

Mom. of hammer before collision
$= 6 \times 5 = 30$ kgm/s $\downarrow$
Mom. of hammer after impact
$= 6 \times 0.5 = 3$ kgm/s $\uparrow$
$\therefore$ Change in mom. $= 30 - (-3)$
$= 33$ kgm/s
$I = Ft$
$33 = F \times 0.01$ $\therefore F = \underline{3.3 \text{ kN}}$

11/18
A batsman strikes a 0.25-kg ball and causes the ball to reverse its direction of motion. If the initial velocity of the ball was 10 m/s and it moved off at 18 m/s in the opposite direction after the impact, determine the average force exerted on the ball if the duration of the impact was 0.01 seconds.



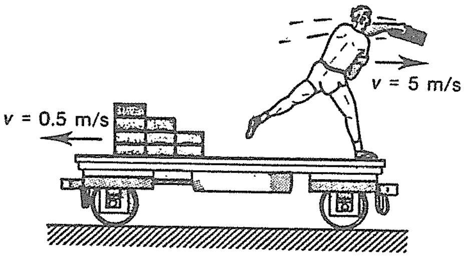
Mom. before impact $= 0.25 \times 10 = 2.5$ kgm/s $\leftarrow$
Mom. after impact $= 0.25 \times 18 = 4.5$ kgm/s $\rightarrow$
$\therefore$ Change in mom $= 2.5 - (-4.5)$
$= 7$ kgm/s

$I = Ft$
$7 = F \times 0.01$
$F = \underline{700 \text{ N}}$

11/19

A man standing on a flatcar provides additional forward propulsion by hurling bricks off the back of the car.

At a certain time the forward velocity of the flatcar together with its load of the man plus 10 bricks is 0.5 m/s. The flatcar plus man has a mass of 200 kg and each brick a mass of 4 kg. Calculate the velocity of the flatcar after the next brick is thrown, if it leaves the man's hand at 5 m/s as shown. (Ignore frictional losses.)



Mass of man, car & 10 Bricks = 240 kg

∴ Mom. = 240 × 0·5 = 120 kgm/s ←

Mom. of man, car & 9 bricks = 236 v kgm/s ←

Mom. of 1 brick = 4×5 = 20 kgm/s →

Total mom. = $236v + (-20) = 120$

$236v = 140$

v = <u>0·59 m/s</u>

11/20

A high energy-rate forming press has a hammer of mass 400 kg. During the operation of the machine the hammer is accelerated vertically downwards and hits the metal slug in the die with a velocity of 150 m/s. If, during a particular forming operation, the hammer is brought to rest in 0.02 seconds, determine the average force exerted by the hammer.

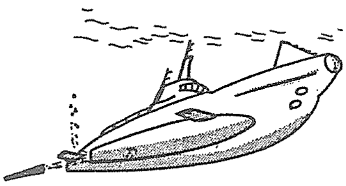
Mom. before impact = 400 × 150 = 60.000 kgm/s

Mom. after impact = 0

∴ Change in mom. = 60.000 kgm/s

$I = Ft = 60000$

F = <u>3 MN</u>



11/21

A submarine of displacement 2000 tonnes fires a torpedo of mass 2 tonnes. If the torpedo is fired from a rear tube with an initial velocity of 30 km/h what momentary increase occurs in the forward velocity of the submarine?

Let x → be the vel. of sub + torpedo, and v → be the increase in vel. after firing.

∴ Sub. vel. after firing = $(x + v)$ km/hr →

and torpedo vel. = $(30 - x)$ km/hr ←

Mom. of $\overrightarrow{\text{Sub + Torp.}}$ = $\overrightarrow{\text{mom. of Sub}}$ − $\overleftarrow{\text{mom. of Torp.}}$

$2002 \times x = 2000(x + v) - 2(30 - x)$

$2002x = 2000x - 2000v - 60 + 2x$

$2000v = 60$

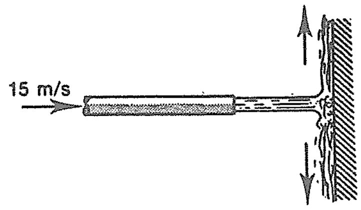
v = <u>0·03 km/hr</u> →

11/22

A water jet from a horizontal pipe has a velocity of 15 m/s when it strikes the vertical wall of a mixing chamber. If 30 litres of water are released every second, determine the force that the water jet exerts on the wall. Note that

(i) the water may be assumed to move parallel to the wall after impact;

(ii) the mass of 1 litre of water may be taken as 1 kg.



$I = Ft = m_1 \vec{v_1} - m_2 \vec{v_2}$ (change in momentum)

$F = \dfrac{30 \times 15 - 30 \times 0}{1}$

$= \underline{450 \text{ N}}$

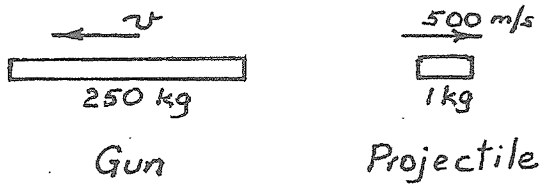
11/23

A gun of mass 250 kg fires a 1-kg projectile with a muzzle velocity of 500 m/s. Determine

(i) the initial recoil velocity of the gun;

(ii) the time taken for the recoil;

(iii) the distance travelled during recoil if the gun moves back against a constant resisting force of 2 kN.



(i) Before firing Mom. = 0

After firing: Gun Mom. = $\overleftarrow{250v}$ kgm/s

Proj. Mom. = $\overrightarrow{1 \times 500}$ kgm/s

$1 \times 500 - 250v = 0$

$\therefore v = \underline{2 \text{ m/s}} \leftarrow$

(ii) $I = Ft$

$= 2000t$

Change in mom. = 500

$2000t = 500$

$t = \underline{0 \cdot 25 \text{ s}}$

(iii) $F = ma$

$2000 = 250a$

$a = -8 \text{ m/s}^2$

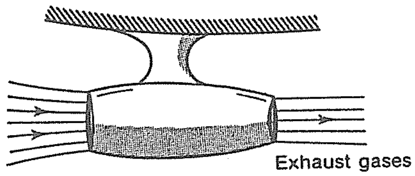
$v^2 = u^2 + 2as$

$s = \underline{0 \cdot 25 \text{ m}}$

$u = 2$ m/s
$v = 0$
$a = -8$ m/s²
$s = ?$
$t = -$

11/24

A jet engine which is stationary in a test rig is consuming air at the rate of 100 kg per second and fuel at the rate of 1 kg per second. If the velocity of the exhaust gases relative to the engine is 600 m/s, what is the thrust of the engine?



Total mass consumed in 1 sec. = 101 kg

Mom. prior to burn = 101×0

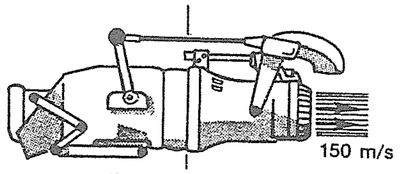
Mom. after burn = 101×600

$\therefore$ Change in mom. = 60600 kgm/s

and Ft (I) = 60600 in 1 sec.

$\therefore F = \underline{60 \cdot 6 \text{ kN}}$

11/25
A marine jet engine in a surf-rescue craft travelling at 36 km/h is consuming 70 litres of water per second and discharging it with a velocity of 150 m/s relative to the boat. What thrust is the engine developing? Take the mass of 1 litre of water as 1 kg.



Forward $\xleftarrow{v}$ $\longrightarrow$ water intake.

As the craft travels forward at 36 km/hr (10 m/s). the water intake is at -10 m/s.

$\therefore$ Mom. of water intake $= 70 \times -10$ kg m/s

& Mom. of water discharge $= 70 \times -150$ kgm/s

$\therefore$ Mom. change $= -700 - (-10500)$

$= 9{,}800$ kgm/s

(I) $Ft = 9.800$

$F = \underline{9 \cdot 8 \text{ kN}}$

11/26
(a) A rocket ejects 20 kg of exhaust gases per second at a velocity of 600 m/s relative to the rocket. Calculate the propulsive force at this instant.
(b) If at a given time the mass of the rocket is 800 kg and it is moving vertically upwards, determine its acceleration.

(a) Relative to rocket, unburnt fuel has no momentum.
After firing fuel gas has a momentum of 20×600 kgm/s

$\therefore$ Change in Mom. $= 12000$ kgm/s

(I) $Ft = 12000$

$F = \underline{12 \text{ kN}}$

(b) $F = m(a + 9 \cdot 8)$

$12000 = 800(a + 9 \cdot 8)$

$a = \underline{5 \cdot 2 \text{ m/s}^2}$

11/27
While in level flight at an altitude of 3500 metres a jet aircraft scoops in 150 kg of air per second and discharges it with a velocity of 800 m/s relative to the aircraft. What thrust is produced by the engine if fuel consumption is 1.25 kg per second and the velocity of the aircraft is 900 km/h?

150 kg of air is travelling with the plane at 250 m/s (900 km/hr)

$\therefore$ Mom. $= 150 \times 250 = 37.500$ kgm/s^2

150 kg of air + 1·25 kg of fuel is discharged at 800 m/s in the opp. direction.

$\therefore$ Mom. of discharge $= 151 \cdot 25 \times 800$ kgm/s

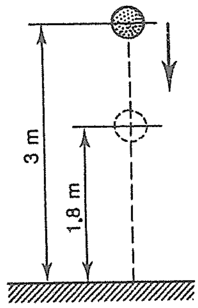
Change in mom. $= (151 \cdot 25 \times 800) - 37.500 = 83{,}500$ kgm/s

(I) $Ft = 83.500$

$F = \underline{83 \cdot 5 \text{ kN}}$

11/28

A ball is dropped from a height of 3 metres on to a concrete path and the height of rebound is measured as 1.8 metres. Determine the coefficient of restitution.



Co-efficient of Restitution (c) equals:

$$\frac{\text{velocity after impact}}{\text{velocity before impact.}}$$

Before

$u = 0$
$v = ?$
$a = 9{\cdot}8\ m/s^2$
$s = 3m$
$t = -$

$v^2 = u^2 + 2as.$
$= 0 + 2 \times 9{\cdot}8 \times 3$
$= 58{\cdot}8$
$v = \underline{7{\cdot}67\ m/s}$

After

$u = ?$
$v = 0$
$a = -9{\cdot}8\ m/s^2$
$s = 1{\cdot}8\ m$
$t = -$

$v^2 = u^2 + 2as$
$0 = u^2 - 2 \times 9{\cdot}8 \times 1{\cdot}8$
$u^2 = 35{\cdot}28$
$u = \underline{5{\cdot}94\ m/s}$

$$C = \frac{5{\cdot}94}{7{\cdot}67}$$

$$= \underline{0{\cdot}77}$$

$$s = r\theta$$
$$v = r\omega$$
$$a = r\alpha$$

12 Circular Motion

12 CIRCULAR MOTION
Angular Displacement (θ). Angular Velocity (ω). Angular Acceleration (α). *Relationships Between Linear and Angular Quantities.* Equations of Rotation. *Graphical Solutions: Motion Diagrams.*

12/7
Express the following angles as radians:
(i) 90°; (ii) 16.5°; (iii) 121°; (iv) 330°.

(i) $\frac{2\pi \times 90}{360} = \frac{\pi}{2} = \underline{1.57 \text{ rad.}}$ (ii) $\frac{16.5 \times 2\pi}{360} = \underline{0.288 \text{ rad.}}$

(iii) $\frac{121 \times 2\pi}{360} = \underline{2.11 \text{ rad.}}$ (iv) $\frac{330 \times 2\pi}{360} = \underline{5.76 \text{ rad.}}$

12/8
Express the following angles in degrees:
(i) 1 rad;
(ii) 19 rad;
(iii) 3π rad.

(i) $1 \text{ rad.} = \frac{1 \times 360}{2\pi} = \underline{57.29^\circ}$

(ii) $19 \text{ rad.} = \frac{19 \times 360}{2\pi} = \underline{1088.62^\circ}$ or $\frac{19}{2\pi} = \underline{3.02 \text{ revs.}}$

(iii) $3\pi \text{ rad.} = \frac{3\pi \times 360}{2\pi} = \underline{540^\circ}$ or $\frac{3\pi}{2\pi} = \underline{1.5 \text{ revs.}}$

12/9
At certain different times during the operation of a machine, its flywheel is rotating with an angular velocity of
(i) 200 rpm;
(ii) 700 rpm;
(iii) 1440 rpm.
Express the angular velocity in each instance as radians per second.

(i) $200 \text{ r.p.m.} = 200 \times 2\pi \text{ rad/m}$
$= \frac{200 \times 2\pi}{60} \text{ rad/s}$
$= \underline{20.94 \text{ rad/s}}$

(ii) $700 \text{ r.p.m.} = \frac{700 \times 2\pi}{60} \text{ rads./s}$
$= \underline{73.3 \text{ rads./s}}$

(iii) $1440 \text{ r.p.m} = \frac{1440 \times 2\pi}{60} \text{ rad/s}$
$= \underline{150.8 \text{ rad/s}}$

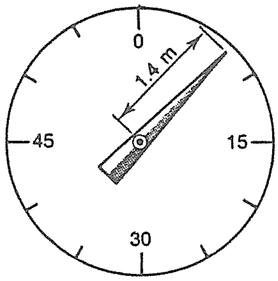
12/10
Convert the following angular velocities to revolutions per minute:
(i) 120 revolutions per second;
(ii) 31 416 radians per minute;
(iii) 50 radians per second.

(i) $120 \text{ r.p.s.} = 120 \times 60 \text{ r.p.m}$
$= \underline{7,200 \text{ r.p.m.}}$

(ii) 31416 rad/m
$= \frac{31416}{2\pi} \text{ rev./m}$
$= \underline{5000 \text{ r.p.m}}$

(iii) 50 rad/s
$= 50 \times 60 \text{ rad/m}$
$= \frac{50 \times 60}{2\pi}$
or $\underline{477.46 \text{ r.p.m.}}$

12/11
The Olympic Pool in Mexico City has an electronic timing device with a large public display incorporating a sweep second hand of 1.4 m effective radius. Determine the angular and linear velocities of the tip of the hand.



$1 \text{ rev/m.} = 2\pi \text{ rad/m}$

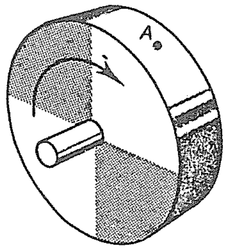
$\therefore \text{Angular vel.} = 2\pi/60 \text{ rad/s} = \underline{0.105 \text{ rad/s}}$

$0.105 \text{ rad/s} = 0.105/2\pi \text{ revs/s}$

$= \dfrac{0.105 \times 2\pi \times 1.4}{2\pi} \text{ m/s}$

$= \underline{0.147 \text{ m/s}}$

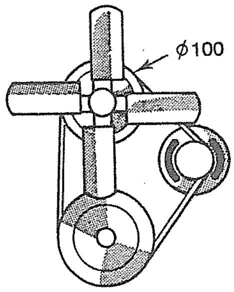
12/12
A flywheel of diameter 0.5 metres is rotating at 1000 rpm. Determine the linear speed of the point *A* on its circumference.



$1000 \text{ r.p.m.} = \dfrac{1000}{60} \text{ r.p.s.}$

$= \dfrac{1000 \times \pi \times 0.5}{60} = \underline{26.2 \text{ m/s}}$

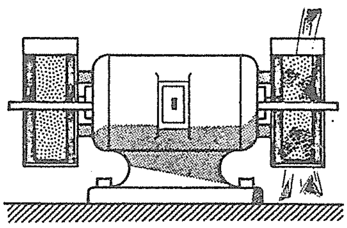
12/13
Determine linear speed (in m/s) of the fan belt of a motor vehicle, when the 100-mm diameter fan pulley is rotating at 4000 revolutions per minute.



$4000 \text{ r.p.m.} = \dfrac{4000}{60} \text{ r.p.s.}$

$= \dfrac{4000 \times \pi \times 100}{60 \times 1000} = \underline{20.94 \text{ m/s}}$

12/14
A grinding wheel of 200 mm diameter is rotating at 1500 rpm when it begins to disintegrate. With what maximum linear velocity will pieces of the wheel be flung off?



$1500 \text{ r.p.m.} = \dfrac{1500}{60} \text{ r.p.s.}$

$= \dfrac{1500 \times \pi \times 200}{60 \times 1000} = \underline{15.7 \text{ m/s}}$

12/15
A water turbine starts from rest and accelerates at a uniform rate to its normal operating speed of 500 rpm in 5 minutes. Determine its angular acceleration (in rad/s²) and its total displacement during acceleration (in radians).

$$500 \text{ r.p.m.} = 500/60 \text{ r.p.s.} = \frac{500 \times 2\pi}{60} \text{ rad/s}$$

$$\alpha = \frac{\omega_1 - \omega_0}{t} = \frac{500 \times 2\pi - 0}{60 \times 5 \times 60} = \underline{0.175 \text{ rad/s}^2}$$

$$s = \omega_0 t + \tfrac{1}{2}\alpha t^2$$

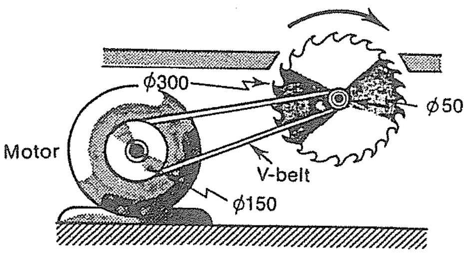
$$= 0 + \tfrac{1}{2} \times 0.175 \times 300^2 \quad \boxed{OR}$$

$$= 7854 \text{ rad.}$$

$$\text{Av. speed} = 500/2 = 250 \text{ r.p.m.} = 250 \times 2\pi \text{ rad/m}$$

$$\therefore s = 250 \times 2\pi \times 5 = \underline{7854 \text{ rad.}}$$

12/16
The elements of a circular saw are shown in the diagram. The driven pulley has a diameter of 50 mm and rotates at 4500 rpm. Given that the saw-blade diameter is 300 mm, determine the linear speed (in m/s) of the tip of one tooth of the saw blade.



$$4500 \text{ r.p.m.} = 4500/60 \text{ r.p.s.}$$

$$\therefore \text{Linear speed of teeth} = \frac{4500 \times \pi \times 300}{60 \times 1000} = \underline{70.7 \text{ m/s}}$$

12/17
A pulley with an initial angular speed of 50 radians per second is accelerated at 10 radians per second squared for 10 seconds. What is its final angular velocity in rps, and what was its displacement during the 10-second period (in revolutions).

$$\omega_t = \omega_0 + \alpha t = 50 + 10 \times 10 = \underline{150 \text{ rad/s}}$$

$$\text{or } 150/2\pi = \underline{23.9 \text{ r.p.s.}}$$

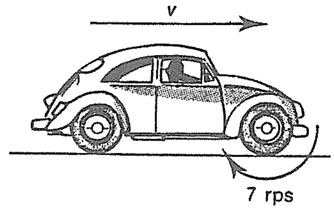
$$s = \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$= 50 \times 10 + \tfrac{1}{2} \times 10 \times 100$$

$$= 1000 \text{ rad.}$$

$$\text{or } 1000/2\pi = \underline{159 \text{ revs.}}$$

12/18
The wheels of a car have diameters of 560 mm. Determine the linear velocity of the car in m/s and km/h if the wheels revolve at 7 revolutions per second.



$$7 \text{ revs/s} = \frac{7 \times \pi \times 560}{1000} \text{ m/s} = \underline{12.3 \text{ m/s}} \text{ or } \underline{44.3 \text{ km/hr}}$$

12/19
Determine the angular and linear velocities of the extremities of the hour and minute hands of the Town Hall clock, given that their respective lengths, as measured from their pivot points, are 1 and 1.3 metres. Use rad/s and m/s for your answers.

HOUR HAND

1 rev or $2\pi \times 1$ m in 12 hrs

$$= \frac{2\pi \times 1}{12 \times 3{,}600} \text{ m/s}$$

$$= \underline{145 \times 10^{-6} \text{ m/s}}$$

1 rev or 2π rad in 12 hrs

$$= \frac{2\pi}{12 \times 3600} \text{ rad/s}$$

$$= \underline{145 \times 10^{-6} \text{ rad/s}}$$

MINUTE HAND

1 rev or $2\pi \times 1.3$ m in 1 hr

$$= \frac{2\pi \times 1.3}{1 \times 3600} \text{ m/s}$$

$$= \underline{22.6 \times 10^{-4} \text{ m/s}}$$

1 rev or 2π rad in 1 hr

$$= \frac{2\pi}{1 \times 3600} \text{ rad/s}$$

$$= \underline{17.5 \times 10^{-4} \text{ rad./s}}$$

12/20
A boy riding his bicycle along a level road slows from 15 km/h to rest in 15 metres. If the bicycle wheels have diameters of 0.76 metres, determine
(i) the average deceleration of the bicycle (in m/s^2);
(ii) the angular retardation of the wheels (in rad/s^2).

15 km/hr = 4.167 m/s

(i) $u = 4.167$ m/s
$v = 0$
$a = ?$
$s = 15$ m

$v^2 = u^2 + 2as$
$0 = 17.36 + 30a$
$a = \underline{-0.58 \text{ m/s}^2}$

(ii) $4.167 \text{ m/s} = \frac{4.167}{\pi \times 0.76}$ rev/s

$$= \frac{4.167 \times 2\pi}{\pi \times 0.76}$$

$$= 10.966 \text{ rad/s}$$

$u = 4.167$ m/s
$v = 0$
$a = -0.58$ m/s^2
$s = -$
$t = ?$

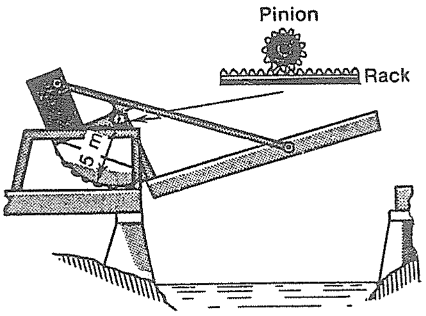
$v = u + at$
$0 = 4.167 - 0.58t$
$t = 7.18$ s.

$$\alpha = \frac{\omega_t - \omega_0}{t}$$

$$= \frac{10.966 - 0}{7.18}$$

$$= \underline{1.53 \text{ rad/s}^2}$$

12/21
The bascule bridge is opened by winding a pinion along a rack on each side of the bridge.
(i) If the pinion is 250 mm in diameter and revolves at 5 rpm, what is the angular velocity of the bridge deck?
(ii) How long will it take to open the bridge through 60°?



(i) $\dfrac{\text{Ratio of Pinion Dia.}}{\text{10 m Wheel Dia.}} = \dfrac{250}{10 \times 1000}$

$\therefore$ If pinion revolves at 5 r.p.m. wheel will revolve at $\dfrac{5 \times 250}{10 \times 1000}$

or $\underline{0 \cdot 125 \text{ r.p.m}}$

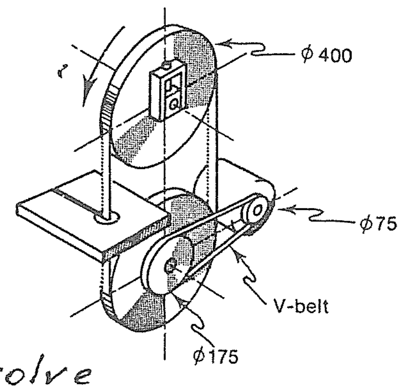
(ii) $0 \cdot 125$ revs. in 1 minute

$= 0 \cdot 125/60$ r.p.s.

$\therefore$ for $60°$ $\left(\frac{1}{6} \text{ rev.}\right)$ $t = \dfrac{1}{6} \div \dfrac{0 \cdot 125}{60}$

$= \underline{80 \text{ s}}$

12/22
A bandsaw motor runs at 1425 rpm. The motor pulley has a diameter of 75 mm, the drive pulley has a diameter of 175 mm and the driving wheels have diameters of 400 mm each. Determine the linear speed of the saw blade as it passes the cutting table.



$\dfrac{\text{Motor Pulley Dia.}}{\text{Saw Pulley Dia.}} = \dfrac{75}{175}$

$\therefore$ Saw Pulley and Driving Wheels revolve at $\dfrac{1425 \times 75}{175} = 610 \cdot 7$ r.p.m.

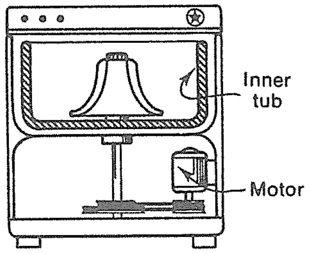
Linear speed of Driving Wheel rim

$= \dfrac{610 \cdot 7 \times \pi \times 400}{60 \times 1000}$ m/s

$= \underline{12 \cdot 8 \text{ m/s}}$

12/23

The spindrier of a washing machine accelerates from rest at 30 rpm per second, reaches its maximum velocity, and immediately begins to decelerate at 10 rpm per second. If the spin cycle takes 4 minutes, determine by using a suitable velocity–time graph the maximum velocity of the spindrier and the time taken to reach this velocity.



30 r.p.m/s. = 1800 r.p.m/m

10 r.p.m/s = 600 r.p.m/m

Acceleration

Time(s)	0	20	40	60	80
Vel. R.P.M.	0	600	1200	1800	2400

Deceleration

Time(s)	240	180	120	60	0
Vel. R.P.M.	2400	1800	1200	600	0

$\omega_0 = 0$ r.p.m.

$\omega_t = ?$

$\alpha = 30$ r.p.m/s

$t =$ (i) 20 s, (ii) 40 s, (iii) 60 s, (iv) 80 s

(i) $\omega_t = \omega_0 + \alpha t = 0 + 30 \times 20 = \underline{600 \text{ r.p.m.}}$

(ii) $= 0 + 30 \times 40 = \underline{1200 \text{ r.p.m}}$

(iii) $= 0 + 30 \times 60 = \underline{1800 \text{ r.p.m.}}$

(iv) $= 0 + 30 \times 80 = \underline{2400 \text{ r.p.m.}}$

$\omega_0 = ?$

$\omega_t = 0$

$\alpha = -10$ r.p.m/s

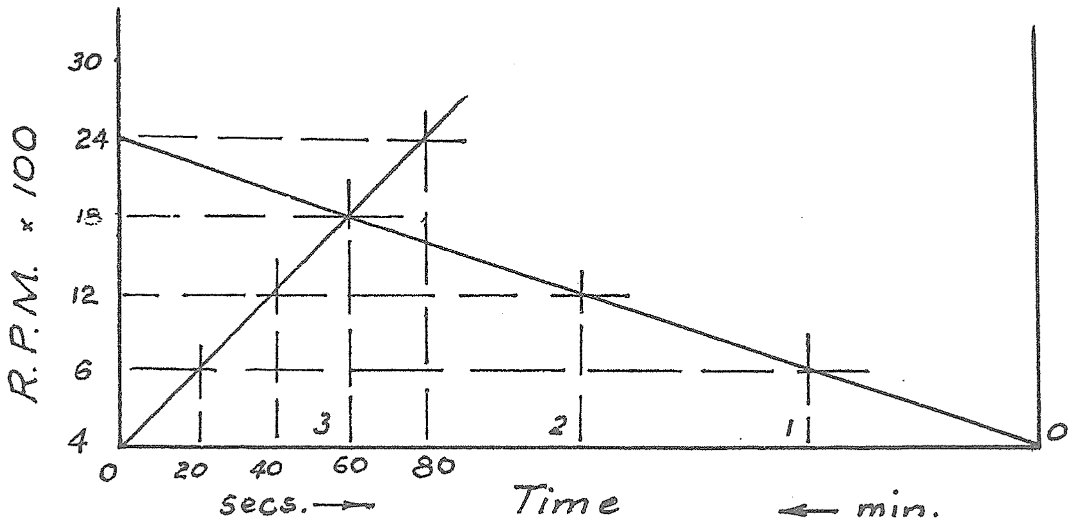
$t =$ (i) 240 s, (ii) 180 s, (iii) 120 s, (iv) 60 s

(i) $\omega_t = \omega_0 + \alpha t$; $0 = \omega_0 - 10 \times 240$; $\omega_0 = \underline{2400 \text{ r.p.m}}$

(ii) $0 = \omega_0 - 10 \times 180$; $\omega_0 = \underline{1800 \text{ r.p.m}}$

(iii) $0 = \omega_0 - 10 \times 120$; $\omega_0 = \underline{1200 \text{ r.p.m}}$

(iv) $0 = \omega_0 - 10 \times 60$; $\omega_0 = \underline{600 \text{ r.p.m}}$



Max. vel. = $\underline{\text{1800 r.p.m after 1 min}}$

12/24
Two gears of equal diameters are rotating on adjoining shafts. One is rotating at a constant angular velocity of 140 rev/min in a clockwise direction. The other accelerates from rest at a constant angular acceleration of 2 rad/s² in an anti-clockwise direction. At what instant can the two gears be smoothly meshed together? Solve the problem analytically and graphically.

1st Gear

$140 \text{ r.p.m} = 140/60 \text{ r.p.s}$

$= \frac{140 \times 2\pi}{60} \text{ rad/s}$

$= 14{\cdot}66 \text{ rad./s}$

2nd. Gear

$\omega_0 = 0$

$\omega_t = 14{\cdot}66 \text{ rad/s}$

$\alpha = 2 \text{ rad/s}^2$

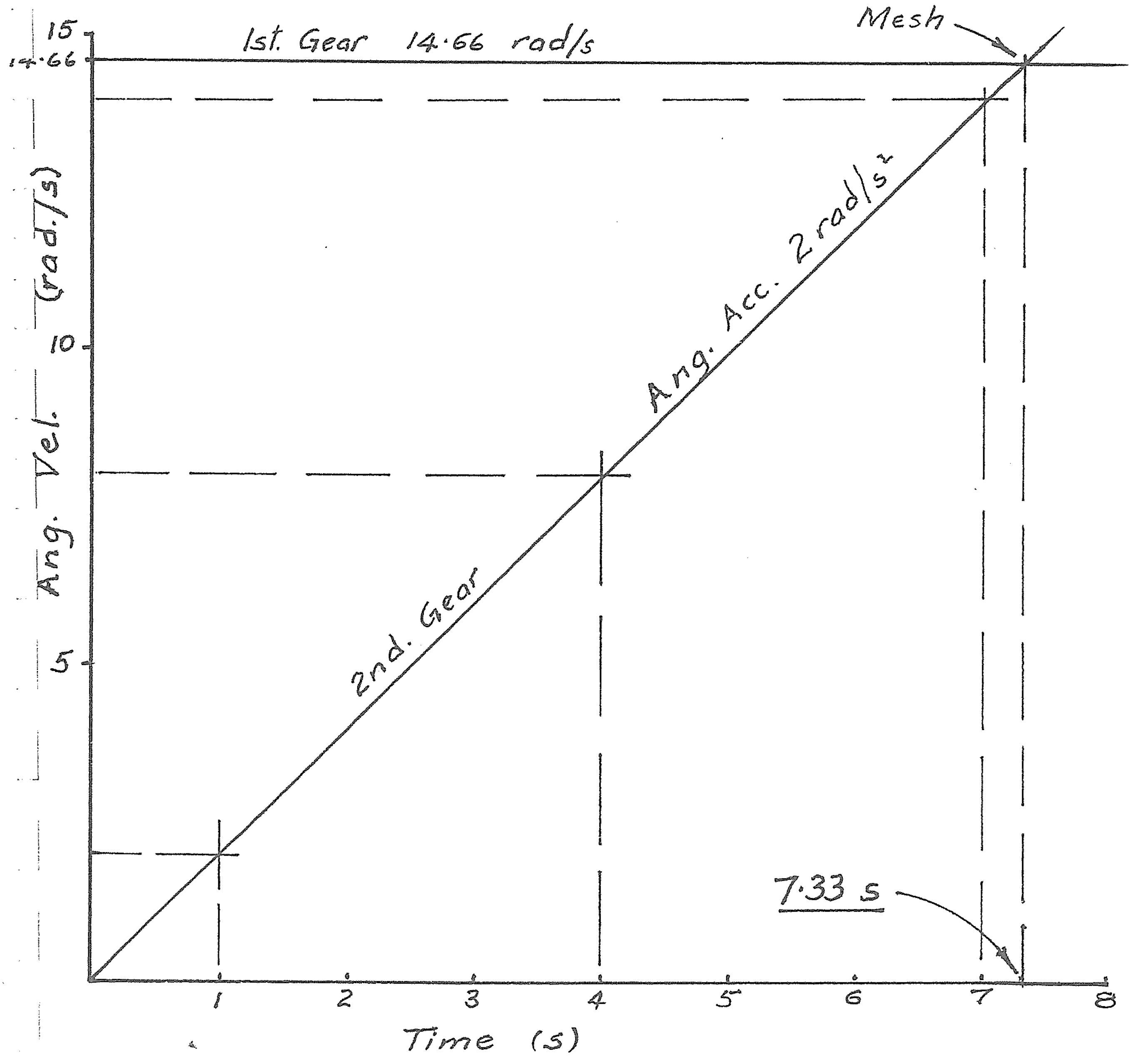
$\theta = \text{—}$

$t = ?$

$\omega_t = \omega_0 + \alpha t$

$14{\cdot}66 = 0 + 2t$

$t = \underline{7{\cdot}33 \text{ s}}$

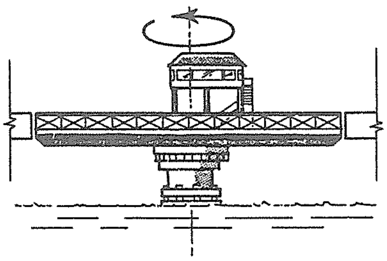


12/25

Pyrmont bridge has a centre section which opens in a horizontal arc in order to allow cargo vessels to enter the docks. The bridge takes $2\frac{1}{2}$ minutes to turn through 90°, its motion consisting of an initial constant angular acceleration for 60 seconds, a period of uniform angular velocity for 50 seconds, followed by deceleration to rest in the last 40 seconds.

Determine, with the aid of a graph of angular velocity against time,

(i) the maximum angular velocity;
(ii) the angular acceleration; and
(iii) the angular deceleration of this section of the bridge.



Let ω_t be max. ang. vel.

$\therefore\ \omega_t/2$: Av. vel. during accel. and decel.

(i) It follows that:

$$\omega_t/2 \times 60 : \theta_1$$
$$\omega_t \times 50 : \theta_2 \qquad \text{But } \theta_1 + \theta_2 + \theta_3 : \tfrac{\pi}{2} \text{ rad.}$$
$$\& \quad \omega_t/2 \times 40 : \theta_3$$
$$\therefore\ 100\,\omega_t : \pi/2$$
$$\text{and } \omega_t : \underline{1{\cdot}57 \times 10^{-2} \text{ rad/s}}$$

(ii) $\omega_o : 0$ $\quad \omega_t : \omega_o + \alpha t$

$\omega_t : \pi/200$ rad/s

$\alpha : ?$ $\quad \dfrac{\pi}{200} : 0 + 60\alpha$

$t : 60$s

$\alpha : \underline{2{\cdot}62 \times 10^{-4} \text{ rad/s}^2}$

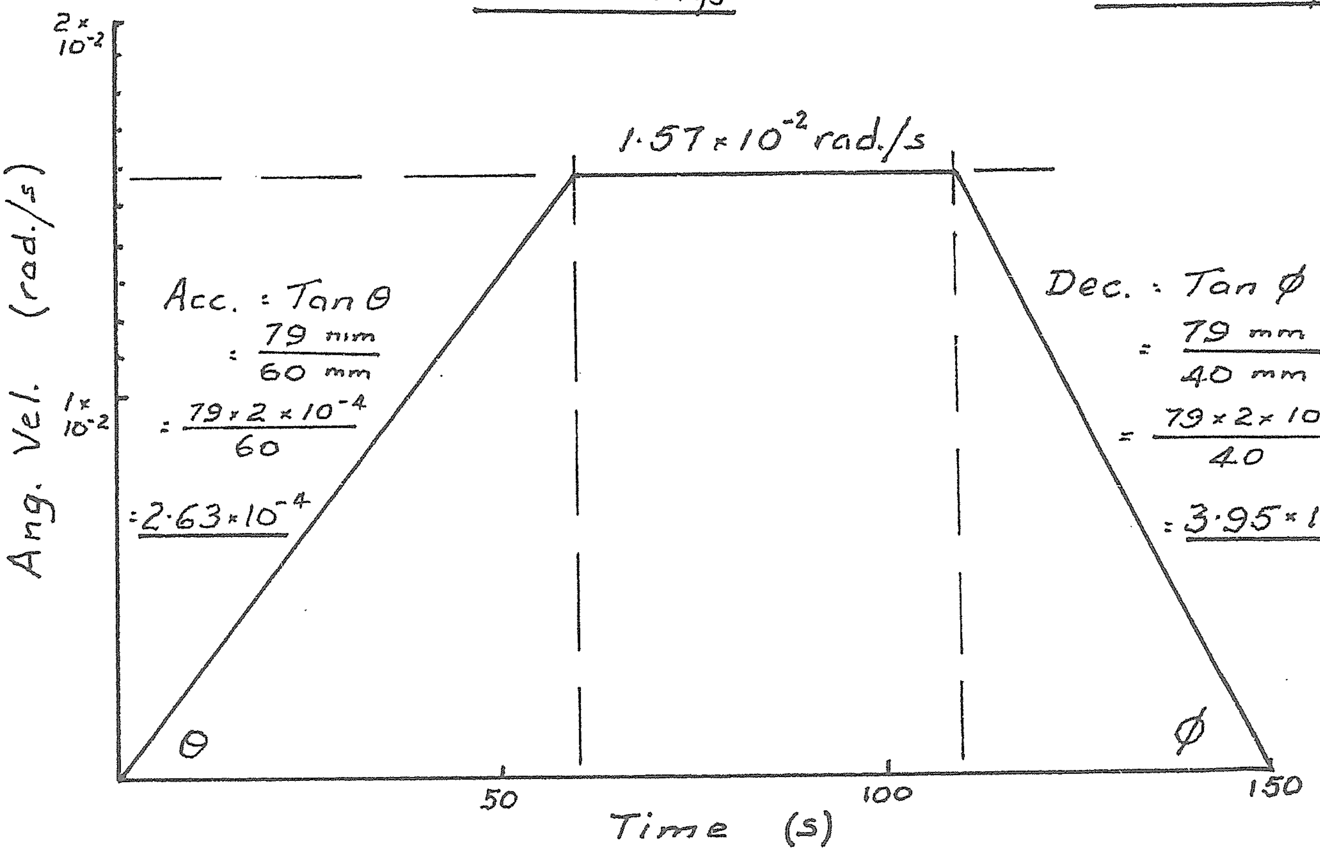
(iii) $\omega_o : \pi/200$ rad/s

$\omega_t : 0$ $\quad \omega_t : \omega_o + \alpha t$

$\alpha : ?$ $\quad 0 : \pi/200 + 40\alpha$

$t : 40$s

$\alpha : \underline{-3{\cdot}93 \times 10^{-4} \text{ rad/s}}$



12/26

Friction in the shaft bearings brings a flywheel to rest in 3 minutes. If the angular speed of the flywheel was 1800 rpm, determine

(i) the deceleration due to friction;

(ii) the total number of revolutions turned during deceleration (that is, displacement);

(iii) the time taken to complete the first half of these revolutions.

(i) $\omega_0 = 1800$ r.p.m.

$\omega_t = 0$

$\alpha = ?$

$t = 3$ min.

$\omega_t = \omega_0 + \alpha t$

$0 = 1800 + 3\alpha$

$\alpha = \underline{-600 \text{ r.p.m/m}}$

(ii) $\omega_0 = 1800$ r.p.m.

$\omega_t = 0$

$\alpha = -600$ r.p.m./m

$\theta = ?$

$t = 3$ min.

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$= 1800 \times 3 + \frac{1}{2} \times -600 \times 9$

$= \underline{2700 \text{ revolutions}}$

(iii) $\omega_0 = 1800$ r.p.m.

$\alpha = -600$ r.p.m/m

$\theta = 1350$ revs.

$t = ?$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$1350 = 1800t - \frac{1}{2} \times 600 t^2$

$2t^2 - 12t + 9 = 0$

$t = \frac{12 \pm \sqrt{144 - 72}}{4}$

$= \underline{52 \cdot 8 \text{ s.}}$ or $307 \cdot 2$ s.

<u>NOTE:</u> It would take 307·2 seconds for the wheel to come to rest (after 2700 revs.), start accelerating at 600 r.p.m/m and complete the next 1350 revs.
i.e. 4050 revs. in all.

13 Rotational Motion

$$a_c = \frac{v^2}{r}$$
$$F_c = \frac{mv^2}{r} = m\omega^2 r$$
$$T = I\alpha$$
$$M = I\omega$$

$g : 9{\cdot}8\ m/s^2$

13 ROTATIONAL MOTION

Calculating Centripetal Acceleration and Centripetal Force. Centrifugal Force. *Superelevation of Roads and Railway Curves.* Torque and Angular Acceleration. *Moment of Inertia. Radius of Gyration (k). Angular Momentum and Impulse.*

13/13
A flywheel of diameter 1.5 metres is rotating at 600 rpm. Determine the acceleration of any point on the outer rim of the flywheel.

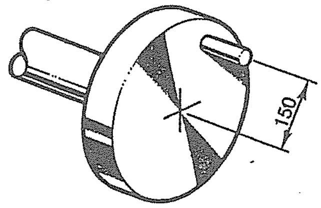
Linear speed of point – $v : r\omega$

$: \frac{0{\cdot}75 \times 600 \times 2\pi}{60}$

$= 47{\cdot}12\ m/s$

$A_c : \frac{v^2}{r}$

$= \frac{47{\cdot}12^2}{0{\cdot}75}$ $= \underline{2961\ m/s^2 \text{ towards the centre}}$



13/14
A flywheel attached to a small engine is revolving at 1000 rpm. A crank pin attached to the face of the flywheel is located 150 mm from the centre of the flywheel. Determine the acceleration acting on this crank pin.

Linear speed of pin – $v : r\omega$

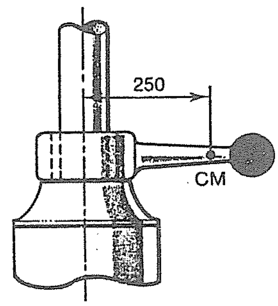
$: \frac{150 \times 1000 \times 2\pi}{60 \times 1000}$

$= \underline{15{\cdot}707\ m/s}$

$A_c = \frac{v^2}{r}$

$= \frac{15{\cdot}707^2 \times 1000}{150}$

$= \underline{1645\ m/s^2}$



13/15
Part of a governor mechanism consists of a vertical shaft about which an arm of mass 2 kg rotates. Determine the force exerted on the shaft bearing when the arm rotates at 10 rad/s.

$v = r\omega$

$= \frac{250 \times 10}{1000}$

$= 2{\cdot}5\ m/s$

$F_c = \frac{mv^2}{r}$

$= \frac{2 \times 2{\cdot}5^2 \times 1000}{250}$

$= \underline{50\ N}$

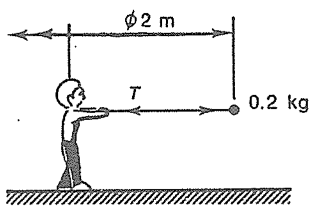
$\boxed{OR}$

$F_c = m\omega^2 r$

$= \frac{2 \times 100 \times 250}{1000}$

$= \underline{50\ N}$

13/16
A lead weight is attached to the end of a piece of string and whirled around in a circle of diameter 2 metres. If the string remains horizontal and the lead has a mass of 0.2 kg, determine
(i) the tension, T, in the string when the angular velocity is 1 revolution per second;
(ii) the minimum angular velocity necessary to break the string, if the maximum tensile load it can sustain is 70 N.



(i) $1 \text{ r.p.s} = 2\pi \text{ rad/s}$

$F_c = m\omega^2 r$

$= 0.2 \times (2\pi)^2 \times 1$

$= \underline{7.89 \text{ N}}$

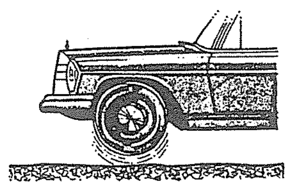
(ii) $F_c = m\omega^2 r$

$70 = 0.2\,\omega^2 \times 1$

$\omega = \underline{18.7 \text{ rad/s}}$

or $\underline{2.97 \text{ r.p.s.}}$

13/17
After being dynamically tested for balance, the front wheel of a car is found to need a 60-g balance weight fixed to the 500 mm diameter rim. What extra load would have been placed on the wheel bearings when the wheel rotated at 500 rpm if the wheel had not been balanced?

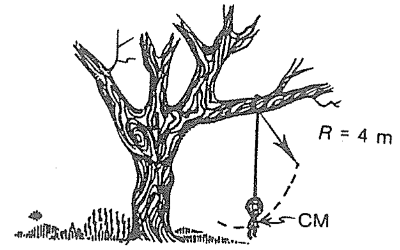


$$500 \text{ r.p.m.} = \frac{500 \times 2\pi}{60} \text{ rad/s}$$

$$F_c = \frac{60}{1000} \times \frac{500 \times 500 \times 4\pi^2}{60 \times 60} \times \frac{250}{1000}$$

$$= \underline{41.1 \text{ N}}$$

13/18
A girl of mass 25 kg is swinging on the end of a rope tied to the branch of a tree. The girl's centre of mass moves in a radius of 4 metres and at the bottom of each swing she moves with a horizontal velocity of 8 m/s. Determine the tension in the rope in this vertical position.



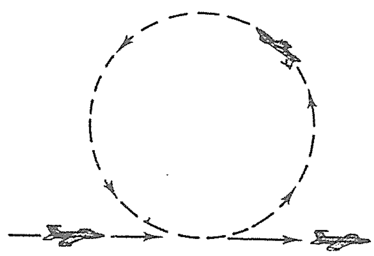
$$F_c = \frac{m v^2}{r}$$

$$= \frac{25 \times 8 \times 8}{4} = \underline{400 \text{ N}}$$

Tension $= F_c + mg$

$= \underline{645 \text{ N}}$

13/19
A plane is flying in a vertical circle (loop) of 200 metres radius. Determine the angular and linear velocities (in rad/s and m/s) of the plane at the highest point of the loop if the pilot experiences "weightlessness" at this stage of the manoeuvre. The pilot has a mass of 90 kg.



For weightlessness $F_c = mg = 882 \text{ N}$

(a) $F_c = m\omega^2 r$

$882 = 90\,\omega^2 \times 200$

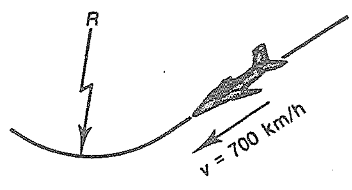
$\omega = \underline{0.22 \text{ rad/s}}$

(b) $F_c = \dfrac{m v^2}{r}$

$882 = \dfrac{90 v^2}{200}$

$v = \underline{44.3 \text{ m/s}}$

13/20
Pilots can withstand accelerations of up to 9 g for short periods without "blacking out" when wearing protective flying suits. Determine the minimum radius of curvature, R, with which a fighter pilot may turn his plane upward out of a dive if the speed of the plane is 700 km/h at the end of the dive.



$$700 \text{ km/hr} = 194{\cdot}4 \text{ m/s}$$

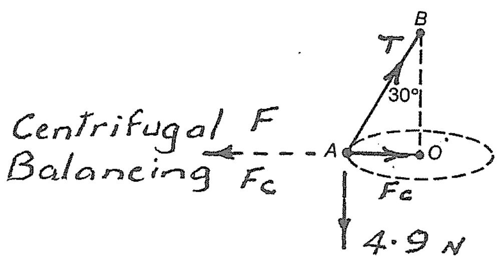
$$F_c = \frac{mv^2}{r}$$

$$9\cancel{m}g = \frac{\cancel{m} \times 194{\cdot}4^2}{r}$$

$$r = \frac{194{\cdot}4^2}{9 \times 9{\cdot}8}$$

$$= \underline{428{\cdot}5 \text{ m}}$$

13/21
A ball of mass 0.5 kg is fastened to one end, A, of a piece of string of length 250 mm, the other end being attached to the fixed pivot, B. The ball rotates in a horizontal circle of radius OA so that the string is always inclined at 30° to the vertical as shown. Determine
(i) the angular velocity of the ball;
(ii) the tension in the string;
(iii) the total acceleration acting on the ball.



(ii) Resolve vert.

$$T\cos 30° = 4{\cdot}9$$

$$T = \underline{5{\cdot}66 \text{ N}}$$

(i) Resolve horiz.

$$\text{Centrifugal F} \therefore F_c = T\sin 30°$$

$$= 2{\cdot}83 \text{ N}$$

$$F_c = m\omega^2 r$$

$$\omega^2 = \frac{F_c}{mr}$$

$$= \frac{2{\cdot}83 \times 1000}{0{\cdot}5 \times 125}$$

$$\therefore \omega = \underline{6{\cdot}73 \text{ rad/s}}$$

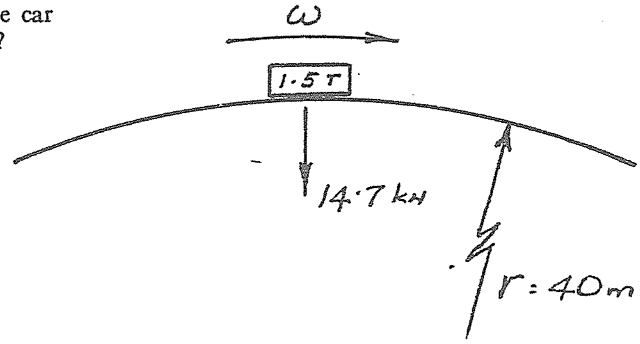
(iii) $$a_c = \frac{v^2}{r}$$

$$v = \frac{6{\cdot}73 \times 125}{1000} = 0{\cdot}84 \text{ m/s}$$

$$a_c = \frac{0{\cdot}84^2 \times 1000}{125}$$

$$= \underline{5{\cdot}66 \text{ m/s}^2}$$

13/22
An automobile of mass 1.5 tonnes travels over the crest of a hill whose road-bed is part of a vertical circle of radius 40 metres.
(i) Determine the maximum (linear) speed (in km/h) of the car over the crest if its wheels are not to leave the road.
(ii) If this speed is exceeded, how does the car move in relation to the crest of the road?



(i) $F_c = m\omega^2 r$

$\omega^2 = \dfrac{F_c}{m \cdot r}$

$= \dfrac{14.7 \times 1000}{1500 \times 40}$

$\omega = 0.495 \text{ rad/s}$

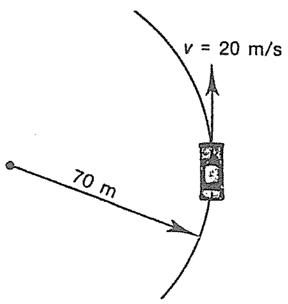
$v = r\omega$

$= 40 \times 0.495$

$= \underline{19.8 \text{ m/s} \text{ or } 71.3 \text{ km/hr}}$

(ii) With $v > 71.3$ km/hr the car will leave the road initially tangentially then in a parabola

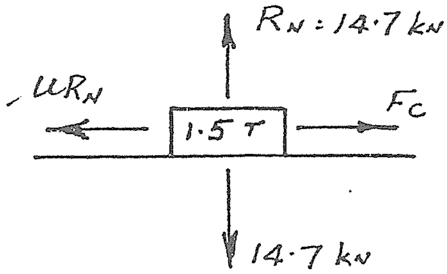
13/23
A car of mass 1.5 tonnes is travelling around a curve of radius 70 metres with a velocity of 20 m/s. Determine
(i) the coefficient of friction between the tyres and the road if the car is on the point of skidding.
(ii) the acceleration acting on the driver in this situation.



(i) $F_c = \dfrac{m v^2}{r}$

$= \dfrac{1500 \times 20 \times 20}{70 \times 1000}$

$= 8.57 \text{ kN}$



Resolve horiz.

$\mu R_N = 8.57$

$\mu = 8.57/14.7$

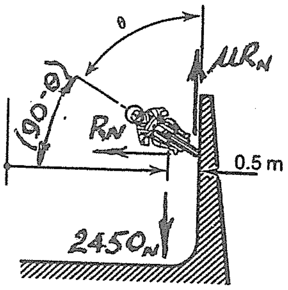
$= \underline{0.58}$

(ii) $a_c = v^2/r = \dfrac{20 \times 20}{70} = \underline{5.7 \text{ m/s}^2}$

13/24

In the "wall of death" act at a sideshow, a stunt rider drives his motor bike in a horizontal circle around the inside of a cylindrical track. The cylindrical "wall of death" has an inside diameter of 10.5 metres and the bike plus rider a total mass of 250 kg. Determine the minimum linear speed (in km/h) that the rider must maintain in order to remain in this position, and the angle θ.

The coefficient of sliding friction between the wall and wheels is 0.6 and the centre of mass of the bike plus rider is 0.5 m radially from the wall as shown.



$\tan(90-\theta) = 0{\cdot}6$

$\therefore \theta = \underline{59{\cdot}04^\circ}$

Resolve vert.

$\mu R_N = 2450$

$R_N = 4083{\cdot}3\ N$

For equilibrium R_N (F_c) = Centrifugal Force

$$F_c = \frac{mv^2}{r}$$

$$4083{\cdot}3 = \frac{250\,v^2}{\left(\frac{10.5}{2} - 0.5\right)}$$

$v^2 = 77{\cdot}58$

$v = 8{\cdot}8\ m/s$ or $\underline{31{\cdot}7\ km/hr}$

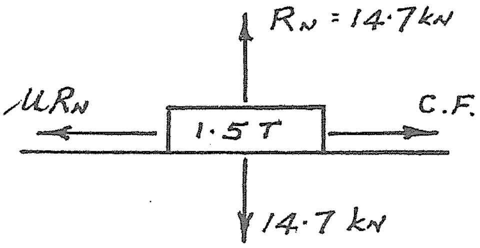
13/25

A car of mass 1.5 tonnes rounds a curve of radius 100 m. Determine its safe maximum speed (that is, speed at which skidding does not occur)

(i) if the roadway is horizontal;

(ii) if the roadway is super-elevated to an angle of 20°.

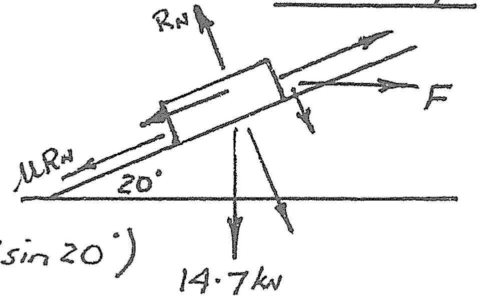
The coefficient of friction between the wheels and roadway is 0.58 (take $\tan^{-1} 0.58 = 30°$).



(i) $C.F = \mu R_N = 8526\ N$

$$C.F = \frac{m v^2}{r}$$

$$v^2 = \frac{8526 \times 100}{1{\cdot}5 \times 1000} \quad \therefore v = 23{\cdot}84\ m/s \text{ or } \underline{85{\cdot}8\ km/hr}$$



(ii) Resolve ⊥ to plane

$R_N = 14{\cdot}7\cos 20^\circ + F\sin 20^\circ$

Resolve // to plane

$F\cos 20^\circ = 14{\cdot}7\sin 20^\circ + \mu(14{\cdot}7\cos 20^\circ + F\sin 20^\circ)$

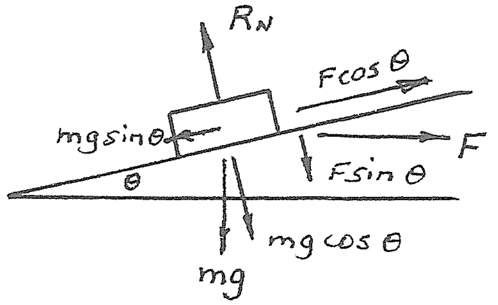
$F(\cos 20^\circ - 0{\cdot}58\sin 20^\circ) = 13{\cdot}04$

$F = 17{\cdot}59\ kN$

$$F_c = \frac{m v^2}{r}$$

$$v^2 = \frac{17{\cdot}59 \times 1000 \times 100}{1{\cdot}5 \times 1000} \quad \therefore v = 34{\cdot}24\ m/s \text{ or } \underline{123{\cdot}3\ km/hr}$$

13/26
Determine the required super-elevation for a railway of standard gauge 1.435 metres if the equilibrium (normal) speed is 72 km/h and the radius of the curve is 600 metres. (*Hint:* Consider that, at equilibrium speed, the flange pressure between the outside wheels and the rails is zero.)



72 km/hr : 20 m/s

Resolve // to plane

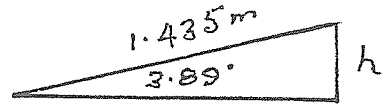
$F \cos \theta = mg \sin \theta$

$\therefore F = mg \tan \theta$

$F_c = \dfrac{m v^2}{r}$

$\cancel{m} g \tan \theta : \dfrac{\cancel{m} \times 400}{600}$

$\theta : 3 \cdot 89°$

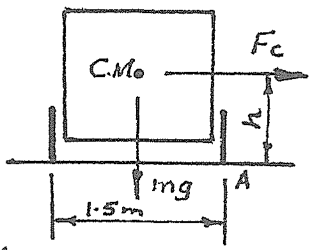


$\sin . 3 \cdot 89° : \dfrac{h}{1435}$

$h : \underline{97 \cdot 35 \text{ mm}}$

13/27
An electric trolley car with an effective track of 1.5 m turns a corner of radius 10 metres at a speed of 20 km/h. If the trolley way is level, determine how high the centre of mass can be without allowing the trolley to tip over.

20 km/hr : 5·55 m/s



$F_c : \dfrac{m v^2}{r}$

$= \dfrac{m \times 30 \cdot 8}{10}$

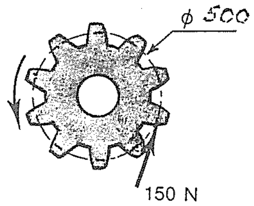
$= 3 \cdot 08 m \text{ N}$

Take moments about A

$3 \cdot 08 m \times h = mg \times 0 \cdot 75$

$h = \underline{2 \cdot 4 \text{ m above road}}$

13/28
Determine the torque produced when the 150-N force acts tangentially on the gear tooth as shown. The effective radius of the gear is 250 mm.

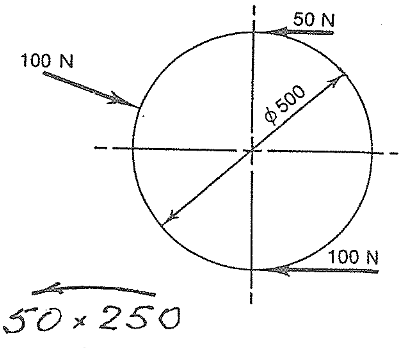


Torque : $F \times$ rad.

$: \dfrac{150 \times 250}{1000}$

$= \underline{37 \cdot 5 \text{ Nm}}$

13/29
Determine the total torque on the flywheel when it is subjected to the three forces as shown.

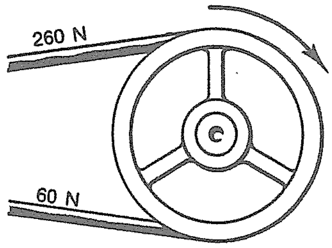


The 100N through wheel centre produces no torque.

Total torque $= \overrightarrow{100 \times 250} - \overleftarrow{50 \times 250}$

$: \underline{12 \cdot 5 \text{ Nm (clockwise)}}$

13/30
A driving pulley attached to an electric motor has a diameter of 400 mm. When rotating at 1000 rpm, the belt tensions are 60 newtons on the slack side and 260 newtons on the tight side. What torque is being produced? (*Hint:* The available tangential force is equal to the difference between the taut and slack tensions of the belt.)

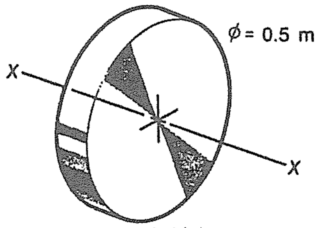


Available force to produce Torque $= 260 - 60$
$= 200\ N$

$\therefore$ Torque produced $= \dfrac{200 \times 200}{1000}$

$= \underline{40\ Nm}$

13/31
Determine the moment of inertia and the radius of gyration of the flywheel about its axis of rotation (X—X), if its mass is 50 kg.



Flywheel thickness = 50 mm

Rad. of Gyr. $k = \dfrac{r}{\sqrt{2}} = \dfrac{0{\cdot}25}{\sqrt{2}} = \underline{0{\cdot}177\ m}$

$I = \dfrac{mr^2}{2}$

$= \dfrac{50 \times 0{\cdot}25^2}{2} = \underline{1{\cdot}56\ kgm^2}$

13/32
An aeroplane propeller has a mass of 60 kg and a radius of gyration of 600 mm. Determine
(i) its moment of inertia;
(ii) the unbalanced torque necessary to cause the propeller to revolve with an acceleration of 25 rad/s^2.

(i) $I = mk^2$

$= \dfrac{60 \times 600 \times 600}{1000 \times 1000}$

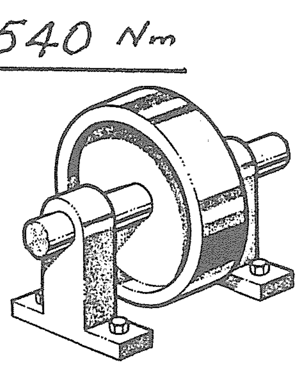
$= \underline{21{\cdot}6\ kgm^2}$

(ii) $T = I\alpha$

$= 21{\cdot}6 \times 25$

$= \underline{540\ Nm}$

13/33
A shaft and flywheel of mass 100 kg is revolving at 500 rpm. The radius of gyration of this shaft and flywheel is 1 metre and the friction torque present in the bearings is 50 N m. How long does it take for the flywheel to come to rest?



$T = mr^2\alpha$

$50 = 100 \times 1^2 \times \alpha$

$\alpha = -0{\cdot}5\ rad/s^2$

$\omega_t = \omega_0 + \alpha t$

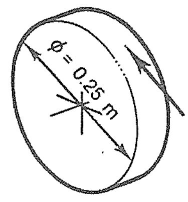
$0 = \dfrac{500 \times 2\pi}{60} - 0{\cdot}5t$

$t = \underline{104{\cdot}7\ s.}$

13/34

A wheel of diameter 0.25 m and moment of inertia 0.5 $kg\,m^2$ is acted on by a constant tangential force of 200 N. Determine

(i) the resultant angular acceleration of the wheel;
(ii) the angular speed (in rad/s) of the wheel after 4 seconds (from rest);
(iii) the number of revolutions turned in the 4-second period.



(i) $T = 200 \times 0.125$
$= 25\ Nm$

& $I = mk^2$
$0.5 = m \times 0.125^2$
$m = 32\ kg$

$T = mr^2\alpha$
$25 = 32 \times 0.125^2 \times \alpha$
$\therefore \alpha = \dfrac{25}{32 \times 0.125^2} = \underline{50\ rad/s^2}$

(ii) $\omega_t = \omega_0 + \alpha t$
$= 0 + 50 \times 4$
$= \underline{200\ rad/s}$

(iii) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$= 0 + \frac{1}{2} \times 50 \times 16$
$= 400$ rad. or $\underline{63.7\ revs.}$

13/35

Determine the angular momentum of a flywheel, with moment of inertia of 20 $kg\,m^2$, when it is revolving at

(i) 100 rad/s;
(ii) 500 rpm.

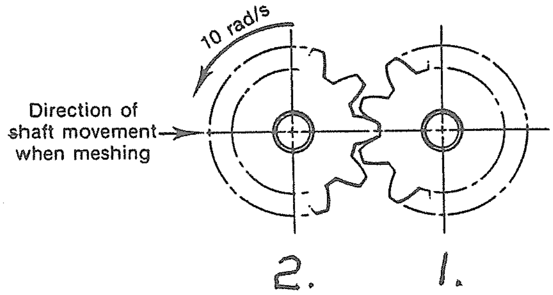
(i) Angular Momentum $M = I\omega$
$= 20 \times 100$
$= \underline{2000\ kgm^2/s}$

(ii) $\omega = \dfrac{500 \times 2\pi}{60}$

$= 52.36\ rad/s$

$M = I\omega$
$= 20 \times 52.36$
$= \underline{1047.2\ kgm^2/s}$

13/36
Two identical gear wheels on parallel shafts have moments of inertia of 1 kg m². At a given instant, one gear wheel is rotating at 10 rad/s clockwise when it is meshed with the other gear wheel. Determine the common rotational velocity of both gear wheels, if the second gear wheel was
(i) stationary;
(ii) rotating at 2 rad/s clockwise;
(iii) rotating at 2 rad/s anti-clockwise at the time of meshing.



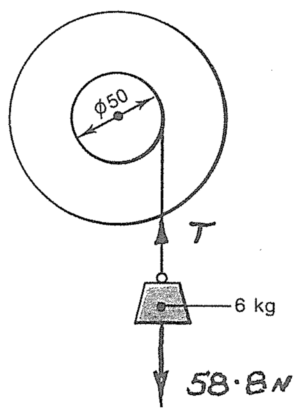
(i) Mom. of Wheel 1. = 0
Mom. of Wheel 2. = $I\omega$
$= 10\ \text{kgm}^2/\text{s}$

$I_1\omega_1 + I_2\omega_2 = 10$
$2I\omega = 10$
$\therefore \omega = \underline{5\ \text{rad/s}}$

(ii) Mom. of Wheel 1. = 1×-2
Mom. of Wheel 2. = 1×10
$I_1\omega_1 + I_2\omega_2 = 10 + (-2)$
$2I\omega = 8$
$\omega = \underline{4\ \text{rad/s}}$

(iii) Mom. of Wheel 1. : $1 \times +2 = 2\ \text{kgm}^2/\text{s}$
Mom. of Wheel 2. = $1 \times 10 = 10\ \text{kgm}^2/\text{s}$
$I_1\omega_1 + I_2\omega_2 = 10 + 2$
$2I\omega = 12$
$\omega = \underline{6\ \text{rad/s}}$

13/37
A wheel and axle is free to rotate about its horizontal axis. The 6-kg weight attached to the cord wrapped around the axle is allowed to fall freely a distance of 4 m from rest. Determine the moment of inertia of the wheel and axle if the weight takes 4 seconds to fall this distance. (Neglect friction.)



Linear accel. of 6 kg mass
$u = 0$ $\quad s = ut + \frac{1}{2}at^2$
$v = -$ $\quad 4 = 0 + \frac{1}{2} \times a \times 16$
$a = ?$ $\quad a = 0.5\ \text{m/s}^2$
$s = 4\ \text{m}$
$t = 4\ \text{s}$

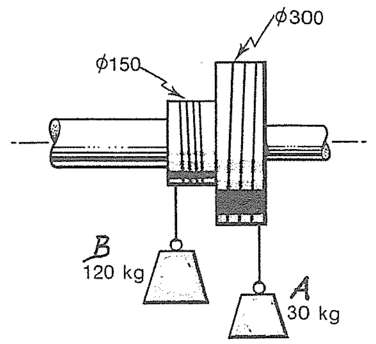
Ang. accel. of Wheel $\alpha = a/r$
$= \dfrac{0.5 \times 1000}{25}\ \text{rad/s}^2$

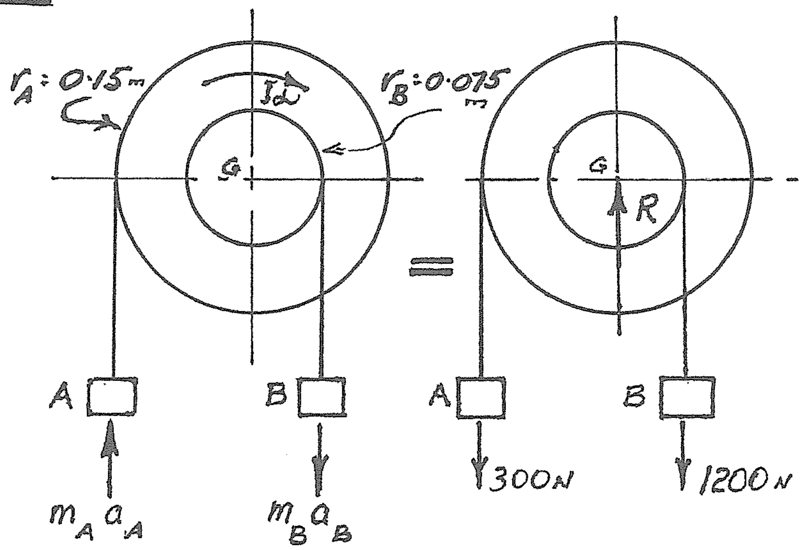
An unbalanced force $(mg - T)$ produces this accel.
$F = ma$
$(6 \times 9.8 - T) = 6 \times 0.5$
$T = 55.8\ \text{N}$

& Torque $\dfrac{55.8 \times 25}{1000}\ \text{Nm}$
$= 1.47\ \text{Nm}$

from Torque = $I\alpha$
$I = \text{Torque}/\alpha$
$= \underline{0.0735\ \text{kgm}^2}$

13/38
The pulley system has a moment of inertia of 30 kg·m². If the weight *A* has a mass of 30 kg and the weight *B* a mass of 120 kg, determine the acceleration of the system.





To find direction of rotation assume equilib--rium : $\Sigma M = 0$

$A \times 150 = 120 \times 75$

$A = 60\ kg$

but $A = 30\,kg$ ∴ rotation will be ↷

Note : The angular accel. (α) is the same for both pulleys.

$a_A = r_A \cdot \alpha$ and $a_B = r_B \cdot \alpha$

or $a_A = 0.15\alpha \uparrow \ldots\ldots (1)$ and $a_B = 0.075\alpha \downarrow \ldots\ldots (2)$

The system consists of 2 equivalent sets of forces causing motion :

1. The nett moment about G caused by the 2 masses.

   i.e. $1200 \times 0.075 - 300 \times 0.15$ or $\underline{45\ Nm}$

2. The Torque ($I\alpha$) plus the moments of the effective tensions in the ropes.

   i.e. $I\alpha + T_A \times 0.15 + T_B \times 0.075$

   or $I\alpha + m_A a_A \times 0.15 + m_B a_B \times 0.075$

∴ From (1) & (2) $I\alpha + m_A \times 0.15\alpha \times 0.15 + m_B \times 0.075\alpha \times 0.075$

$= 30\alpha + 30 \times 0.15 \times 0.15\alpha + 120 \times 0.075 \cdot 0.075\alpha$

$= 31.35\alpha$

Thus $31.35\alpha = 45$

$\alpha = \underline{1.435\ rad/s^2}$

14 Work and Energy

$U = Fs$
$PE = mgh$
$KE = \frac{1}{2} mv^2$

1 Nm = 1 J
g = 9.8 m/s²

14 WORK AND ENERGY
Work Done by a Variable Force. Energy. Conservation of Energy, Efficiency. Work Done By or Against a Torque. Kinetic Energy of Rotation.

14/15
State the type of energy possessed by the following objects:
(i) a loose brick balanced on a wall of height 2 metres;
(ii) a bullet travelling at 300 m/s;
(iii) water in a dam that is used to drive a turbine;
(iv) a rotating flywheel.

(i) Potential
(ii) Kinetic
(iii) Potential
(iv) Kinetic (Rotational)

14/16
(i) Determine the work done against gravity when a mass of 50 kg is lifted 6 metres vertically.
(ii) How much of this energy is lost when the mass falls 1 metre from this height?

(i) $U = Fs$
$= 50 \times 9.8 \times 6$
$= \underline{2,940 \text{ J}}$

(ii) Energy lost $= mg(h - h_1)$
$= 50 \times 9.8 (6 - 5)$
$= \underline{490 \text{ J}}$
or $\frac{2940}{6} = \underline{490 \text{ J}}$

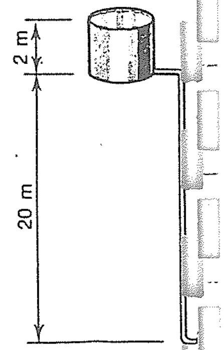
14/17
A boy of mass 70 kg climbs a cliff 30 metres high.
(i) How much (useful) work has he done?
(ii) What is his potential energy in relation to his starting point?
(iii) If he dislodges a stone of mass 1 kg from the clifftop with what KE will it hit the ground?

(i) Work $= Fs$
$= 70 \times 9.8 \times 30$ Nm
$= \underline{20.58 \text{ kJ}}$

(ii) Energy = Work done
$= \underline{20.58 \text{ kJ}}$

(iii) P.E of stone $= 1 \times 9.8 \times 30$
$= 294$ J
∴ K.E. on impact $= \underline{294 \text{ J}}$

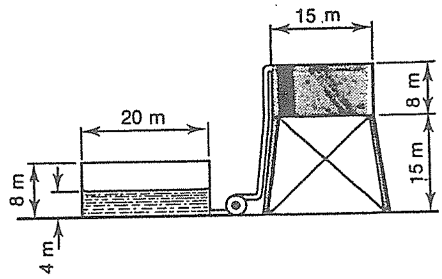
14/18
(i) A cylindrical water tank, 2 m high, contains 20 000 litres of water when full. What is the potential energy of the water relative to an outlet point 20 m vertically below the bottom tank?
(ii) If the tank is refilled by means of a pump situated at the same level as the outlet, what is the useful work done by the pump?
Take the mass of 1 litre of water as 1 kg.



(i) Mass of water = 20,000 kg
Av. height = 21 m
P.E. = mgh
$= 20000 \times 9.8 \times 21$
$= \underline{4.116 \text{ MJ}}$

(ii) Work done = P.E
$= \underline{4.116 \text{ MJ}}$

14/19
A tank, 20 m long by 8 m wide by 8 m deep, sits on level ground, and contains water to a depth of 4 metres. A second tank, 15 m long by 8 m deep by 8 m wide, sits on a stand so that the bottom of the tank is 15 m above ground. Find the work required to pump all the water from the first tank to the second tank. The density of water is 1000 kg/m^3. Neglect frictional losses.



Vol. of water in bottom tank $= 20 \times 8 \times 4 = 640\ m^3$

$\therefore$ Mass of water in bottom tank $= 640 \times 10^3$ kg.

<u>Note</u>: The lower tank water is 4 m up the pipe before pumping $\therefore$ Av. pumping height is:

$$15 - 2 + 8 = 21 m.$$

$$U = Fs$$

$$= \frac{640 \times 10^3 \times 9.8 \times 21}{10^6} \text{ MJ}$$

$$= \underline{131.7 \text{ MJ}}$$

14/20
A rubber ball of mass 0.5 kg falls 4 metres vertically before striking a horizontal surface, from which it rebounds to a height of 3 metres. Determine
(i) its potential energy before it falls;
(ii) its kinetic energy at the instant of impact;
(iii) its kinetic energy at the instant of rebound.
Explain what happens to the energy "lost" during the impact.

(i) P.E. $= mgh$
$= 0.5 \times 9.8 \times 4$
$= \underline{19.6 \text{ J}}$

(ii) $u = 0$, $v = ?$, $a = 9.8\ m/s^2$, $s = 4$ m.

$v^2 = u^2 + 2as$
$= 0 + 2 \times 9.8 \times 4$
$= 78.4$

K.E. $= \frac{1}{2} m v^2$
$= \frac{1}{2} \times 0.5 \times 78.4$
$= \underline{19.6 \text{ J}}$

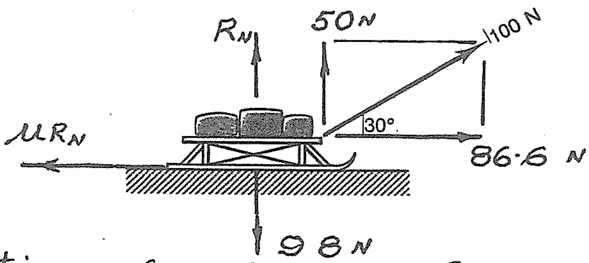
(iii) $u = ?$, $v = 0$, $a = -9.8\ m/s^2$, $s = 3$ m

$v^2 = u^2 + 2as$
$u^2 = 2 \times 9.8 \times 3$
$= 58.8$

$\therefore$ K.E. $= \underline{14.7 \text{ J}}$

4.9 J has been "lost" in deformation, heat, sound etc.

14/21
A sled of mass 10 kg is pulled 10 metres along level ground. The tension in the tow rope is 100 N and it is inclined at 30° to the ground as shown. Determine
(i) the total work done;
(ii) the work done against friction, if the coefficient of friction between sled and ground is 0.3.



(i) Applied force in direction of motion = 86·6 N

$U = Fs$

$= \underline{866\ J}$

(ii) Resolve vert.

$R_N + 50 = 98$

$R_N = 48$

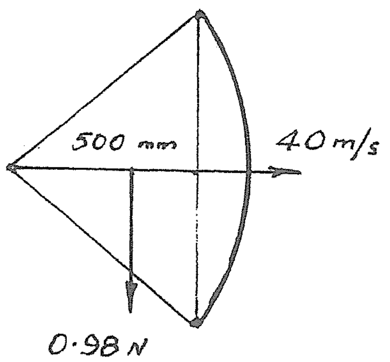
$\therefore \mu R_N = 14{\cdot}4\ N$

$\therefore U = F \times s$

$= 14{\cdot}4 \times 10$

$= \underline{144\ J}$

14/22
An archer pulls back a bowstring a distance of 500 mm and then releases his arrow which has a mass of 100 g. Determine the average force exerted on the arrow if its release velocity is 40 m/s. Solve this problem by the work-energy method.



$K.E. = \frac{1}{2} \times \frac{100 \times 40 \times 40}{1000}$

$= 80\ J$

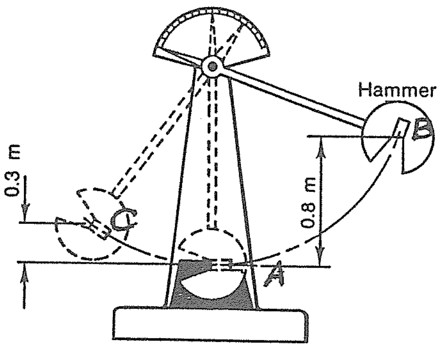
$\therefore$ Work done $= 80\ J$

Work $= Fs$

$80 = F \times 0{\cdot}5$

$F = \underline{160\ N}$

14/23
The 6-kg pendulum of a Charpy notched-bar impact testing machine falls freely through a vertical height of 0.8 metres before it hits the specimen. Determine
(i) its kinetic energy immediately before impact;
(ii) its kinetic energy immediately after impact, if it rises 0.3 m vertically after striking the specimen;
(iii) the work done on the specimen by the impact.



(i) K.E. at A = P.E. at B

$= 6 \times 9{\cdot}8 \times 0{\cdot}8$

$= \underline{47{\cdot}04\ J}$

(ii) K.E. at A = P.E. at C

$= 6 \times 9{\cdot}8 \times 0{\cdot}3$

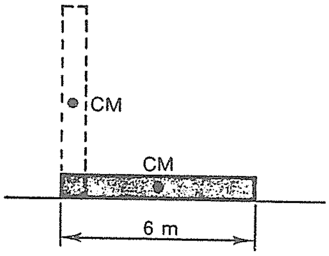
$= \underline{17{\cdot}64\ J}$

(iii) Work done at impact = Energy lost

$= 47{\cdot}04 - 17{\cdot}64$

$= \underline{29{\cdot}4\ J}$

14/24
A concrete post of uniform cross-section has a mass 1 tonne and a length of 6 m. Determine the amount of work done on the pole when it is lifted from a horizontal to a vertical position.



Work done lifting post : work done by force (9·8 kN) over 3 m

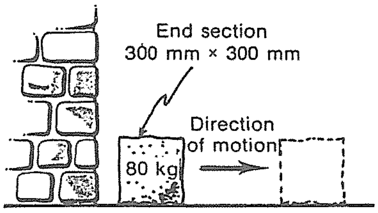
$U = F \times S$

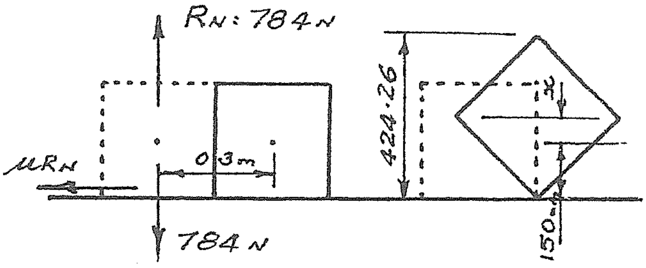
$= 9{\cdot}8 \times 3$

$= \underline{29{\cdot}4\ \text{kJ}}$

14/25
A man building a wall wishes to move a block of dimension stone measuring 300 mm by 300 mm by 1 m across a concrete path. Given that the block has a mass of 80 kg, will he use more energy if he slides it or rolls it?

The coefficient of friction between path and stone is 0.6. (*Hint:* Consider the energy used to displace the block by a distance equal to its 300-mm width by rolling and by sliding.)





Work done sliding $= F \times S$

$= 470{\cdot}4\,(\mu R_N) \times 0{\cdot}3$

$= \underline{141{\cdot}12\ \text{J}}$

Work done rolling equals work done raising centre of mass x m. where x : 62·13 mm.

$U = F \times S$

$= \dfrac{784 \times 62{\cdot}13}{1000}$

$= \underline{48{\cdot}7\ \text{J}}$ $\quad \therefore$ More energy to slide

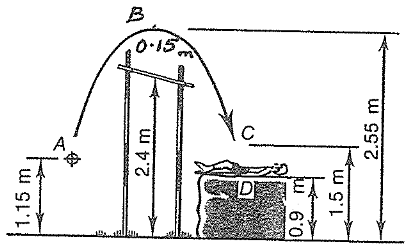
14/26
The high-jump world record was recently broken be Nee-Hi-Me at 2.4 m.

(i) If the athlete had a mass of 70 kg, and raised his centre of mass through 1.4 m from *A* to *B* during the jump, what was his gain in potential energy?

(ii) If he then fell on a pile of plastic mattress material at *C*, which compressed from 1.5 m down to 0.9 m at *D*, what was his kinetic energy at *C*, and how much energy altogether was absorbed by the plastic? Neglect any kinetic energy he may have had at *B*.

(iii) Assuming his deceleration from *C* to *D* was uniform, what was its value?

Assume throughout that his centre of mass was 150 mm above the bar at *B*, above the uncompressed mattress at *C*, and above the compressed mattress at *D*, and neglect all horizontal components of velocity.



(i) P.E. at B : mgh

$: 70 \times 9{\cdot}8 \times 1{\cdot}4$

$: \underline{960{\cdot}4\ \text{J}}$

14/26 (cont.)

(ii) $u : 0$ $\qquad v^2 : u^2 + 2as$

$v : ?$ $\qquad : 0 + 2 \times 9.8 \times 0.9$

$a : 9.8\ m/s^2$ $\qquad : 17.64$

$s : 0.9\ m$ $\qquad$ K.E. $: \frac{1}{2} m v^2$

$= \frac{1}{2} \times 70 \times 17.64$

$: \underline{617.4\ J}$

Energy absorbed : Energy lost from C–D : $\underline{617.4\ J}$

(iii) $u : 4.2\ m/s\ (\sqrt{17.64})$ $\qquad v^2 = u^2 + 2as$

$v = 0$ $\qquad 0 : 17.64 + 2 \times 0.6\, a$

$a : ?$ $\qquad a = -\underline{14.7\ m/s^2}$

$s : (1.5 - 0.9)\ m$

$t : -$

Decel. from C – D = $\underline{14.7\ m/s^2}$

14/27
The planned excavation for the basement of a building is 20 m by 40 m by 5 m deep. If the spoil has a mass of 1800 kg per cubic metre, how much work will be done in raising the spoil to the original ground level? (*Note:* No allowance is to be made for heaping up the spoil once it is excavated from the hole.)

Volume of Spoil : $20 \times 40 \times 5\ m^3$

Mass of Spoil : $20 \times 40 \times 5 \times 1800\ kg$

$\therefore$ Mass force : $\dfrac{20 \times 40 \times 5 \times 1800 \times 9.8}{1000}\ kN$

$= 70.56\ MN$

The average height to which the spoil must be raised $= 5/2 = 2.5\ m.$

$\therefore$ Work done : 70.56×2.5

$= \underline{176.4\ MJ}$

14/28

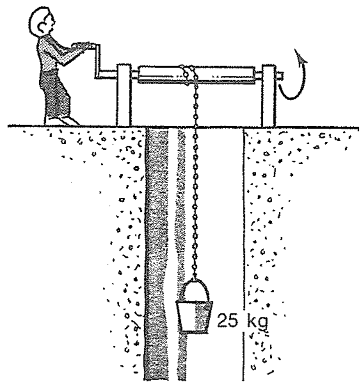
During the operation of a windlass, 30 metres of chain are to be wound on to the drum.

(i) Determine the work done on the chain if the chain has a mass of 0.5 kg/m.

(ii) A bucket containing spoil is attached to the end of the chain. If the bucket plus spoil has a mass of 25 kg, determine the work done in lifting the bucket through the 30 metres.

(iii) What is the total work done on both bucket and chain?

(iv) How much energy does the operator expend, if the windlass has an efficiency of 45%?



(i) Mass of chain $= 30 \times 0.5$ kg

Half of this is the effective mass to be wound onto the windlass $= \frac{1}{2} \times 30 \times 0.5$

Mass force $= \frac{1}{2} \times 30 \times 0.5 \times 9.8$ N

and Work done $= \frac{1}{2} \times 30 \times 0.5 \times 9.8 \times 30$

$= \underline{2{,}205 \text{ Nm}}$

(ii) Work done on bucket $= 25 \times 9.8 \times 30$

$= \underline{7{,}350 \text{ Nm}}$.

(iii) Total work $= 9555$ Nm or $\underline{9.6 \text{ kJ}}$

(iv) Energy used (input) $=$ output/efficiency

$$= \frac{9555 \times 100}{45}$$

$= \underline{21.2 \text{ kJ}}$

14/29

A mortar shell of mass 15 kg was fired from a mortar barrel of length 3 metres. If its velocity on leaving the barrel was 600 m/s, determine the average force exerted by the charge on the shell as it was fired.

$u = 0$

$v = 600$ m/s

$a = ?$

$s = 3$ m

$t = $ —

$v^2 = u^2 + 2as$

$360{,}000 = 0 + 2 \times 3a$

$a = 60{,}000$ m/s^2

$F = ma$

$= 15 \times 60{,}000$

$= \underline{900 \text{ kN}}$

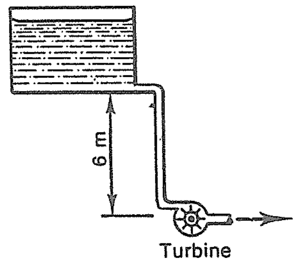
<u>OR</u>:

Work done by force on shell = Change in K.E. of shell

i.e. $Fs = \frac{1}{2} \times 15 \times 600 \times 600$

$$F = \frac{1}{2} \times \frac{15 \times 600 \times 600}{3 \times 1000} = \underline{900 \text{ kN}}$$

14/30
A water tank 5 m long by 4 m wide by 4 m deep contains water to a depth of 3.5 m. A water turbine is in position 6 m below the bottom of the tank. Determine the output work of this turbine when all of the water in the tank flows through it, if the overall efficiency of the system is 60 %. The mass of 1 cubic metre of water is 1000 kg.



Volume of water : $5 \times 4 \times 3{\cdot}5 \ m^3$

Mass of water : $5 \times 4 \times 3{\cdot}5 \times 1000$ kg

$\therefore$ Mass force : $5 \times 4 \times 3{\cdot}5 \times 1000 \times 9{\cdot}8$ N

: 686 kN

Average height of water above turbine : 7·75 m.

$\therefore$ Work potential : $686 \times 7{\cdot}75$: 5,320 kJ

and turbine output : $\dfrac{5320 \times 60}{100}$: $\underline{3{\cdot}18 \text{ MJ}}$

14/31
A car of mass 1 tonne travelling at 60 km/h has its speed uniformly reduced to 20 km/h in 5 seconds.
(i) Assuming that the reduction in speed is solely due to the brakes being applied, calculate the energy absorbed by the brakes.
(ii) Determine the average braking force acting on the car.

60 km/hr : 16·67 m/s

20 km/hr : 5·57 m/s

(i) Energy absorbed : Loss in kinetic energy

$= \frac{1}{2} m v_1^2 - \frac{1}{2} m v_2^2$

$: 500 (16{\cdot}67^2 - 5{\cdot}57^2)$

: $\underline{123{\cdot}6 \text{ kJ}}$

(ii) $u : 16{\cdot}67$ m/s

$v = 5{\cdot}55$ m/s

$a : ?$

$s = -$

$t = 5$ s.

$v = u + at$

$5{\cdot}55 = 16{\cdot}67 + 5a$

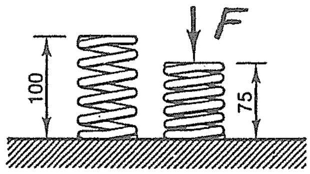
$a : -2{\cdot}22 \ m/s^2$

$F = ma$

$= 1000 \times 2{\cdot}22$

: $\underline{2{\cdot}22 \text{ kN}}$

14/32
The mean force required to compress a given spring from its free length of 100 mm to 75 mm is 300 N. Determine the energy stored in it in the compressed state.



Work done by F $= F \times s$

$: \dfrac{300 \times 25}{1000}$

$= \underline{7{\cdot}5 \text{ J}}$

14/33
A spring has a free length of 100 mm. When a force of 40 N is applied, the spring compresses 10 mm; an additional force of 40 N causes the spring to compress a further 10 mm.
(i) Assuming the spring behaves in accordance with Hooke's law, how much work would be done on it if it is compressed from its free length of 100 mm to a length of 50 mm?
(ii) What would be the strain energy contained in the spring in the compressed state?

(i) Modulus of Stiffness of spring $= 40/10$

$= 4\ N/mm$

$\therefore$ Av. force to compress spring 50 mm $= \dfrac{4 \times 50}{2}$

$= 100\ N$

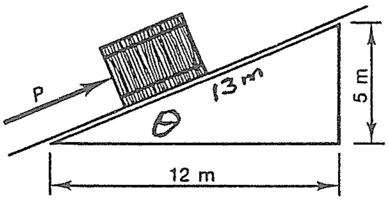
and work done $= \dfrac{100 \times 50}{1000}$

$= \underline{5\ J}$

(ii) Strain Energy stored = compression work done

$= \underline{5\ J}$

14/34
A crate of mass 50 kg is pushed 13 metres up an incline that rises 5 metres vertically over a horizontal distance of 12 metres. If the frictional resistance is constant at 120 N, calculate the work done on the crate.



Resolve parallel to plane

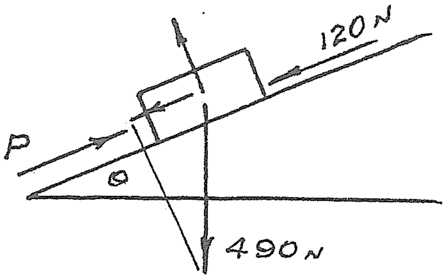
$P = 120 + 490 \sin 22.62°$

$= 308.5\ N$

Work done $= F \times S$

$= 308.5 \times 13$

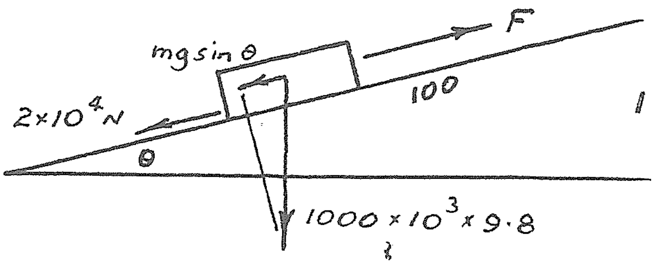
$= \underline{4.01\ kJ}$



$\tan\theta = 5/12$

$\theta = 22.62°$

14/35
A diesel locomotive pulls a train of mass 1000 tonnes up a 1% grade at constant speed. If the average resistance to motion is 20 N per tonne, calculate
(i) the drawbar pull supplied by the locomotive;
(ii) the work done in moving the train 2 kilometres up the track.



(i) $F = mg\sin\theta + 2 \times 10^4$

$= 1000 \times 10^3 \times 9.8 \times 0.01 + 2 \times 10^4$

$= 11.8 \times 10^4\ N$

$= \underline{118\ kN}$

(ii) Work $= F \times S$

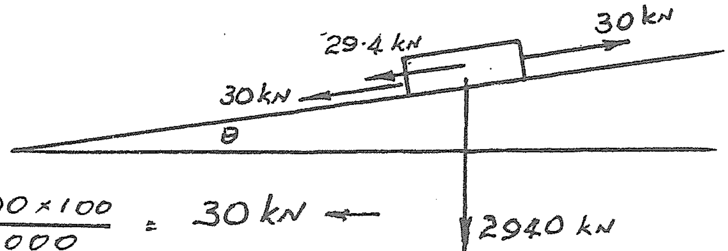
$= 118 \times 10^3 \times 2 \times 10^3$

$= 236 \times 10^6$

$= \underline{236\ MJ}$

14/36

A train of mass 300 tonnes, moving at 72 km/h along a horizontal track begins to climb an incline of 1% slope. During the climb the engine exerts a constant tractive force (drawbar pull) of 30 kN and the frictional resistances to motion remain constant at 100 N per tonne. How far will the train move up the incline?



Frictional Resistance $= \frac{300 \times 100}{1000} = 30$ kN ←

Resolved Mass force down the plane $= 2940 \sin\theta$

$= 29.4$ kN ←

Drawbar pull $= 30$ kN →

∴ Nett force $= 29.4$ kN ←

$F = ma$

$29.4 \times 1000 = 300 \times 1000\, a$

$a = 0.098\ m/s^2$ ←

$u = 72\ km/hr = 20\ m/s$

$v = 0$

$a = -0.098\ m/s^2$

$s = ?$

$v^2 = u^2 + 2as$

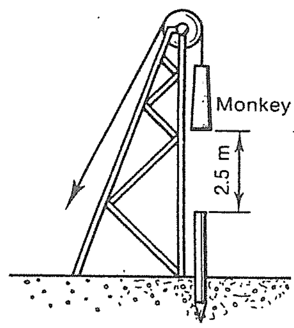
$0 = 400 - 2 \times 0.098\, s$

$s = \underline{2040\ m}$

14/37

A pile driving hammer (monkey) of mass 500 kg falls 2.5 metres from rest onto a pile of mass 140 kg. Assuming there is no rebound and the pile is driven 150 mm into the ground, determine the following:

(i) the velocity of the hammer just before impact;
(ii) the common velocity of the pile and hammer after impact;
(iii) the work performed in sinking the pile 150 mm into the ground;
(iv) the average resisting force of the ground in bringing the pile and driver to rest.



(i) $u = 0$

$v = ?$

$a = 9.8\ m/s^2$

$s = 2.5\ m$

$v^2 = u^2 + 2as$

$= 0 + 2 \times 9.8 \times 2.5$

$v = \underline{7\ m/s}$

(ii) $m_1 v_1 + m_2 v_2 = m_3 v_3$

$500 \times 7 + 140 \times 0 = 640\, v_3$

$v_3 = \underline{5.47\ m/s}$

(iii) $u = 5.47\ m$

$v = 0$

$a = ?$

$s = 150/1000\ m$

$v^2 = u^2 + 2as$

$0 = 29.9 + \frac{300\,a}{1000}$

$a = -99.7\ m/s^2$

14/37 (cont.)

$u = 5{\cdot}47$ m/s

$v = 0$

$a = -99{\cdot}7$ m/s^2

$s = -$

$t = ?$

$v = u + at$

$0 = 5{\cdot}47 - 99{\cdot}7t$

$t = 0{\cdot}055$ s

Mom. after impact $= mv$

$= 640 \times 5{\cdot}47$ kgm/s

When pile and monkey come to rest, mom. = 0

Impulse I = change in mom.

$= 640 \times 5{\cdot}47$

$= 3{,}500$ Ns

$I = F \times t$

$3500 = 0{\cdot}055F$

$\therefore F = 63{\cdot}65$ kN

Total force = $63{\cdot}65$ + mass force

$= 63{\cdot}65 + 6{\cdot}27$

$= 69{\cdot}92$ kN

Work $= F \times s$

$= \dfrac{69{\cdot}92 \times 1000 \times 150}{1000}$

$= \underline{10{\cdot}5\text{ kJ}}$

(iv) Acceleration of pile and monkey $= 99{\cdot}7 + 9{\cdot}8$

$= 109{\cdot}5$ m/s^2

$F = ma$

$= 640 \times 109{\cdot}5$

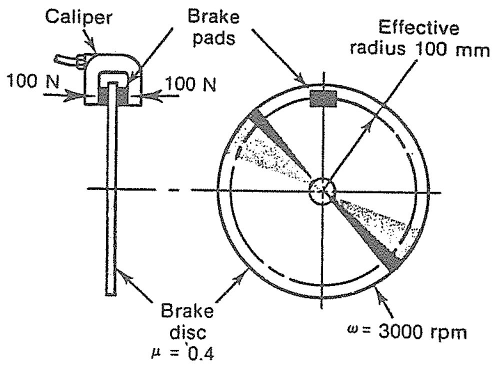
$= \underline{70{\cdot}1\text{ kN}}$

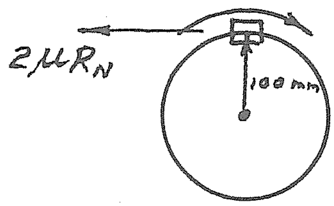
14/38

Two views of the basic units (disc and brake pads) of a disc brake are shown in the diagram. The mean effective radius of the disc is 100 mm, the force on each brake pad is 100 N and the coefficient of friction between the pads and disc is 0.4. If the disc is rotating at 3000 rpm, determine:

(i) the moment resisting the rotation of the disc;

(ii) the energy absorbed through the brake pads if the disc takes 20 revolutions to come to rest.





(i) Moment of Resistance

$= F \times d$

$= \dfrac{2 \times 0.4 \times 100 \times 100}{1000}$

$= \underline{8\ Nm}$

(ii) Energy = Work done

$= F \times s$

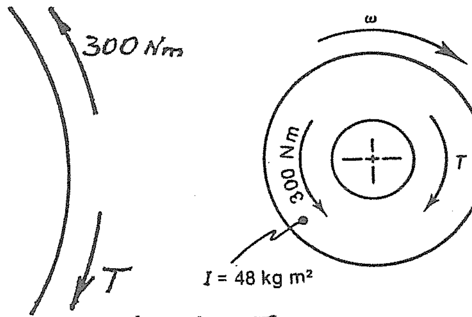
$= 2\mu R_N \times \pi D \times 20$

$= \dfrac{2 \times 0.4 \times 100 \times \pi \times 200 \times 20}{1000}$

$= \underline{1005\ J}$

14/39

A rotating pulley and shaft has a moment of inertia of 48 kg m². Given that the bearing friction is equivalent to a resisting couple of 300 N m, determine the torque necessary to accelerate the shaft from rest to an angular speed of 600 rpm in 12 revolutions from rest. Solve by a work-energy method and also by using kinetics of rotation.



Rotation Method.

$\omega_0 : 0$

$\omega_t : \dfrac{2\pi \times 600}{60} : 62.83\ rad/s$

$\alpha : ?$

$\theta : 2\pi \times 12 : 75.4\ rad.$

$\omega_t^2 : \omega_0^2 + 2\alpha\theta$

$3948 : 0 + 2 \times 75.4\alpha$

$\alpha : 26.15\ rad/s^2$

$T : I\alpha$

$: 48 \times 26.15 + 300$

$= \underline{1.56\ kNm}$

Work-Energy Method

Work $U : T\theta$

Rot. K.E. $= \frac{1}{2} I \omega^2$

$T\theta = \frac{1}{2} I \omega^2$

$T = \dfrac{1 \times I \times \omega^2}{2 \times \theta}$

$= \dfrac{1 \times 48 \times 62.83^2}{2 \times 75.4}$

$= 1256\ Nm.$

Total Torque req. $= 1256 + 300$

or $\underline{1.56\ kNm}$

14/40
A flywheel has a radius of gyration of 1.5 m and a mass of 10 tonnes. Determine
(i) its kinetic energy when it revolves at 500 rpm;
(ii) the time taken to bring it to a rest when a resisting torque of 680 N m is applied to it.

(i) Linear vel. $v = \dfrac{500 \times 2\pi \times 1.5}{60}$

$= 78.54 \text{ m/s}$

$K.E. = \frac{1}{2} m v^2$

$= \dfrac{1 \times 10 \times 1000 \times 78.54^2}{2 \times 1000 \times 1000}$

$= \underline{30.84 \text{ MJ}}$

(ii) Torque $= F \times r$

$680 = F \times 1.5$

$F = 453.3 \text{ N}$

$F = ma$

$453.3 = 10000\, a$

$a = 0.045 \text{ m/s}^2$

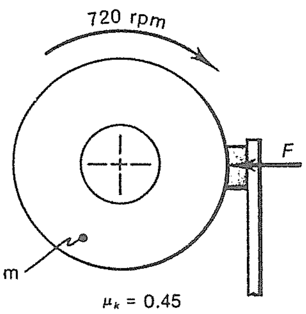
$u = 78.54 \text{ m/s}$
$v = 0$
$a = -0.045 \text{ m/s}^2$
$s = -$
$t = ?$

$v = u + at$

$0 = 78.54 + 0.045t$

$t = \underline{1745 \text{ s}}$

14/41
A shaft and drum rotating at 720 rpm has a moment of inertia of 34 kg m². A brake block acting on the surface of the drum brings it to rest in 12 revolutions, the coefficient of friction present being 0.45. Determine the normal force, F, between the brake block and drum, given that the drum diameter is 1 metre.

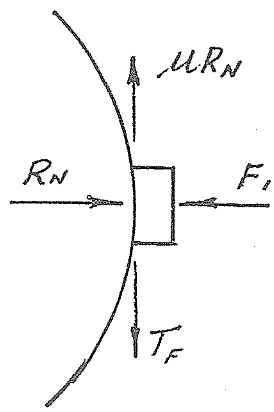


$\omega_0 = 2\pi \times 720/60 \text{ rad/s}$
$\omega_t = 0$
$\alpha = ?$
$\theta = 12 \times 2\pi \text{ rad.}$

$\omega_t^2 = \omega_0^2 + 2\alpha\theta$

$0 = 5685 + 151\alpha$

$\alpha = -37.65 \text{ rad/s}^2$



$T = I\alpha$

$= 34 \times 37.65$

$= 1280 \text{ Nm}$

$\therefore T_F = 1280/0.5$

$= 2560 \text{ N}$

Resolve:

$T_F = \mu R_N$

$2560 = 0.45 R_N$

$\therefore R_N = 5.68 \text{ kN}$

and $F_1 = \underline{5.68 \text{ kN}}$

14/42

A shaft having a moment of inertia of 16 kg m² is accelerated from 1200 rpm to 1320 rpm during 2 revolutions. Determine

(i) the increase in kinetic energy of the shaft;

(ii) the torque required to provide this acceleration, given that a friction couple of 80 N m is acting to resist rotation.

$$\omega_0 = \frac{2\pi \times 1200}{60} = 125.7 \text{ rad/s}$$

$$\omega_t = \frac{2\pi \times 1320}{60} = 138.2 \text{ rad/s}$$

$$\alpha = ?$$

$$\theta = 2\pi \times 2 = 12.57 \text{ rad}$$

(i) Increase in K.E.

$$= \tfrac{1}{2} I \omega_t^2 - \tfrac{1}{2} I \omega_0^2$$

$$= \tfrac{1}{2} I (\omega_t + \omega_0)(\omega_t - \omega_0)$$

$$= \frac{1 \times 16 \times 263.9 \times 12.5}{2 \times 1000} = \underline{26.4 \text{ kJ}}$$

(ii) $\omega_t^2 = \omega_0^2 + 2\alpha\theta$

$138.2^2 = 125.7^2 + 2 \times 12.57\alpha$

$\alpha = 131.2 \text{ rad/s}^2$

$T = I\alpha$

$= 16 \times 131.2$

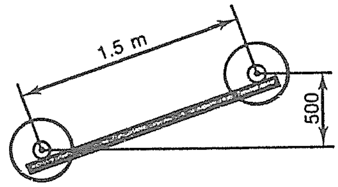
$= 2099.6 \text{ Nm}$

$\therefore T_{req.} = 2099.6 + 80 = \underline{2.18 \text{ kNm}}$

14/43

In an experiment to determine the moment of inertia of a shaft and flywheel, the assembly was allowed to roll freely down an inclined plane formed by two highly polished parallel steel bars set the required distance apart. The flywheel and shaft rolled, without slipping, 1.5 metres down the incline and at the same time fell through a vertical distance of 500 mm, the time of motion being 5 seconds.

Neglecting friction, determine the approximate moment of inertia of the flywheel and shaft, given that its mass was 20 kg and the shaft diameter was 50 mm.



Shaft dia. $= 50$ mm

$\therefore$ Circumf. $= \dfrac{\pi \times 50}{1000}$ mm

and Revs. $= \dfrac{1.5 \times 1000}{\pi \times 50}$

$= 9.55$ revs or 60 rad.

$\omega_0 = 0$

$\omega_t = -$

$\alpha = ?$

$\theta = 60$ rads

$t = 5$ s.

$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2$

$60 = 0 + \tfrac{1}{2}\alpha \times 25$

$\alpha = 4.8 \text{ rad/s}^2$

$U = F \times S$

$= \dfrac{20 \times 9.8 \times 500}{1000}$

$= 98 \text{ Nm}$

$U = T\theta$

$98 = T \times 60$

$T = 1.63 \text{ Nm}$

$T = I\alpha$

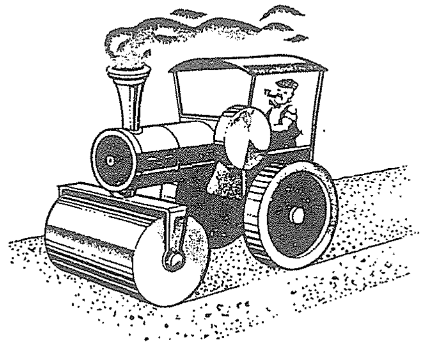
$1.63 = I \times 4.8$

$I = \underline{0.34 \text{ kgm}^2}$

14/44

Fred's road roller has a total mass of 10 tonnes. The front roller assembly has a mass of 2 tonnes, a diameter of 1.2 m, and a radius of gyration of 0.45 m. The rear wheels and axle taken together have a mass of 2.5 tonnes and a radius of gyration of 0.6 m, the diameter of the rear wheels being 1.5 m. If the road roller is moving with a constant velocity of 10 km/h, determine

(i) the kinetic energy of rotation of the front roller assembly;
(ii) the kinetic energy of rotation of the rear wheels and axle;
(iii) the total kinetic energy of the road roller;
(iv) the braking force necessary to bring it to rest in 10 metres on a horizontal stretch of road.



(i) 10 km/hr = 2·78 m/s : Front Roller circ. = $\pi \times 1{\cdot}2$ m

At 2·78 m/s, Fr. Roller revolves $\frac{2{\cdot}78}{\pi \times 1{\cdot}2}$ revs/s or 4·63 rad/s

$I = mk^2$

$= 2 \times 1000 \times 0{\cdot}45^2$

$= 405 \text{ kgm}^2$

$\therefore$ K.E. $= \frac{1}{2} I \omega^2$

$= \frac{1 \times 405 \times 4{\cdot}63^2}{2}$

$= \underline{4{\cdot}34 \text{ kJ}}$

(ii) Rear Roller circ. = $\pi \times 1{\cdot}5$ m

At 2·78 m/s Rear roller revolves $\frac{2{\cdot}78}{\pi \times 1{\cdot}5}$ revs/s or 3·71 rad/s

$I = mk^2$

$= 2{\cdot}5 \times 1000 \times 0{\cdot}6^2$

$= 900 \text{ kgm}^2$

$\therefore$ K.E. $= \frac{1}{2} I \omega^2$

$= \frac{1 \times 900 \times 3{\cdot}71^2}{2}$

$= \underline{6{\cdot}19 \text{ kJ}}$

(iii) Total K.E. $= \frac{1}{2} m v^2 +$ Fr. Roller K.E. + Rear Roller K.E.

$= \left(\frac{1 \times 10 \times 1000 \times 2{\cdot}78^2}{2 \times 1000} \right) + 4{\cdot}34 + 6{\cdot}19$

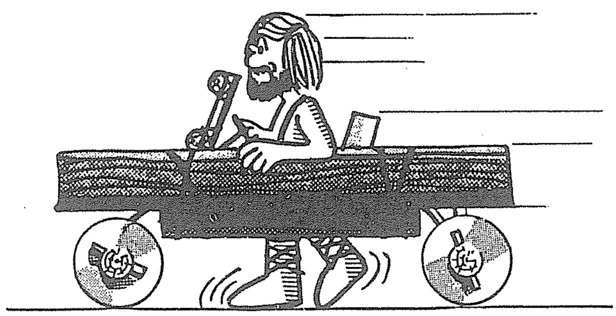
$= \underline{49{\cdot}17 \text{ kJ}}$

(iv) Work done to stop roller = K.E. of roller at 2·78 m/s (10 km/hr)

$\therefore 49{\cdot}17 = \frac{F \times 10 \times 1000}{1000}$

$F = \underline{4{\cdot}917 \text{ kN}}$

$$P = \frac{U}{t} = \frac{Fs}{t}$$



15 Power

$g = 9.8\ m/s^2$

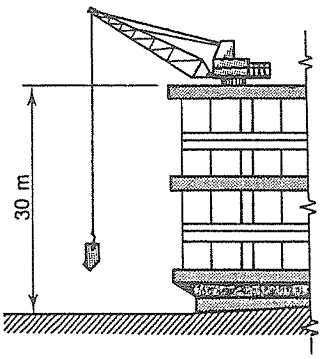
15 POWER
Power. Power of Vehicles on Gradients. Power Requirements of Hoists. *Efficiency.* Rotary Power. *Frictional Losses in Bearings.*

15/12
A jet engine can develop 25 kN of thrust when travelling at 650 km/h. Determine the power that the engine is delivering.

$$P = \frac{F \times s}{t}$$

$$= \frac{25 \times 10^3 \times 650 \times 1000}{3600 \times 1000 \times 1000}$$

$$= \underline{4.51\ MW}$$



15/13
Determine the average power that would be necessary to raise a hopper of concrete through 30 metres in 2 minutes with constant velocity, given that the combined mass of hopper and concrete is 750 kg.

$$Work = F \times s$$

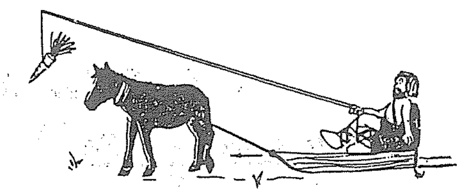
$$= 750 \times 9.8 \times 30$$

$$P = W/t$$

$$= \frac{750 \times 9.8 \times 30}{120}$$

$$= \underline{1.84\ kw}$$

15/14
A man of mass 65 kg sits on a sled being pulled by a mule. The mule exerts a constant horizontal pull of 250 newtons and the sled moves with a constant velocity of 0.5 m/s. At what rate is (i) the man working? (ii) the mule working?



(i) Nil.

(ii) $P = W/t$

$$= \frac{250 \times 0.5}{1}$$

$$= \underline{125\ W}$$

15/15
The output of a certain machine is equivalent to 15 kW. If the efficiency of the machine is 80%, what is the necessary input power?

$$Eff. = \frac{output}{input} \times 100$$

$$Input = \frac{output \times 100}{Eff.}$$

$$= \frac{15 \times 100}{80} = \underline{18.75\ kW}$$

15/16

A crane lifts a girder of mass 12 tonnes vertically upwards through 30 metres. During the first 5 m of lift, the load is accelerated uniformly to a velocity of 1 m/s. Thereafter the lift is continued at that velocity. Find:

(i) the tension in the cable during the period of acceleration;
(ii) the tension in the cable during the period of uniform velocity;
(iii) the capacity of the motor necessary to complete this operation, without overloading;
(iv) the total work done in the lifting operation.

Friction and inertial effects to be neglected for (iii) and (iv).

$u = 0$
$v = 1\ m/s$
$a = ?$
$s = 5m$

$v^2 = u^2 + 2as$
$1 = 0 + 10a$
$a = 0{\cdot}1\ m/s^2$

(i) $F = ma$
$= 12 \times 10^3 \times 0{\cdot}1$
$= 1200\ N$
$\therefore T = 1200 + 12 \times 10^3 \times 9{\cdot}8$
$= \underline{118{\cdot}8\ kN}$

(ii) $T =$ mass force
$= 12 \times 10^3 \times 9{\cdot}8$
$= \underline{117{\cdot}6\ kN}$

(iii) $P = ?$
$F = 118{\cdot}8 \times 10^3 N$
$v = 1\ m/s$

$P = Fv$
$= 118{\cdot}8\ kw$ or $\underline{120\ kW}$

(iv) Total work = Work during acc. + work for const. vel.

Work(Acc) $= F \times s$
$= 118{\cdot}8 \times 5$
$= 594\ kJ$

Work (const. vel.) $= F \times s$
$= 117{\cdot}6 \times 25$
$= 2940\ kJ$

$\therefore$ Total work $= 3534\ kJ$ or $\underline{3{\cdot}5\ MJ}$

15/17

A centrifugal pump raises water through a vertical height of 20 metres. If the pump motor supplies 20 kilowatts of power and the efficiency of the pump is 80%, determine the quantity of water that can be pumped in one hour. Take the mass of 1 litre of water as 1 kg.

Motor output = 20 kw
$\therefore$ Pump output = 16 kw (80% Eff)

$$P = \frac{F \times s}{t}$$

$$16 \times 10^3 = \frac{F \times 20}{3.600}$$

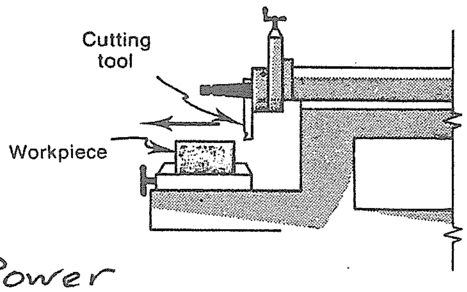
$$F = \frac{16 \times 10^3 \times 3600}{20}\ N$$

$$= \underline{2{\cdot}88\ MN}$$

$\therefore$ Vol. $= \underline{294\ \text{kilolitres}}/hr$

$\therefore$ Mass of water $= \dfrac{2{\cdot}88 \times 1000}{9{\cdot}8}$ kg.

15/18
During a cutting stroke of a shaping machine it is found that the average force on the cutting tool is 1.5 kN. Given that the cutting speed is 0.3 m/s and the overall efficiency is 70%, determine the suitable power rating for its motor.



Work per second = Power

$= 1.5 \times 10^3 \times 0.3$

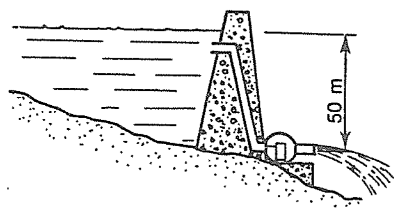
$$\text{Eff.} = \frac{\text{output}}{\text{input}} \times 100$$

$$\therefore \text{Motor input} = \frac{1.5 \times 10^3 \times 0.3 \times 100}{70}$$

$= 642.86$ watts

A suitable motor rating would be 750 watts

15/19
The water used to drive a turbo-generator plant falls in a pipe through an effective height of 50 m to the turbine, from which it is released at a velocity of 1.3 m/s. If the combined losses in the pipe to the turbine and in the turbine itself are 25% of the potential power, what is the power available to drive the generator, when 60 000 litres of water pass through the turbine every minute? Take the mass of 1 litre of water as 1 kg.



60.000 litres = 60.000 kg

$\therefore$ Mass force = 588 kN

$U = F \times S$

$$\therefore P = \frac{588 \times 50}{60} \text{ kw.}$$

With a pipe and turbine eff. of 75%

$$\text{Power available} = \frac{588 \times 50 \times 75}{60 \times 100}$$

$= 367.5$ kw

Alternatively

P.E. = mgh

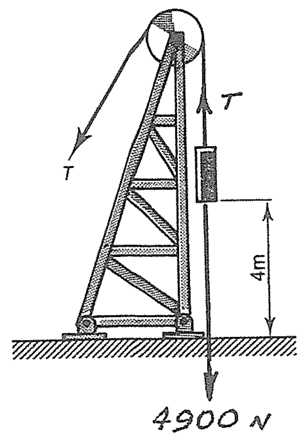
$$\therefore \text{Power} = \frac{60.000 \times 9.8 \times 50 \times 75}{60 \times 100}$$

$= 367.5$ kw

15/20
A pile-driver hammer of mass 500 kg is raised vertically a distance of 4 m in 6 seconds. The hoisting equipment has an efficiency of 80%. Determine the power of the motor necessary to operate the hoist, assuming that the lift is at constant velocity and friction is negligible.

Resolve vert. to find T in rope

$T = 4900$ N



Work done on hammer $= 4900 \times 4$ J

$\therefore$ Power output of motor/hoist

$$= \frac{4900 \times 4}{6} \text{ W.}$$

With an efficiency of 80% for hoist motor output

$$= \frac{4900 \times 4 \times 100}{6 \times 80 \times 1000}$$

$$= \underline{4.08 \text{ kW}}$$

15/21
A pump raises 13 cubic metres of water per hour through a height of 100 metres. Calculate the power required to drive the pump if 60% of its power is used to do useful work. The mass of 1 cubic metre of water = 1 tonne.

$13\,m^3$ per hr.

$= 13$ tonnes per hr.

$= 13000$ kg per hr.

$= 13000/3600$ kg/s

$$P = \frac{F \times s}{t}$$

$$= \frac{13000 \times 9.8 \times 100 \times 100}{3600 \times 1000 \times 60} \text{ kW}$$

$$= \underline{5.898 \text{ kW}}$$

15/22
An engine delivering 4 kW is used to pump water from a shaft 30 m deep. How many litres will be raised in 24 hours.
(i) if friction losses are negligible;
(ii) if the overall efficiency of the system is 85%?
Take the mass of 1 litre of water as 1 kg.

(i) $$\text{Power} = \frac{F \times s}{t}$$

$$4000 = \frac{F \times 30}{24 \times 3600}$$

$$F = \frac{4000 \times 24 \times 3600}{30}$$

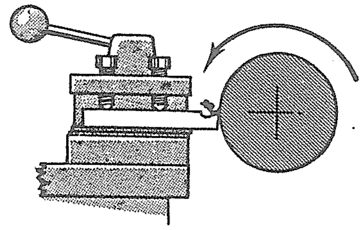
$$= 11.52 \text{ MN}$$

$$\therefore \text{Mass} = \frac{11.52 \times 10^6}{9.8} \text{ kg. or } \underline{1176 \text{ kilolitres}}$$

(ii) $$\frac{1176 \times 85}{100}$$

$$= \underline{999.18 \text{ kilolitres}}$$

15/23
In a test on a lathe cutting tool; it was observed that the power required for a certain size cut at an effective 150 mm radius and a speed of 200 rev/min was 4 kW. If power losses in the machine accounted for 1 kW, determine the tangential force on the cutting tool.



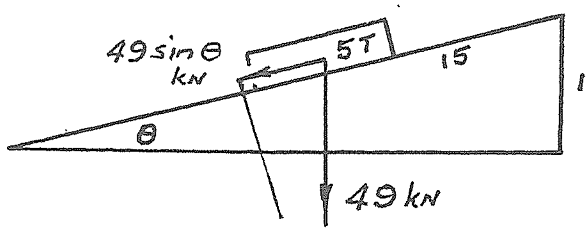
$$P = \frac{F \times S}{t}$$

$$3 \times 1000 = \frac{\cancel{2}\pi \times \cancel{150} \times 2\cancel{00} \times F}{1\cancel{000} \times \cancel{60}\,\cancel{4}\,2}$$

$$F = \frac{3000 \times 2}{2\pi}$$

$$= \underline{955\ N}$$

15/24
During a test on a motor vehicle with a total load of 5 tonnes, its road speed was steady at 50 km/h. The vehicle was travelling up an incline of 1 in 15 and instruments indicated that the engine was delivering 55 kW. Determine:
(i) the power which produced no useful output;
(ii) the power which did produce useful output;
(iii) the efficiency of the engine at this instant.



Negecting any losses, Power req. $= \dfrac{F \times S}{t}$

$$= \frac{49 \sin\theta \times 50 \times 1000}{3600}$$

$$= 45.37\ kW$$

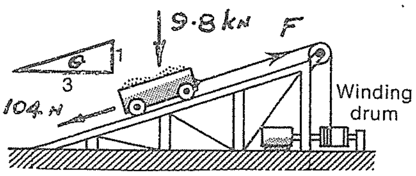
Engine delivers 55 kw

(i) Power lost $=$ $\underline{9.63\ kw}$ (ii) Power avail. $=$ $\underline{45.37\ kw}$

and (iii) η $=$ output/input $\times 100$

$$= \underline{82.5\%}$$

15/25
A steel cable is used to haul a truck up an incline as shown. The mass of the truck and load is 1 tonne and the frictional resistance to motion is 104 N. Determine the power necessary to haul the truck up the slope at 10 km/h.



$$\text{Tan } \theta = 0.3333$$

$$\theta = 18.43^\circ$$

$$F = 9800 \sin 18.43^\circ + 104$$

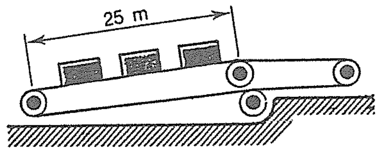
$$= 3202\ N$$

$$P = \frac{F \times S}{t}$$

$$= \frac{3202 \times 10 \times 1000}{3600 \times 1000}$$

$$= \underline{8.89\ kw}$$

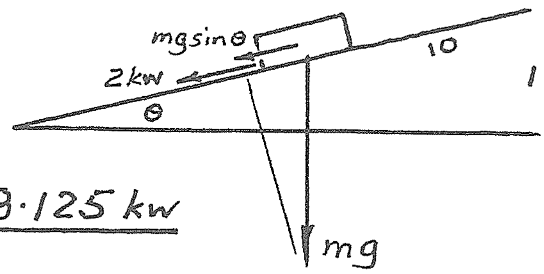
15/26
An electrically powered conveyer carries 30 000 packages per hour a distance of 25 m up an incline of 1 in 10. Each package has a mass of 30 kg and the power absorbed by friction in the drive is 2 kW.
(i) Determine the power of the motor driving this conveyer.
(ii) What is the efficiency of the conveyer?



(i) $U = 30000 \times 30 \times 9.8 \times 0.1 \times 25$ J

$$P = \frac{30{,}000 \times 30 \times 9.8 \times 0.1 \times 25}{3600 \times 1000}$$

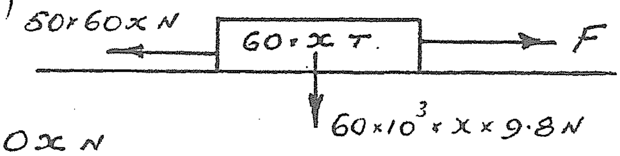
$= 6.125$ kW $+ 2$ kW lost $= \underline{8.125 \text{ kW}}$



(ii) Efficiency $\eta = \dfrac{\text{output}}{\text{input}} \times 100$

$$= \frac{6.125}{8.125} \times 100 = \underline{75.4\%}$$

15/27
A diesel locomotive can deliver a maximum of 3 MW of power to the drawbar. Given that rolling resistances are 50 N per tonne, how many 60-tonne freight cars can the locomotive pull
(i) along a level track at 100 km/h;
(ii) up a 2% grade at the same speed?

(i)


Resolve horiz.: $F = 50 \times 60x$ N

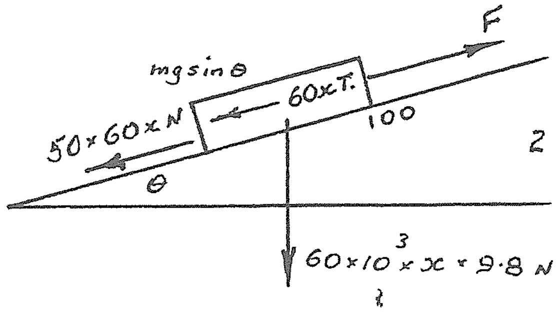
$$P = \frac{F \times s}{t}$$

$$3 \times 10^{6} = \frac{50 \times 60x \times 10^3 \times 10^3}{36 \times 10^2} \quad \therefore x = \frac{3 \times 10 \times 36}{5 \times 6} = \underline{36 \text{ trucks}}$$

(ii) Resolve // plane

$F = 50 \times 60x + 60 \times 10^3 \times x \times 9.8 \times 2/100$

$= x(3000 + 11760)$ N



$$P = \frac{F \times s}{t}$$

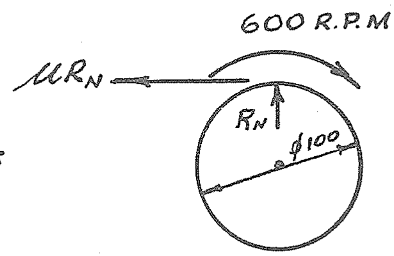
$$3 \times 10^{6} = \frac{x \times 14760 \times 10^3 \times 10^3}{36 \times 10^2}$$

$$x = \frac{3 \times 10^2 \times 36}{1476}$$

$= 7.3$

or $\underline{7 \text{ trucks}}$

15/28
A cylindrical bearing 100 mm in diameter is subject to a load of 8 tonnes. Determine the power absorbed by friction between the bearing and its journal when the shaft is rotating at 600 rpm. The coefficient of friction is 0.03.



$$600 \text{ R.P.M} = \frac{600 \times \pi \times 100}{60 \times 1000} \text{ m/s}$$

$$= \pi \text{ m/s}$$

$$P = F \times S / t$$

$$= \frac{\mu R_N \times \pi}{1}$$

$$= \frac{0.03 \times 8 \times 1000 \times 9.8 \times \pi}{1000} \text{ kW}$$

$$= \underline{7.38 \text{ kW}}$$

15/29
Determine the torque on a shaft which transmits 600 kW when running at 200 rpm.

Let the radius of the shaft be R metres

$\therefore$ Dist. travelled in 1 sec $= \dfrac{200 \times 2\pi R}{60}$ m

Power transmitted $= 600 \times 10^3$ Nm/s

$$P = \frac{F \times S}{t}$$

$$600 \times 10^3 = \frac{F \times 200 \times 2\pi R}{60}$$

$$F = \frac{600 \times 10^3 \times 60}{200 \times 2\pi R}$$

$$= \frac{28.64 \times 10^3}{R} \text{ N}$$

$$\text{Torque} = F \times r$$

$$= \frac{28.64 \times 10^3 \times \cancel{R}}{\cancel{R}}$$

$$= \underline{28.64 \text{ kNm}}$$

15/30
A guillotine relies for its operation on the kinetic energy stored in its flywheel. The flywheel has a mass of 400 kg, a radius of gyration of 1.3 metres, and is rotating at 300 rpm. During a one-second shearing operation, it slows to 270 rpm. Determine
(i) the loss in kinetic energy of the flywheel during the cut;
(ii) the power used during this cut, if the machine has an overall efficiency of 60%.

$$I = mk^2$$
$$= 400 \times 1.3^2$$
$$= 676 \text{ kgm}^2$$

(i) At 300 R.P.M

$$\omega = \frac{300 \times 2\pi}{60}$$
$$= 10\pi \text{ rad/s}$$

and Rotary K.E. $= \frac{1}{2} I\omega^2$
$$= \frac{1 \times 676 \times 100\pi^2}{2}$$
$$= 333.6 \text{ kJ}$$

At 270 R.P.M

$$\omega = \frac{270 \times 2\pi}{60}$$
$$= 9\pi \text{ rad/s}$$

and Rotary K.E. $= \frac{1}{2} I\omega^2$
$$= \frac{1 \times 676 \times 81\pi^2}{2}$$
$$= 270 \text{ kJ}$$

$\therefore$ Loss in K.E. during operation $= \underline{63.6 \text{ kJ}}$

(ii) Work done = Loss in K.E.
$$= 63.6 \text{ kJ}$$

Since this takes place in 1 second the power used in the operation = 63.6 kW

With an efficiency of 60% the power used is found by:

$$\eta = \frac{\text{output}}{\text{input}}$$

$$\text{input} = \frac{\text{output}}{\eta}$$
$$= \frac{63.6 \times 100}{60}$$
$$= \underline{106 \text{ kW}}$$

15/31
An electrically operated hoist drum of diameter 1.2 metres has a moment of inertia of 100 kg m^2. The friction in the bearings of the hoist is equivalent to a couple of 1 kN m. Determine the power of the motor required to drive this hoist if it is to be capable of lifting a maximum load of 2 tonnes vertically with an acceleration of 1 m/s^2 for 4 seconds. Assume that motor efficiency is 87%.

With a 1 m/s^2 accel. upwards there must be an unbalanced force, F, acting on the 2T load.

$$F = (P - 19600)\ N$$

$$F = ma$$
$$P - 19600 = 2000 \times 1$$
$$P = 21{,}600\ N$$

The total torque, T, acting on the drum equals
Torque (T_1) to overcome P plus
Torque (T_2) to overcome drum inertia plus
Torque (T_3) to overcome friction.

$T_1 = 21600 \times 0.6 = 12960$ Nm
$T_2 = I\alpha = 100 \times 1/0.6 = 166.67$ Nm
$T_3 = 1000$ Nm

$\therefore$ Total Torque $T = 14126.67$ Nm

$u = 0$
$v = ?$ $\quad v = u + at \quad$ and ang. vel $\omega = v/r$
$a = 1\ m/s^2 \quad = 4\ m/s \qquad\qquad = 6.67$ rad/s
s
$t = 4s$

Power required by drum $= T\omega = 14126.67 \times 6.67$
$= 94.18$ kW

From $\eta = \dfrac{\text{output}}{\text{input}}$

Motor rating $= \dfrac{94.18 \times 100}{87} = 108.25$ kW

or <u>110 kW</u>

15/32
A pressing machine has a flywheel of moment of inertia 50 kg m^2 which provides part of the energy for the pressing operation. This flywheel is driven by a 3-kW motor and each pressing requires 5 kJ of energy and takes 1 second to complete. Determine the reduction in speed of the flywheel in rpm and the maximum number of pressings possible per minute, if the initial speed of the flywheel before each pressing is 240 rpm.

240 R.P.M. (ω_1) = 25·13 rad/s

K.E. of Flywheel = $\frac{1}{2}.I.\omega_1^2$

$= \frac{1}{2} \times 50 \times 25{\cdot}13^2$

$= 15{,}788$ J

Power supplied by motor = 3000 W

∴ Energy " " " per. sec = 3000 J

Each pressing requires 5000 J, therefore the flywheel must supply (5000 - 3000) or 2000 J each second and this equals the change in K.E. each second. If ω_2 is the flywheel speed at the completion of a pressure then:

$$15{,}788 - \tfrac{1}{2} \times 50 \times \omega_2^2 = 2000$$

$$25\,\omega_2^2 = 13{,}788$$

$$\omega_2 = 23{\cdot}5 \text{ rad/s}$$

or 224 R.P.M.

∴ Change in speed = 240 - 224 or <u>16 R.P.M</u>

1 watt = 1 Joule/sec.

Total energy required per sec. = 5000 J

Motor can supply only 3000 J each sec.

(the flywheel supplies the remaining 2000 J)

∴ Motor capacity is 3000/5000 of the energy requirement each second.

i.e. the motor will make 3/5 or 0·6 of a pressing each second or:

<u>36 pressings each minute</u>

16

Direct Stress and Strain

Note: 1 MPa = 1 N/mm^2

= 10^6 N/m^2

1 GPa = 10^3 N/mm^2

= 10^9 N/m^2

$$\sigma = \frac{P}{A}$$

$$\varepsilon = \frac{\Delta L}{L} = \frac{e}{L}$$

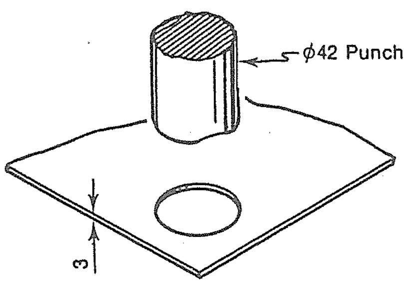
$$E = \frac{\sigma}{\varepsilon} = \frac{PL}{Ae}$$

$$\text{SE/unit volume} = \frac{\sigma^2}{2E}$$

16 DIRECT STRESS AND STRAIN
Intensity of Stress. Types of Stress. Strain. Hooke's Law. *The Tension Test. Other Mechanical Tests Involving Direct Stress. Engineering Stress.* Resilience. Factor of Safety. Direct Stresses in Compound Bars.

g = 9.8 m/s^2

16/13
A press tool is required to punch out discs 42 mm diameter from steel sheet 3 mm thick. If the ultimate shearing stress of the sheet steel is 320 megapascals, calculate:
(i) the minimum force required to punch out a disc,
(ii) the average compressive stress set up in the punch.



(i) $\sigma = P/A$

$$320 = \frac{P}{\pi \times 42 \times 3}$$

$$P = \frac{320 \times \pi \times 42 \times 3}{10^3}$$

$$= \underline{126.7 \text{ kN}}$$

(ii) $\sigma = P/A$

$$= \frac{126.7 \times 10^3 \times 4}{\pi \times 42 \times 42}$$

$$= \underline{91.5 \text{ MPa}}$$

16/14
A 20 mm diameter bar has a tensile load of 36kN steadily applied to it. It is observed that a gauge length of 200 mm on the bar increased to 200.3 mm under the full load. If the elastic limit was not exceeded, determine
(i) the stress in the bar;
(ii) the strain in the bar;
(iii) the modulus of elasticity of the material.

(i) $\sigma = P/A$

$= \dfrac{36 \times 10^3 \times 4}{\pi \times 20 \times 20}$

$= \underline{114 \cdot 59 \text{ MPa}}$

(ii) $\varepsilon = e/L$

$= \dfrac{0 \cdot 3}{200}$

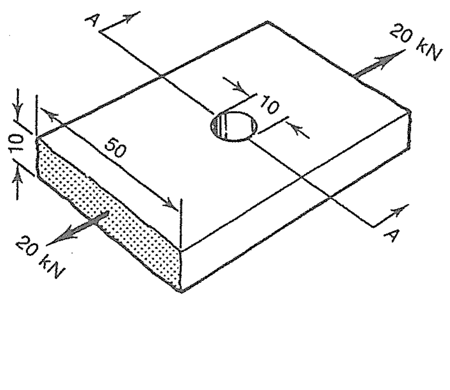
$= \underline{1 \cdot 5 \times 10^{-3}}$

(iii) $E = \sigma/\varepsilon$

$= \dfrac{114 \cdot 59}{1 \cdot 5 \times 10^{-3}}$

$= \underline{76 \cdot 4 \text{ GPa}}$

16/15
A steel tie has a cross-section as shown in the diagram. Determine the stress in the section *A-A* if the tie is subjected to a tensile load of 20 kN.



Area $= (50 \times 10) - (10 \times 10)$

$= 400 \text{ mm}^2$

$\sigma = L/A$

$= \dfrac{20 \times 10^3}{400}$

$= \underline{50 \text{ MPa}}$

16/16
A press can exert a maximum force of 3.5 MN. What maximum thickness of sheet metal can be sheared by a cylindrical punch of diameter 100 m, if the ultimate shear strength of the sheet material is 400 MPa? What maximum compressive stress will be developed in the punch during this operation?

$A = \pi D \times \text{Thickness}$

$= \pi D \times T$

$\sigma = P/A$

$A = P/\sigma$

$\pi \times 100 \times T = \dfrac{3.5 \times 10^6}{400}$

$T = \dfrac{3.5 \times 10^6}{400 \times \pi \times 100}$

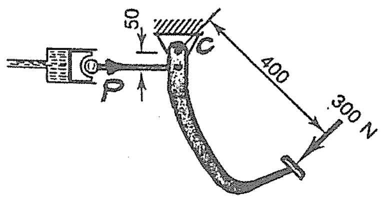
$= \underline{27.9 \text{ mm}}$

$\sigma = P/A$

$= \dfrac{3.5 \times 10^6 \times 4}{\pi \times 100 \times 100}$

$= \underline{445.6 \text{ MPa}}$

16/17
A hydraulic master cylinder is shown in the diagram.
(i) Determine the compressive force in the push-rod when the foot-pressure on the hydraulic brake pedal is 300 N.
(ii) What is the compressive stress in the piston if the master cylinder has a cross-sectional area of 600 mm²?



(i) Take mom. about C

$P \times 50 = 300 \times 400$

$P = \underline{2.4 \text{ kN}}$

(ii) $\sigma = P/A$

$= \dfrac{2.4 \times 10^3}{600}$

$= \underline{4 \text{ MPa}}$

16/18
A tie rod in a machine is 3.140 m long and 40 mm in diameter. What is the extension under a load of 100 kN? The elastic modulus of the material is 200 GPa.

$L = 3140 \text{ mm}$

$A = \dfrac{\pi \times 40 \times 40}{4} \text{ mm}^2$

$P = 100 \times 10^3 \text{ N}$

$E = 200 \times 10^3 \text{ MPa}$

$\sigma = \dfrac{100 \times 10^3 \times 4}{\pi \times 40 \times 40}$

$\varepsilon = \dfrac{e}{3140}$

$E = \sigma/\varepsilon$

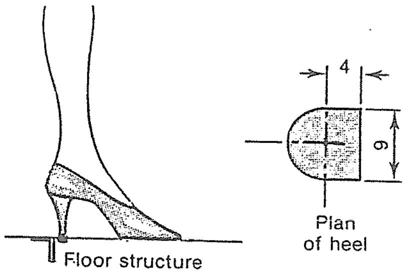
$200 \times 10^3 = \dfrac{100 \times 10^3 \times 4 \times 3140}{\pi \times 40 \times 40 \times e}$

$e = \dfrac{100 \times 10^3 \times 4 \times 3140}{200 \times 10^3 \times \pi \times 40 \times 40}$

$= \underline{1.25 \text{ mm}}$

16/19

Some years ago airlines banned the wearing of stiletto heels by women because of damage to the aluminium alloy skins and floors of aircraft. Determine the approximate mass of a woman passenger whose heel just perforates the aluminium floor, given that the floor is 0.35 mm thick and its ultimate shear stress is 45 MPa. Assume that her full weight-force acted on one heel. The size of her heel is as shown in the diagram.



Shear Area : $\left(9 + 8 + \frac{\pi \times 9}{2}\right) \times 0.35$

$= 10.9\ mm^2$

$\sigma = P/A$

$45 = P/10.9$

$P = 490.6\ N$

$\therefore$ Mass must not exceed 50 kg

16/20

Calculate the length of a steel tie rod of cross-sectional area 300 mm² which, when suspended vertically, contains a maximum stress of 100 kPa due to its own weight-force. The weight density of steel is 60 kN/m³.

Wt. density $= 60 \times 10^3\ N/m^3$

$= \frac{60 \times 10^3}{10^9}\ N/mm^3$

Vol. $= 300L\ mm^3$

$\therefore$ Mass force $= \frac{300 \times L \times 60 \times 10^3}{10^9}\ N$

$\sigma = P/A$

$\frac{100}{10^3} = \frac{\cancel{300} \times L \times 6\cancel{0} \times \cancel{10^3}}{10^{\cancel{9}5} \times \cancel{300}}$

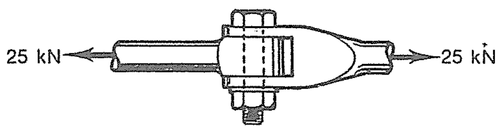
$L = \frac{100 \times 10^5}{6 \times 10^3}\ m$

$= 1.67\ m$

16/21

Given that the shear stress in the bolt is not to exceed 150 MPa and that the maximum axial load to be applied to the rod coupling is 25 kN, calculate the minimum diameter bolt that should be used.

What size bolt should be used if a factor of safety of 2 is introduced in the design calculations?



The bolt is in double shear.

$\therefore \sigma = P/2A$

$150 = \frac{25 \times 10^3}{2A}$

$\therefore A = 83.34\ mm^2$

and $d = 10.3\ mm$

A 12 mm (M12) bolt would be suitable. Threading reduces the effective dia.

For a S.F. of 2 the bolt must carry twice the load i.e. 50 kN.

$\therefore$ Twice the C.S.A. or $166.67\ mm^2$ is reqd.

d would equal 14.56 mm

A 16 mm (M16) bolt would be suitable.

16/22
A wire 2.5 m long and of uniform circular cross-section is stretched 5 mm by the application of a load of 2 kN. Calculate the wire diameter, if the modulus of elasticity of the wire material is known to be 200 GPa.

$$E = \frac{\sigma}{\varepsilon}$$

$$= \frac{P \times L}{A \times e}$$

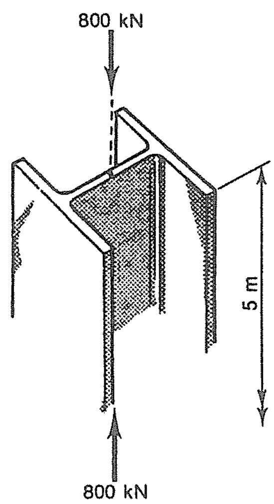
$$A = \frac{P \times L}{E \times e}$$

$$= \frac{2 \times 10^3 \times 2.5 \times 10^3}{200 \times 10^3 \times 5}$$

$$= 5\ mm^2$$

$$\therefore d = \underline{2.52\ mm}$$

16/23
A 5-metre length of 310 UBP 79 (steel universal bearing pile, mass 79 kg/m) has a cross-sectional shape as shown in the diagram. Determine the stress in the column (i) near the top and (ii) near the bottom, when it is supporting an axial compressive load of 800 kN. The cross-section of 310 UBP 79 has an area of 10 000 mm².



Column Mass = 395 kg
Column Mass Force = 3.87 kN

(i) Near the top only the 800 kN load is operating

$$\therefore \sigma = P/A$$

$$= \frac{800 \times 10^3}{10000}$$

$$= \underline{80\ MPa}$$

(ii) Near the bottom the column is supporting an extra 3.87 kN (its weight force).

$$\therefore \sigma = P/A$$

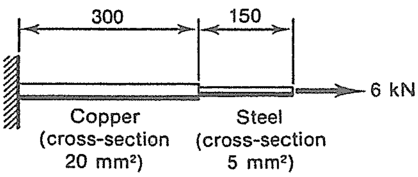
$$= \frac{803.87 \times 10^3}{10000} = \underline{80.39\ MPa}$$

16/24

A compound bar is shown in the diagram. Determine:

(i) the tensile stress in the steel section of the bar and the tensile stress in the copper section of the bar;

(ii) the elongation of the bar if the modulus of elasticity of the steel is 200 GPa and that of the copper is 100 GPa.



$P_{cu.} = P_{st.} = \text{Total Load } 6\text{ kN.}$

(i) $\sigma_c = P_c / A_c = \dfrac{6 \times 10^3}{20} = \underline{300 \text{ MPa}}$

$\sigma_s = P_s / A_s = \dfrac{6 \times 10^3}{5} = \underline{1200 \text{ MPa.}}$

(ii) **Copper Section**

$\sigma_c = \dfrac{6 \times 10^3}{20}$ MPa

$\varepsilon = \dfrac{e}{L} = \dfrac{e}{300}$ mm

$E = \dfrac{P \times L}{A \times e}$

Steel Section

$\sigma_s = \dfrac{6 \times 10^3}{5}$

$\varepsilon = \dfrac{e}{L} = \dfrac{e}{150}$

$E = \dfrac{P \times L}{A \times e}$

Note: All units must be "N" and "m"

$\therefore e = \dfrac{6 \times 10^3 \times 300 \times 10^6}{20 \times 10^3 \times 100 \times 10^9} = 0.0009 \text{ m or } 0.9 \text{ mm}$

$e = \dfrac{6 \times 10^3 \times 150 \times 10^6}{5 \times 10^3 \times 200 \times 10^9} = 0.0009 \text{ m or } 0.9 \text{ mm}$

$\therefore$ Total ext. $= \underline{1.8 \text{ mm}}$

16/25

When a bolt is in tension, the load on the nut is transmitted through the root area of the bolt. An ISO metric bolt 24 mm in diameter (root area = 353 mm²) carries a tensile load. Find the percentage error in the calculated value of the stress if the shank area is used instead of the root area.

Let the tensile load $= P$

Then stress in shank area $= \dfrac{P \times 4}{\pi \times 24^2}$

and stress in root area $= \dfrac{P}{353}$

Difference in stress $= \left(\dfrac{P}{353} - \dfrac{4P}{576}\right)$ MPa

$= P\left(2.84 \times 10^{-3} - 2.21 \times 10^{-3}\right)$

% Error $= \dfrac{\cancel{P}(0.63 \times 10^{-3}) \times 353 \times 100}{\cancel{P}}$

$= \underline{22.2\%}$

16/26

A quick-acting vice is operated by the air cylinder as shown in the diagram. The air pressure used to clamp the small object A in this vice is 75 kPa. Determine:

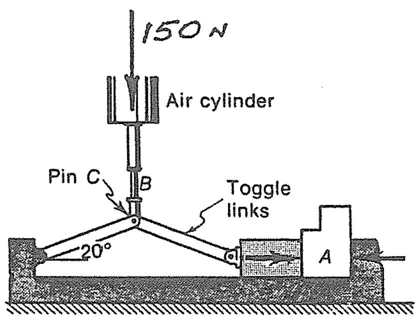
(i) the clamping force exerted on the object A;

(ii) the compressive stress in the air-cylinder piston, given that its cross-sectional area is 2000 mm^2;

(iii) the stress in the activating rod at B, given that it does not bend significantly and that its cross-sectional area is 300 mm^2 at this section;

(iv) the compressive stress in the toggle links, given that they are 15 mm square; and

(v) the shear stress in the pin at C, given that its diameter is 8 mm.



$$\sigma = P/A$$

$$P = \sigma \times A$$
$$= 75 \times 10^3 \times 2000$$
$$= 150\ N$$

(i) Half of the 150N load will push to the left and the other half to the right

$\therefore$ Value of force in Toggles $= 75/\sin 20°$

$= 219{\cdot}3\ N$

Resolve horiz.

Pressure force at A $= 219{\cdot}3 \cos 20° = \underline{206\ N}$

(ii) Stress in piston $= 150/2000$

$= 0{\cdot}075$ MPa or $\underline{75\ kPa}$

(iii) Stress in B $= 150/300$

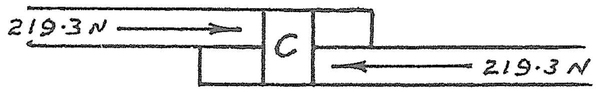
$= 0{\cdot}5$ MPa or $\underline{500\ kPa}$

(iv) Force in Toggles $= 219{\cdot}3\ N$ and C.S.A. $= 225\ mm^2$

$\therefore$ Stress in Toggles $= 219{\cdot}3/225$ MPa

$= 0{\cdot}975$ MPa or $\underline{975\ kPa}$

(v)



L.H. Toggle exerts shear stress on Pin C of:

$$\frac{219{\cdot}3 \times 4}{\pi d^2} = \underline{4{\cdot}36\ MPa}$$

R.H. Toggle also exerts $\underline{4{\cdot}36\ MPa}$

16/27

The permissible extension of a steel tie is 100 mm and the fractional strain in it is not to exceed 0.002. Calculate the maximum length of the tie.

$$\text{Strain } \varepsilon = e/L$$

$$0.002 = 100/L$$

$$L = \frac{100}{0.002 \times 1000} = \underline{50 \text{ m}}$$

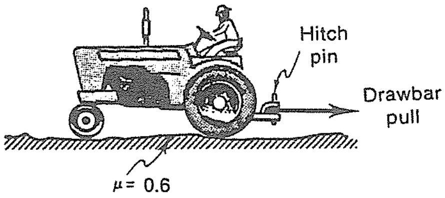
16/28

The tractor shown in the diagram has a mass of 5 tonnes. Determine

(i) the maximum drawbar pull of the tractor if the coefficient of friction between wheels and the ground is 0.6;

(ii) the diameter of the drawbar hitch pin, given that the ultimate shear stress of the material of manufacture is 180 MPa and a factor of safety of 3 is to be used in its design.

Consider that the total weight force of the tractor acts on the two rear wheels when maximum drawbar pull is applied.



(i) The max. friction force to prevent the wheels spinning $= \mu R_N = \dfrac{0.6 \times 5 \times 10^3 \times 9.8}{1000}$

$= 29.4$ kN

If the drawbar pull exceeds this the wheels will spin $\therefore$ Max. Pull $= \underline{29.4 \text{ kN}}$

(ii) Stress $= P/A$

Area $= \dfrac{29.4 \times 10^3}{180}$

and Dia. $= \sqrt{\dfrac{29.4 \times 10^3 \times 4}{180 \times \pi}} = 14.4$ mm

For a Safety Factor of 3 to be introduced.

Safety Factor $= \dfrac{\text{U.T.S.}}{\text{Allowable Working Stress}}$

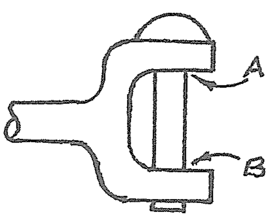
$\therefore$ Allowable Work. Stress $= \dfrac{\text{U.T.S}}{\text{S.F}}$

$= 60$ MPa

$\sigma = P/A$

$A = \dfrac{29.4 \times 10^3}{60}$ mm²

This represents Areas at A & B



$\therefore$ Effective shear area $= \dfrac{29.4 \times 10^3}{60 \times 2}$

$= 245$ mm²

and dia. $= 17.66$ mm or $\underline{18 \text{ mm}}$

16/29
A pull of 1.25 kN is applied to a wire which operates a railway signal. The wire is 60 metres long and has a cross-section of 20 mm^2. If the movement of the end of the wire in the signal box is 250 mm, determine the length of movement of the end of the wire at the signal. The modulus of elasticity of the wire material is 200 GPa.

$$E = \sigma / e$$

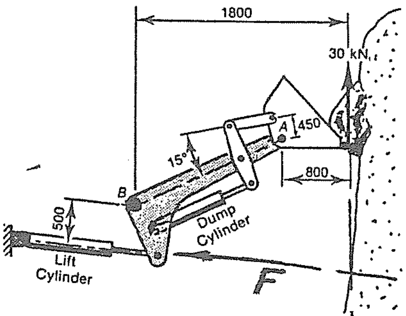
$$200 \times 10^3 = \frac{1{\cdot}25 \times 10^3 \times 60 \times 10^3}{20 \times x}$$

$$x = \frac{1{\cdot}25 \times 10^3 \times 60 \times 10^3}{20 \times 200 \times 10^3}$$

$$= 18{\cdot}75 \text{ mm}$$

This represents a loss in movement
$\therefore$ Movement at signal $= 250 - 18{\cdot}75 = \underline{231{\cdot}25 \text{ mm}}$

16/30
One side of the mechanism for a hydraulically operated loader is shown in the diagram. In removing a large rock from the face of the excavation, the loader bucket exerts a vertically upward force of 30 kN. Assuming that this force acts equally on both sides of the loader mechanism, determine
(i) the compressive stress in the lift-cylinder rod, given that its effective diameter is 50 mm;
(ii) the shear force on the bucket swivel pin A and the stress present within it, given that its diameter is 60 mm.



(i) Moments about B

$$F \times 500 = 30 \times 10^3 \times 1800$$

$$F = \frac{30 \times 10^3 \times 1800}{500}$$

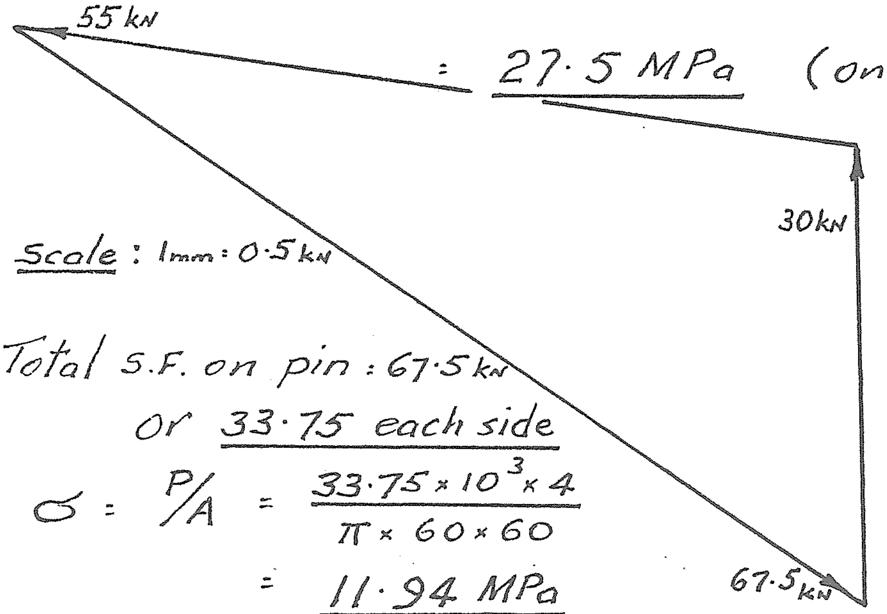
$$= 108 \text{ kN}$$

i.e. 54 kN on each side

$$\therefore \text{Stress} = \frac{54 \times 10^3 \times 4}{\pi \times 50 \times 50}$$

$$= \underline{27{\cdot}5 \text{ MPa}} \quad \text{(on each side)}$$

(ii)



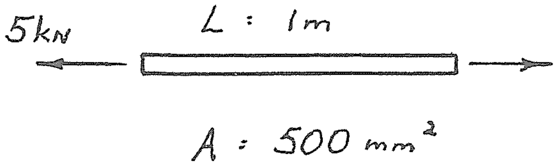
Scale : 1mm = 0·5 kN

Total S.F. on pin = 67·5 kN
or $\underline{33{\cdot}75 \text{ each side}}$

$$\sigma = P/A = \frac{33{\cdot}75 \times 10^3 \times 4}{\pi \times 60 \times 60}$$

$$= \underline{11{\cdot}94 \text{ MPa}}$$

16/31
A tensile load of 5 kN is applied to an aluminium alloy bar of length 1 metre and of cross-sectional area 500 mm². Given that E for this material is 70 GPa, calculate
(i) the extension of the bar;
(ii) the total strain energy present in the deformed bar;
(iii) the modulus of resilience of the alloy;
provided that its proportional limit was not exceeded by the application of this load.



(i) $E = \dfrac{PL}{A\,x}$

$70 \times 10^3 = \dfrac{5 \times 10^3 \times 1 \times 10^3}{500 \times x}$

$x = \dfrac{5 \times 10^3 \times 1 \times 10^3}{500 \times 70 \times 10^3}$

$= \underline{0.143 \text{ mm}}$

(ii) Strain Energy = Work done

Av. Force used $= \dfrac{5 \times 10^3}{2}$

$\therefore$ Work (S.E.) $= \dfrac{5 \times 10^3 \times 0.143}{2 \times 10^3}$

$= \underline{0.357 \text{ J}}$

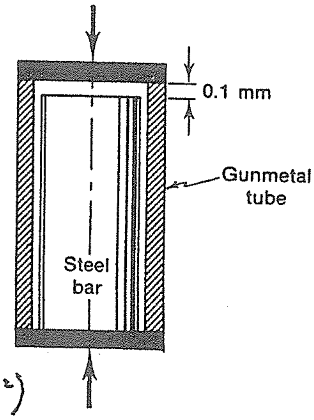
(iii) Bar Area $= 500/10^6 \text{ m}^2$

Bar Vol. $= \dfrac{500 \times 1}{10^6} \text{ m}^3$

Mod. of Resilience = S.E./unit Vol.

$= \dfrac{0.357 \times 10^6}{1 \times 500} = \underline{714 \text{ J/m}^3}$

16/32
A 400-mm long steel bar of diameter 20 mm is placed concentrically inside a gunmetal tube of 22-mm inside diameter and 4-mm wall thickness. The length of the gunmetal tube exceeds the length of the steel bar by 0.1 mm. A compressive load is then applied to the ends of the gunmetal tube through the rigid plates as shown. Find
(i) the load needed to compress the tube to the same length as the steel bar;
(ii) the stress in the gunmetal and in the steel when an axial compressive load of 50 kN is applied.
$E_{steel} = 200$ GPa $\qquad E_{gunmetal} = 100$ GPa



(i) Area of G.M. Tube $= \dfrac{\pi(D^2 - d^2)}{4}$

$= \dfrac{\pi(900 - 484)}{4} = 326.73 \text{ mm}^2$

$E = \dfrac{PL}{A\,x} \qquad \therefore P = \dfrac{E \times A \times x}{L}$

$= \dfrac{100 \times 10^3 \times 326.73 \times 0.1}{400.1 \times 10^3}$

$= \underline{8.16 \text{ kN}}$

16/32 (cont.)

(ii) Area of G.M. = $326{\cdot}73\ mm^2$ & Area of steel = $314{\cdot}16\ mm^2$

Pre-stress in G.M. $= E \times \varepsilon$ OR P/A

$$= \frac{100 \times 10^3 \times 0{\cdot}1}{400{\cdot}1} \qquad = \frac{8{\cdot}16 \times 10^3}{326{\cdot}73}$$

$$= 24{\cdot}99\ MPa \qquad = 24{\cdot}97\ MPa$$

NOTE: $\boxed{\text{Total } P = (P_{Gm} + P_{st.})}$: $\boxed{x_{G.M.} = x_{st}}$: $\boxed{L_{G.M.} = L_{st.}}$

$$E_{st} = \frac{P_{st} \times L_{st.}}{A_{st.} \times x_{st.}} \qquad E_{G.M} = \frac{P_{GM} \times L_{GM}}{A_{Gm} \times x_{Gm}}$$

or $$\frac{L_{st}}{x_{st}} = \frac{E_{st} \times A_{st}}{P_{st}}$$ and $$\frac{L_{Gm}}{x_{Gm}} = \frac{E_{Gm} \times A_{Gm}}{P_{GM}}$$

But $$\frac{L_{st}}{x_{st}} = \frac{L_{Gm}}{x_{Gm}} \quad \therefore \frac{E_{st.} \times A_{st.}}{P_{st.}} = \frac{E_{Gm} \times A_{Gm}}{P_{Gm}} \quad \text{------ (1)}$$

and $$P_{st} + P_{Gm} = 50 \quad \text{---------- (2)}$$

from (2) subst. $(50 - P_{Gm})$ for $P_{st.}$ in (1)

$$\therefore \frac{2\cancel{00} \times \cancel{10^3} \times 314{\cdot}16}{50 \times \cancel{10^3} - P_{Gm} \times \cancel{10^3}} = \frac{1\cancel{00} \times \cancel{10^3} \times 326{\cdot}73}{P_{Gm} \times \cancel{10^3}}$$

$$628{\cdot}32\ P_{Gm} = 16{,}300 - 326{\cdot}73\ P_{Gm}$$

$$P_{Gm} = 17{\cdot}1\ kN$$

and $$P_{st.} = 50 - 17{\cdot}1 = 32{\cdot}9\ kN$$

$$\sigma_{G.M} = \frac{17{\cdot}1 \times 10^3}{326{\cdot}73} + 24{\cdot}99$$

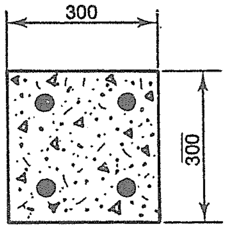
$$= \underline{77{\cdot}3\ MPa}$$

$$\sigma_{st.} = \frac{32{\cdot}9 \times 10^3}{314{\cdot}16}$$

$$= \underline{104{\cdot}7\ MPa}$$

16/33
A concrete column reinforced by 4 steel rods is designed to carry a total load of 30 tonnes. If the combined cross-sectional area of the steel rods is 2000 mm^2, calculate the stress in the steel and in the concrete.

$E_{steel} = 200$ GPa $\qquad E_{concrete} = 14$ GPa



Area of Steel $= 2 \times 10^3 \text{ mm}^2$

Area of Concrete $= 9 \times 10^4 - 2 \times 10^3 = 88 \times 10^3 \text{ mm}^2$

$\boxed{\text{Total } P = P_{st} + P_c}$: $\boxed{x_{st} = x_c}$: $\boxed{L_{st} = L_c}$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} \quad \& \quad \varepsilon = \frac{x_{st}}{L_{st}} \qquad \sigma_c = \frac{P_c}{A_c} \quad \& \quad \varepsilon = \frac{x_c}{L_c}$$

$$\therefore E_{st} = \frac{P_{st} \times L_{st}}{A_{st} \times x_{st}} \qquad \therefore E_c = \frac{P_c \times L_c}{A_c \times x_c}$$

$$\text{or} \quad \frac{L_{st}}{x_{st}} = \frac{E_{st} \times A_{st}}{P_{st}} \qquad \text{and} \quad \frac{L_c}{x_c} = \frac{E_c \times A_c}{P_c}$$

$$\text{Since} \quad \frac{L_{st}}{x_{st}} = \frac{L_c}{x_{st}} \qquad \frac{E_{st} \times A_{st}}{P_{st}} = \frac{E_c \times A_c}{P_c} \quad \text{----} \ (1)$$

$$\text{and} \quad P_{st} + C = 294 \quad \text{-------} \ (2)$$

From (2) subst. $(294 \times 10^3 - P_c)$ for P_{st} in (1)

$$\frac{200 \times \cancel{10^3} \times 2 \times \cancel{10^3}}{294 \times 10^3 - P_c} = \frac{14 \times \cancel{10^3} \times 88 \times \cancel{10^3}}{P_c}$$

$$\frac{400}{(294 \times 10^3 - P_c)} = \frac{1232}{P_c}$$

$$P_c = 222 \text{ kN}$$

$$\text{and} \quad P_{st} = 294 - 222 = 72 \text{ kN}$$

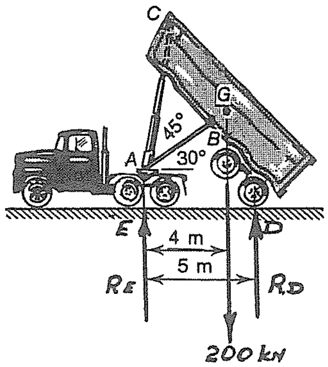
$$\sigma_{st} = \frac{72 \times 10^3}{2 \times 10^3} \qquad \sigma_c = \frac{222 \times 10^3}{88 \times 10^3}$$

$$= \underline{36 \text{ MPa}} \qquad = \underline{2 \cdot 52 \text{ MPa}}$$

16/34

A large tipping trailer is shown. Its 15-tonne load of soil is stable but on the verge of tipping. Given that G, the combined centre of mass of the trailer plus load, is 500 mm above the pin B, determine the forces in the hydraulic ram AC and in the two ties AB (note that one lies directly behind the other). The mass of the empty trailer is 5 tonnes.

The ties are made from thick-walled pipe, outside diameter 75 mm, inside diameter 55 mm. Determine the tensile stress in these ties when in the position shown. Given that the elastic modulus for the tie material is 200 GPa, determine their strains.

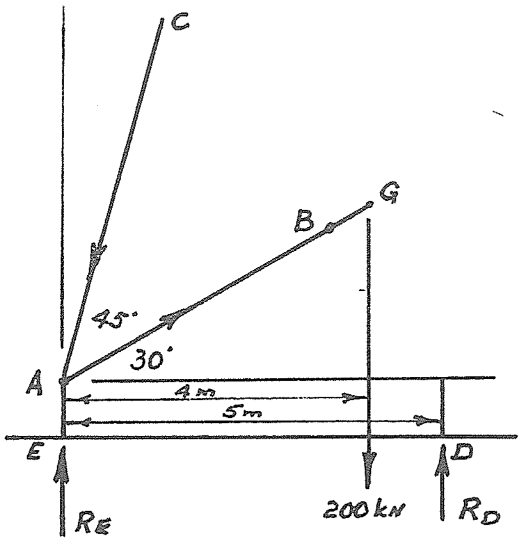


Take moments about A

$R_D \times 5 = 200 \times 4 \quad \therefore \; R_D = 160\text{ kN}\uparrow$

Resolve forces vert.

$R_E + 160 = 200 \quad \therefore \; R_E = 40\text{ kN}\uparrow$



Consider the joint A

Resolve vert.

$AC \sin 75^\circ = 40 + AB \sin 30^\circ \cdots\cdots (1)$

Resolve horiz.

$AC \cos 75^\circ = AB \cos 30^\circ \cdots\cdots\cdots (2)$

From (2) $\quad AB = \dfrac{AC \cos 75^\circ}{\cos 30^\circ}$

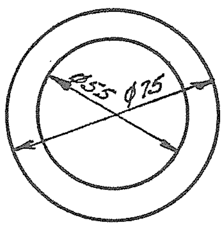
Subst. in (1) $\quad AC \sin 75^\circ = 40 + \dfrac{AC \cos 75^\circ \times \sin 30^\circ}{\cos 30^\circ}$

$AC = \underline{48{\cdot}98\text{ kN (C)}}$

Solving (3) $\quad AB = \underline{14{\cdot}64\text{ kN (T)}}$ or $\underline{7{\cdot}32\text{ kN (T) each}}$

C.S.A. for each Tie $= \frac{\pi}{4}(D^2 - d^2)$

$= 2042\text{ mm}^2$



Stress in each Tie $= \dfrac{7{\cdot}32 \times 10^3}{2042}$ MPa

$= \underline{3{\cdot}57\text{ MPa}}$

$E = \dfrac{\sigma}{e}$

$e = \dfrac{\sigma}{E} = \dfrac{3{\cdot}57 \times 10^6}{200 \times 10^9}$

$= \underline{1{\cdot}78 \times 10^{-5}}$

17 Beams and Bending

(Use $g = 10\,\text{m/s}^2$ unless otherwise specified.)

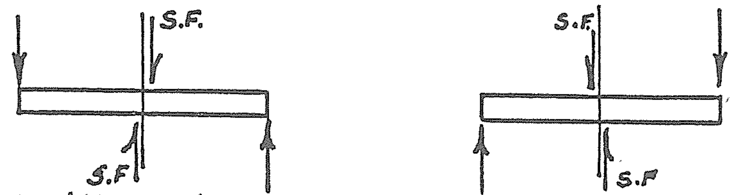
17 BEAMS AND BENDING
Types of Beams. Shear Force and Bending Moment. *Sign Convention for Shear Force and Bending Moment. Shear Force Diagrams: General Method for Constructing Shear Force Diagrams. Bending Moment Diagrams: General Method for Constructing Bending Moment Diagrams. Shear Force and Bending Moments for Uniformly Distributed Loads.* Stresses due to Bending. *Centroids and Second Moment of Area. Units of Second Moment of Area.* Calculation of Stresses due to Bending.

SIGN CONVENTIONS

N.B. Shear Force and Bending Moments are the BALANCING forces required for equilibrium.

1. Shear Force

(a) A DOWNWARD S.F. for the RIGHT-HAND SECTION is +
(b) A DOWNWARD S.F. for the LEFT-HAND SECTION is −
(c) An UPWARD S.F. for the RIGHT-HAND SECTION is −
(d) An UPWARD S.F. for the LEFT-HAND SECTION is +



Positive Shear   Negative Shear

2. Bending Moments

(a) A CLOCKWISE B.M. for the RIGHT-HAND SECTION is +
(b) A CLOCKWISE B.M. for the LEFT-HAND SECTION is −
(c) An ANTI-CLOCKWISE B.M. for the RIGHT-HAND SECTION is −
(d) An ANTI-CLOCKWISE B.M. for the LEFT-HAND SECTION is +

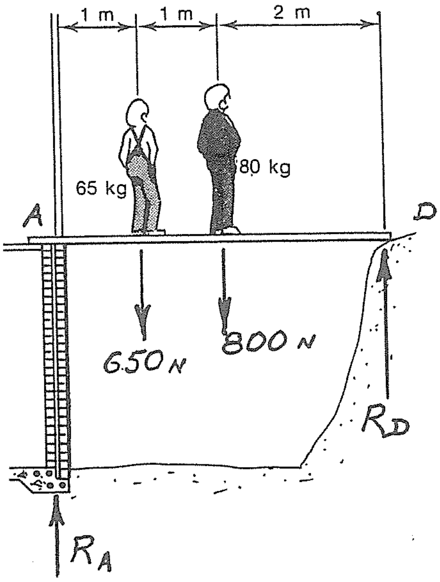
OR

(i) Forces producing SAG at the SECTION are + B.M.
(ii) Forces producing HOG at the SECTION are − B.M.

$$\frac{M}{I} = \frac{E}{r} = \frac{f}{y}$$

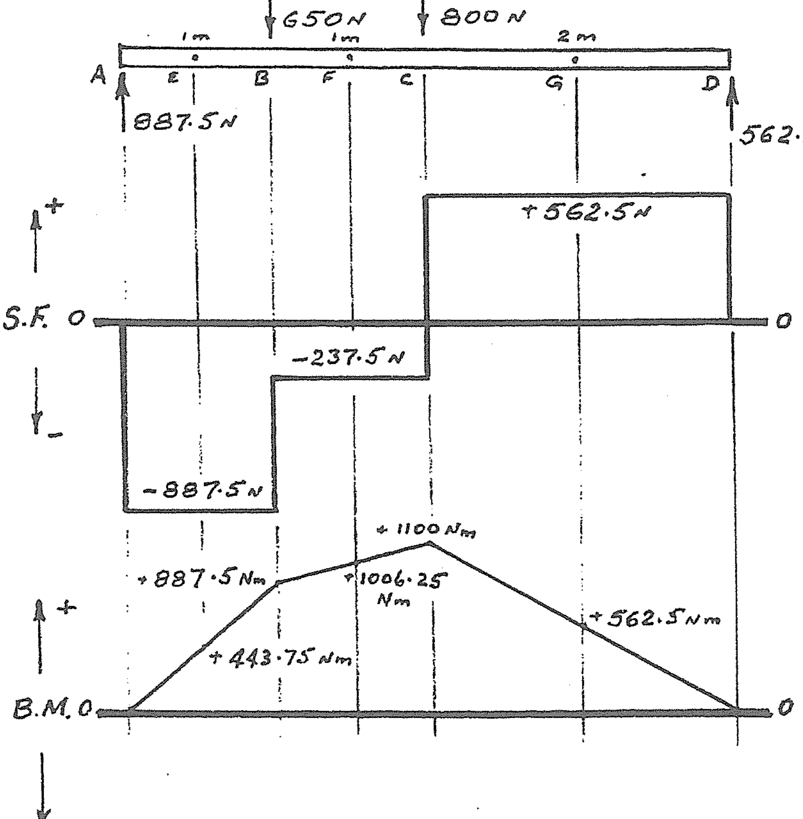
Where:
M : Bending Moment : Nm
I : Moment of Inertia : m^4 ($10^{12}\,mm^4 = 1\,m^4$)
E : Young's Modulus : Pa ($1\,GPa = 10^9\,Pa$)
r : Radius of Curvature : m
f : Stress : Pa or N/m^2
y : Distance from Neutral Axis : m

17/7
A plank is used to provide temporary access over a small excavation to the door of a house. Consider this plank as a simply supported beam and determine the distribution of shear force and bending moments within it when two people of masses 65 kg and 80 kg respectively stand on the plank as shown. Ignore the mass of the plank.



Take moments about A to find:

R_D : 562.5 N & R_A : 887.5 N



SHEAR FORCE

Left of Section

	N
At A :	= 0
Rt. of A :	= -887.5
Before B :	= -887.5
After B : -887.5 + 650	: -237.5
Before C : -887.5 + 650	: -237.5
After C : -887.5 + 650 + 800	: +562.5
Before D : -887.5 + 650 + 800	: +562.5
At D : -887.5 + 650 + 800 - 562.5	: 0

Right of Section

	N
At D :	= 0
Left of D :	: +562.5
Before C :	= +562.5
After C : +562.5 - 800	: -237.5
Before B : +562.5 - 800	: -237.5
After B : +562.5 - 800 - 650	: -887.5
Before A : +562.5 - 800 - 650	: -887.5
At A : +562.5 - 800 - 650 + 887.5	: 0

BENDING MOMENT

Left of Section

	Nm
At A :	: 0
At B : 1 × 887.5	: +887.5
At C : 2 × 887.5 - 1 × 650	: +1125
At D : 4 × 887.5 - 3 × 650 - 2 × 800	: 0

Rt. of Section

	Nm
At D :	: 0
At C : 2 × 562.5	: +1125
At B : 3 × 562.5 - 1 × 800	: +887.5
At A : 4 × 562.5 - 2 × 800 - 1 × 650	: 0

17/8

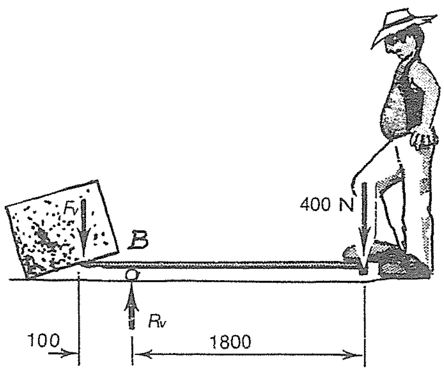
A workman lifting the edge of a dimension stone stands on one end of his crowbar. If the man, by exerting a force of 400 N vertically down, just raises the edge of the stone, determine

(i) the vertical force F_V acting between bar and stone;

(ii) the vertical component R_V of the reaction at the fulcrum;

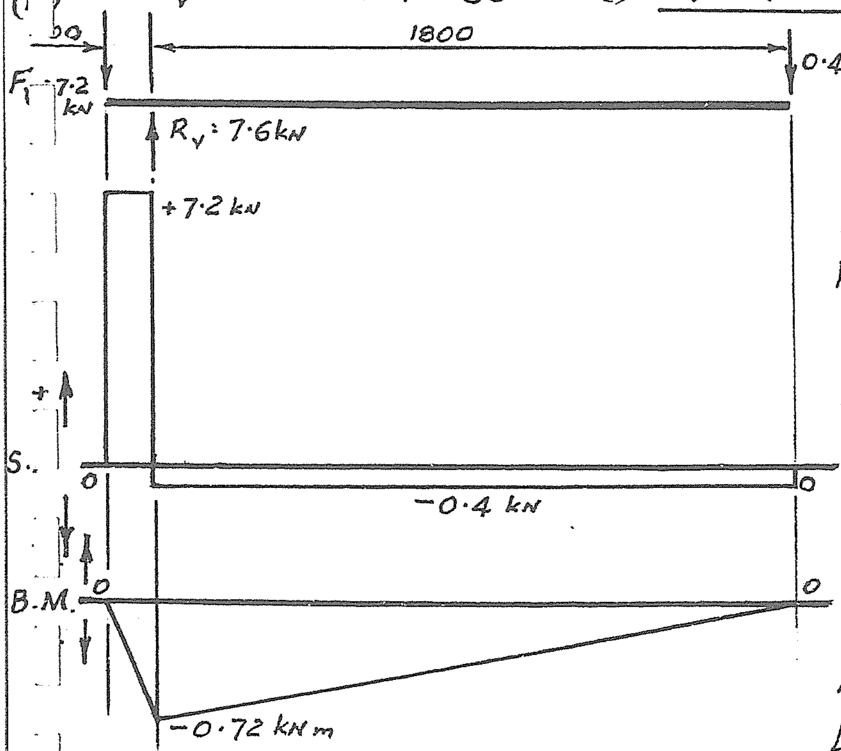
(iii) the distribution of shear force and bending moments along the length of the crowbar.

Neglect the mass of the crowbar since it is small compared to the mass of the stone being lifted.



Take moments about B

(i) $F_V \times 100 = 0{\cdot}4 \times 1800$ (i) $F_V = 7{\cdot}2$ kN ↓ and (ii) $R_V = 7{\cdot}2 + 0{\cdot}4 = 7{\cdot}6$ kN ↑



SHEAR FORCE

Left of Section		kN
At A :	:	0
Rt. of A :	=	+7·2 ↑
Before B :	:	+7·2 ↑
After B : +7·2 − 7·6	:	−0·4 ↓
Before C : +7·2 − 7·6	:	−0·4 ↓
At C : +7·2 − 7·6 + 0·4	:	0

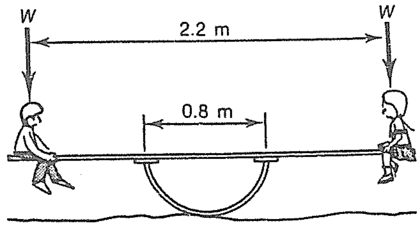
Rt. of Section		kN
At C :	:	0
Left of C :	:	−0·4 ↓
Before B :	:	−0·4 ↓
After B : +7·6 − 0·4	:	+7·2 ↑
Before A : +7·6 − 0·4	:	+7·2 ↑
At A : +7·6 − 0·4 − 7·2	=	0

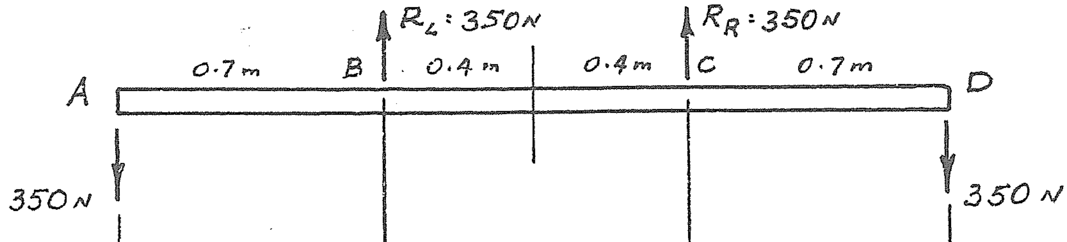
BENDING MOMENT

Left of Section		kNm
At A :	:	0
At B : 100 × 7·2	:	−7·2
At C : (1900 × 7·2) − (1800 × 7·6)	:	0

Rt. of Section		kNm
At C :	:	0
At B : − (1800 × 0·4)	:	−0·72
At A : −(1900 × 0·4) + (100 × 7·6)	:	0

17/9
A see-saw of effective length 2.2 metres is horizontal and supporting two children as shown. If each child has a mass of 35 kg, determine the distribution of shear force and bending moments along the length of the see-saw, if the mass of the plank is negligible.

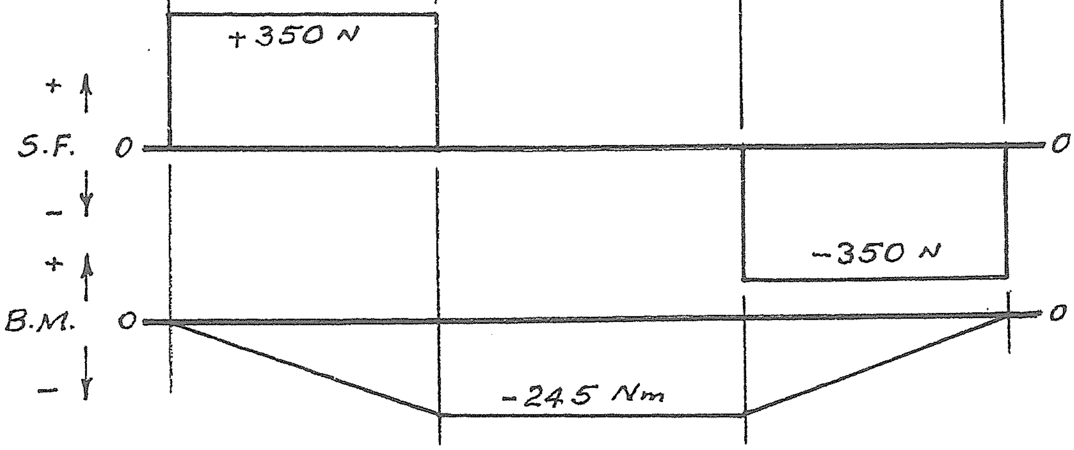




Take moments about B

$350 \times 0.7 + R_R \times 0.8 = 350 \times 1.5$

$\therefore \underline{R_R = 350\ N} \uparrow$ and $\underline{R_L = 700 - 350 = 350\ N} \uparrow$



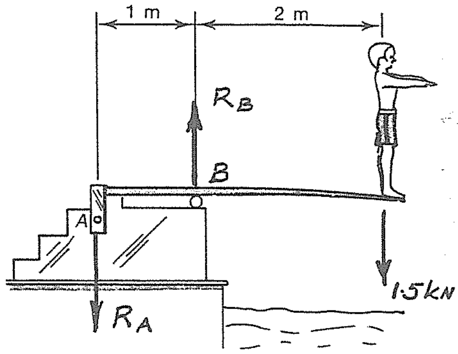
SHEAR FORCE

Left of Section	N	Right of Section	N
At A :	: 0	At A : -350 +350 +350 -350 =	0
From A to B : +350	: +350 ↑	From A to B : +350 +350 -350 =	+350 ↓
From B to C : +350 -350	: 0	From B to C : +350 -350 :	0
From C to D : +350 -350 -350	: -350 ↓	From C to D : -350 :	-350 ↑
At D : +350 -350 -350 +350	: 0	At D :	: 0

BENDING MOMENTS

Left of Section	Nm	Right of Section	Nm
At A :	: 0	At A : +0.7×350 +1.5×350 -2.2×350 :	0
At B : -0.7×350	: -245	At B : +0.8×350 -1.5×350	: -245
At C : -1.5×350 +0.8×350	: -245	At C : -0.7×350	: -245
At D : -2.2×350 +1.5×350 +0.7×350 :	0	At D :	: 0

17/10
A diving board is 3 metres long and is cantilevered as shown in the diagram. By jumping on its free end, a diver exerts a force of 1.5 kN vertically down. Determine the maximum bending moment induced in the diving board due to this load and specify its position from the strap at *A*.
A partial free-body diagram of the board is drawn to assist you.

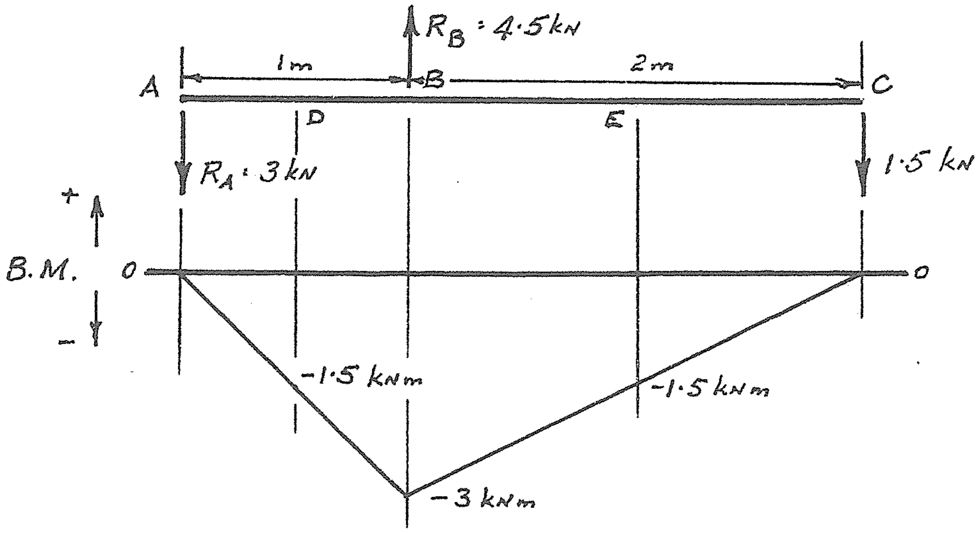


Take moments about B

$$R_A \times 1 = 1{\cdot}5 \times 2$$
$$R_A = 3\text{ kN}\downarrow$$

Resolve vert.

$$R_A + 1{\cdot}5 = R_B$$
$$R_B = 4{\cdot}5\text{ kN}\uparrow$$



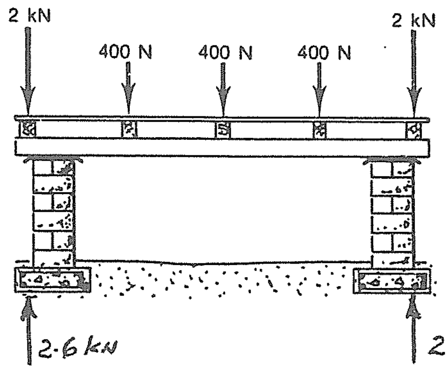
BENDING MOMENT

Left of Section		kNm
At A :		= 0
At D :	$-0{\cdot}5 \times 3$	= $-1{\cdot}5$
At B :	-1×3	= -3
At E :	$-2 \times 3 + 1 \times 4{\cdot}5$	= $-1{\cdot}5$
At C :	$-3 \times 3 + 2 \times 4{\cdot}5$	= 0

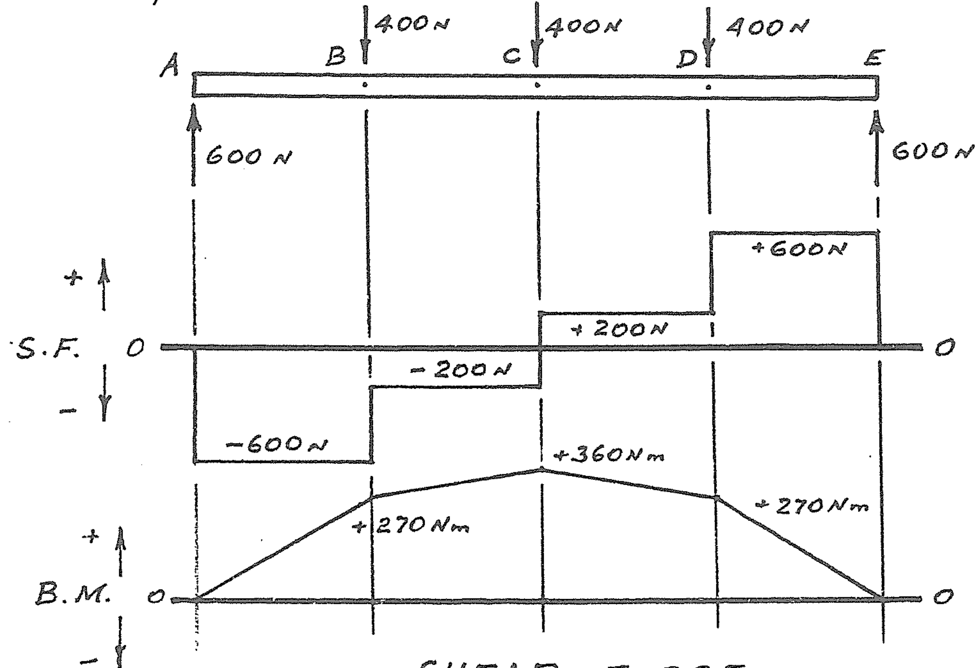
$\therefore$ Max. B.M. = <u>3 kNm Clockwise at B</u>

17/11
A section through a timber floor giving the loads transmitted to the bearer by the floor joists is shown in the diagram. Neglecting the mass of the bearer, determine the distribution of shear force and bending moments along this bearer.

Joist centres 450 mm



Take moments to find pier reactions of 2·6 kN↑ leaving a NETT upward force of 600 N at the outside joists.



SHEAR FORCE

Left of Section — N

At A : = 0

From A to B : = -600 ↓

From B to C : $-600 + 400$ = -200 ↓

At C : $-600 + 400 + (0.5 \times 400)$ = 0

From C to D : $-600 + 400 + 400$ = $+200$ ↑

From D to E : $-600 + 400 + 400 + 400$ = $+600$ ↑

At E : $-600 + 400 + 400 + 400 - 600$ = 0

BENDING MOMENT

Rt. of Section — Nm

At A : $-450 \times 400 - 900 \times 400 - 1350 \times 400 + 1800 \times 600$ = 0

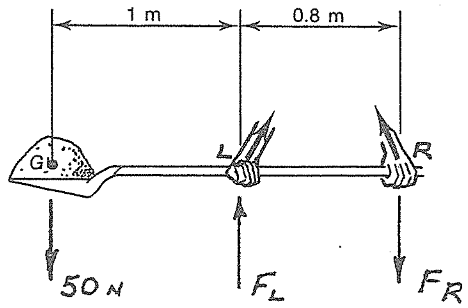
At B : $-450 \times 400 - 900 \times 400 + 1350 \times 600$ = $+270$

At C : $-450 \times 400 + 900 \times 600$ = $+360$

At D : $+450 \times 600$ = $+270$

At E : = 0

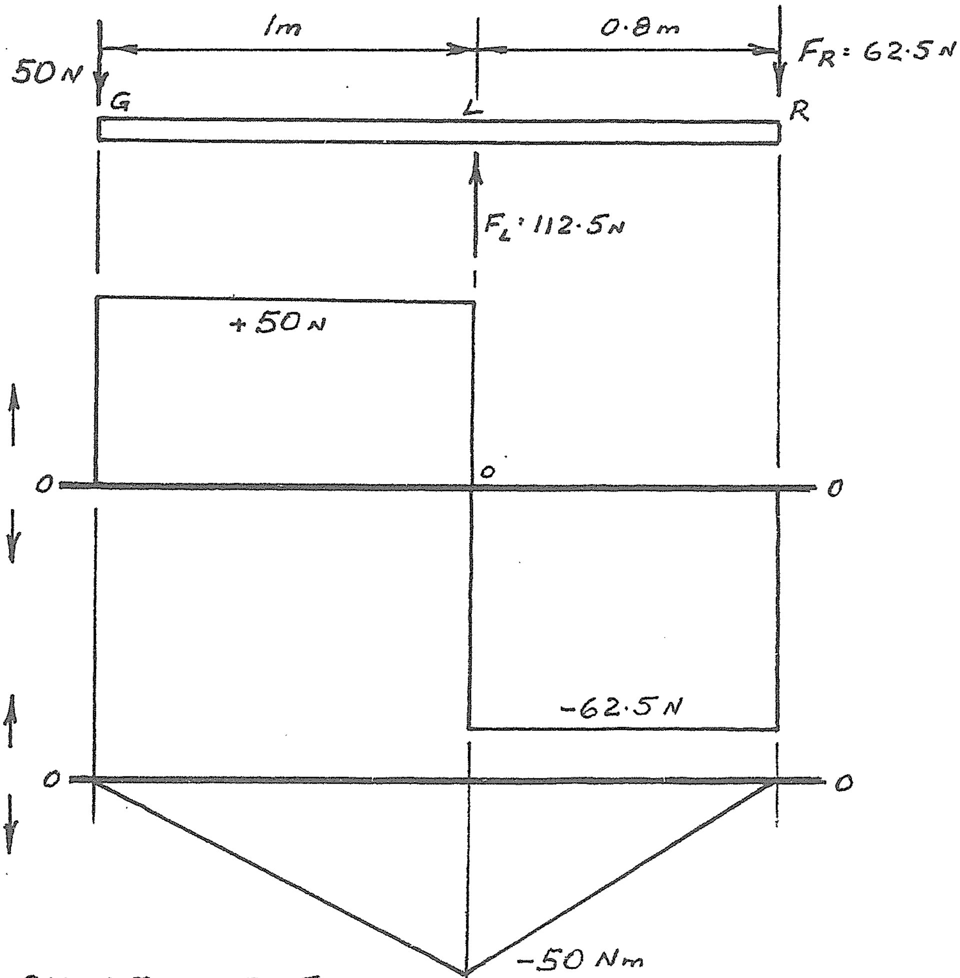
17/12
A long-handled shovel is held horizontally as shown. Given that its 5-kg load of soil has its centre of mass at G, determine the distribution of shear force and bending moments present in the handle due to the load.



Take moments to find:

$F_L = 112.5\,N \uparrow$

and $F_R : 62.5\,N \downarrow$

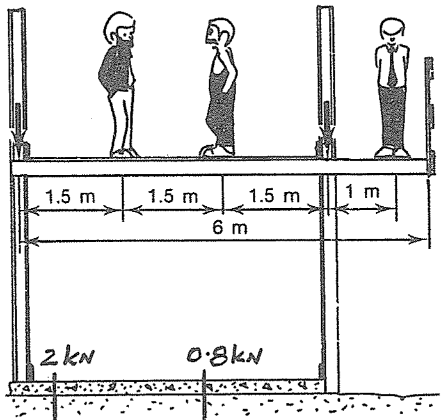


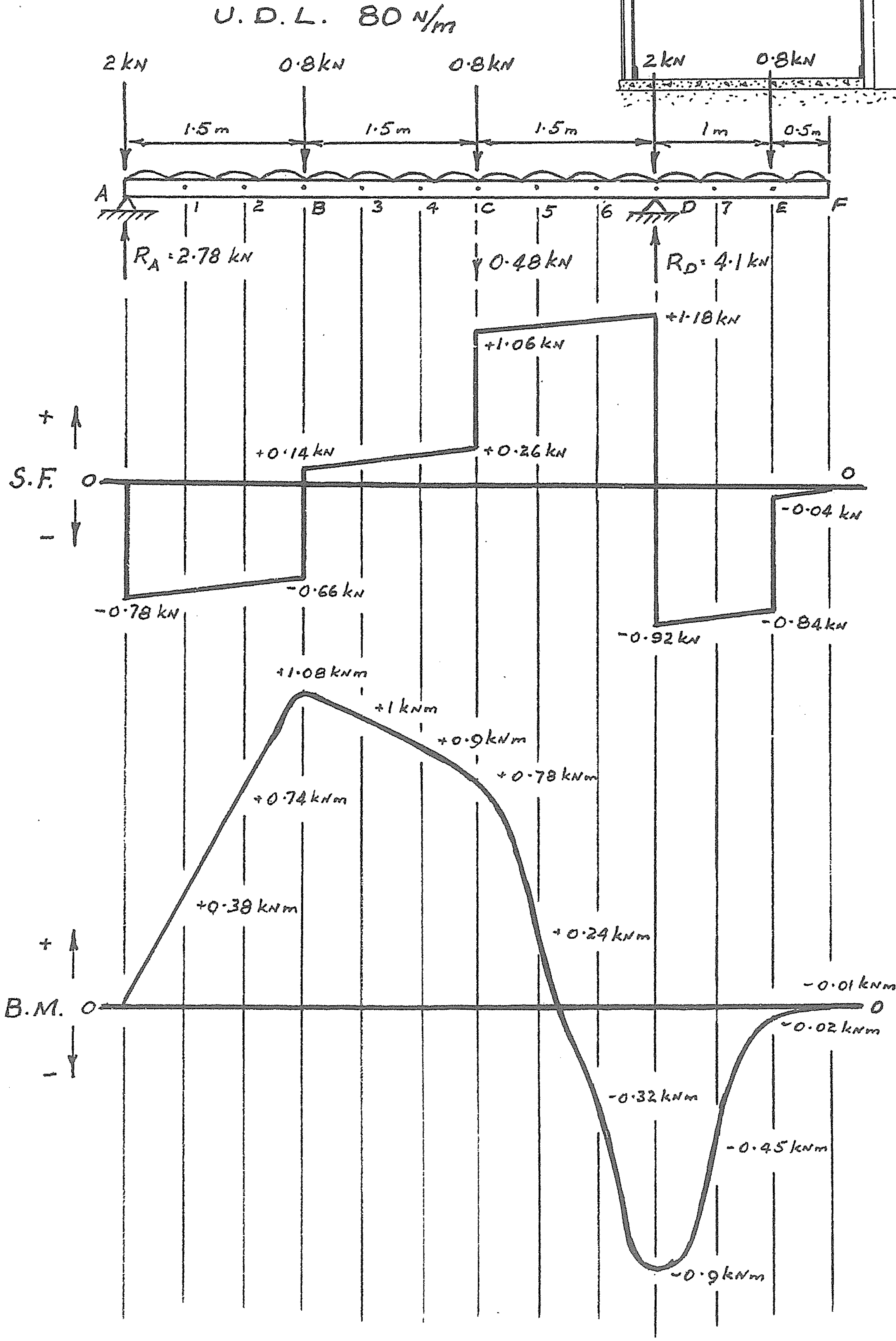
SHEAR FORCE Rt. of Section		N	BENDING MOMENT Left of Section		Nm
At G :	$-50 + 112.5 - 62.5$	: 0	At G :		: 0
From G–L :	$+112.5 - 62.5$	: +50 ↓	At L :	-1×50	: −50
At L :	$+(0.555 \times 112.5) - 62.5$	: 0	At R :	$-1.8 \times 50 + 0.8 \times 112.5$	= 0
From L–R :	-62.5	: −62.5 ↑			
At R :		: 0			

Note: At L, the 112.5 N force is divided in the proportion 1 : 1.8 (50 N and 62.5 N) to give a zero value for S.F. at L.

17/13

Overhanging floor joists are used to cantilever a timber deck at first floor level in a house structure. The weight of a module of the floor, as supported by the one joist shown in the diagram, is known to be 80 N/m. Given that the walls exert vertical loads of 2 kN and that three 80-kg people stand centrally on this joist, determine the distributions of shear force and bending moments along the length of the joist. (Ignore the mass of the deck railing.)





Take moments to find $R_A = 2.78$ kN & $R_D = 4.1$ kN

SHEAR FORCE : Left of Section — kN

At A : $= 0$

Rt. of A : down 2
up 2.78 $= 0.78\uparrow$ $= -0.78\downarrow$

Near B : down $2 + (1.5 \times 0.08) = 2.12$
up 2.78 $\therefore 0.66\uparrow$ $= -0.66\downarrow$

Rt. of B : down $2 + (1.5 \times 0.08) + 0.8 = 2.92$
up 2.78 $\therefore 0.14\downarrow$ $= +0.14\uparrow$

Near C : down $2 + 0.8 + (3 \times 0.08) = 3.04$
up 2.78 $\therefore 0.26\downarrow$ $= +0.26\uparrow$

Rt. of C : down $2 + 0.8 + 0.8 + (3 \times 0.08) = 3.84$
up 2.78 $\therefore 1.06\downarrow$ $= +1.06\uparrow$

Near D : down $2 + 0.8 + 0.8 + (4.5 \times 0.08) = 3.96$
up 2.78 $\therefore 1.18\downarrow$ $= +1.18\uparrow$

Rt. of D : down $2 + 0.8 + 0.8 + 2 + (4.5 \times 0.08) = 5.96$
up $2.78 + 4.1 = 6.88$ $\therefore 0.92\uparrow$ $= -0.92\downarrow$

Near E : down $2 + 0.8 + 0.8 + 2 + (5.5 \times 0.08) = 6.04$
up $2.78 + 4.1 = 6.88$ $\therefore 0.84\uparrow$ $= -0.84\downarrow$

Rt. of E : down $2 + 0.8 + 0.8 + 2 + (5.5 \times 0.08) + 0.8 = 6.84$
up $2.78 + 4.1 = 6.88$ $\therefore 0.04\uparrow$ $= -0.04\downarrow$

At F : down $2 + 0.8 + 0.8 + 2 + (5.5 \times 0.08) + 0.8 + (0.5 \times 0.08) = 6.88$ $= 0$
up $2.78 + 4.1 = 6.88$

17/13 (cont.)

BENDING MOMENTS
Left of Section

kNm

At A : = 0

At 1 : $0.5\times2 + 0.25\times0.5\times0.08 = 1.01$

$0.5\times2.78 = 1.39$ = 0.38 +0.38

At 2 : $1\times2 + 0.5\times1\times0.08 = 2.04$

$1\times2.78 = 2.78$ = 0.74 +0.74

At B : $1.5\times2 + 0.75\times1.5\times0.08 = 3.09$

$1.5\times2.78 = 4.17$ = 1.08 +1.08

At 3 : $2\times2 + 0.5\times0.8 + 1\times2\times0.08 = 4.56$

$2\times2.78 = 5.56$ = 1.00 +1.00

At 4 : $2.5\times2 + 1\times0.8 + 1.25\times2.5\times0.08 = 6.05$

$2.5\times2.78 = 6.95$ = 0.9 +0.9

At C : $3\times2 + 1.5\times0.8 + 1.5\times3\times0.08 = 7.56$

$3\times2.78 = 8.34$ = 0.78 +0.78

At 5 : $3.5\times2 + 2\times0.8 + 0.5\times0.8 + 0.5\times3.5\times3.5\times0.08 = 9.49$

$3.5\times2.78 = 9.73$ = 0.24 +0.24

At 6 : $4\times2 + 2.5\times0.8 + 1\times0.8 + 0.5\times4\times4\times0.08 = 11.44$

$4\times2.78 = 11.12$ = 0.32 −0.32

At D : $4.5\times2 + 3\times0.8 + 1.5\times0.8 + 0.5\times4.5\times4.5\times0.08 = 13.41$

$4.5\times2.78 = 12.51$ = 0.9 −0.9

At 7 : $5\times2 + 3.5\times0.8 + 2\times0.8 + 0.5\times2$

$+ 0.5\times5\times5\times0.08 = 16.4$

$5\times2.78 + 0.5\times4.1 = 15.95$ = 0.45 −0.45

At E : $5.5\times2 + 4\times0.8 + 2.5\times0.8 + 1\times2$

$+ 0.5\times5.5\times5.5\times0.08 = 19.41$

$5.5\times2.78 + 1\times4.1 = 19.39$ = 0.02 −0.02

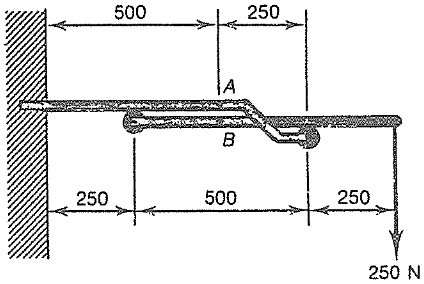
At F : $6\times2 + 4.5\times0.8 + 3\times0.8 + 1.5\times2$

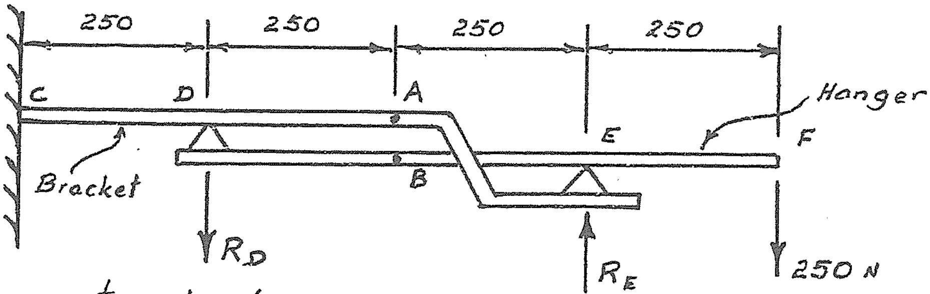
$+ 0.5\times0.8 + 0.5\times6\times6\times0.08 = 22.84$

$6\times2.78 + 1.5\times4.1 = 22.83$ = 0.01 −0.01*

* The error of 0.01 kNm instead of zero at F is the result of using approximate reaction values of 2.78 kN and 4.1 kN

17/14
An adjustable hanger is positioned as shown. Determine the shear force and bending moment at the points *A* and *B* when this hanger supports a load of 250 N. Since the mass of the hanger is very small, it may be neglected in your calculation.





Take moments about C

$$R_D \times 250 + 250 \times 1000 = R_E \times 750$$

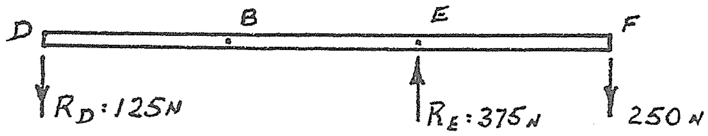
$$3R_E - R_D = 1000$$

Forces on the hanger

$$R_E = R_D + 250$$

Subst. $3(R_D + 250) - R_D = 1000$ ∴ $R_D = 125\,N$ ↓ & $R_E = 375\,N$ ↑

HANGER



SHEAR FORCE — Rt. of Section — BENDING MOMENT

At B: 250 ↓

375 ↑ = 125 N ↑

∴ S.F. at B = +125 N ↓

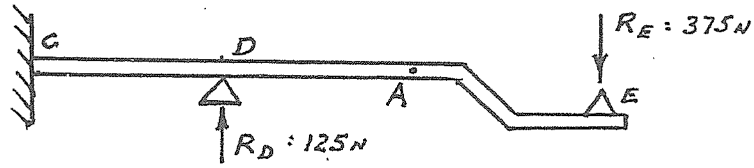
At B: $250 \times 0.5 = 125$ Nm

$375 \times 0.25 = 93.75$ Nm

$= 31.25$ Nm

∴ B.M. at B = −31.25 Nm

BRACKET



SHEAR FORCE — Rt. of Section — BENDING MOMENT

At A: 375 N ↓

0 = 375 N ↓

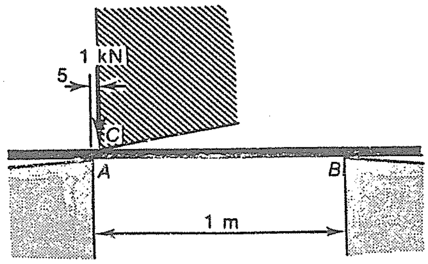
∴ S.F. at A = −375 N ↑

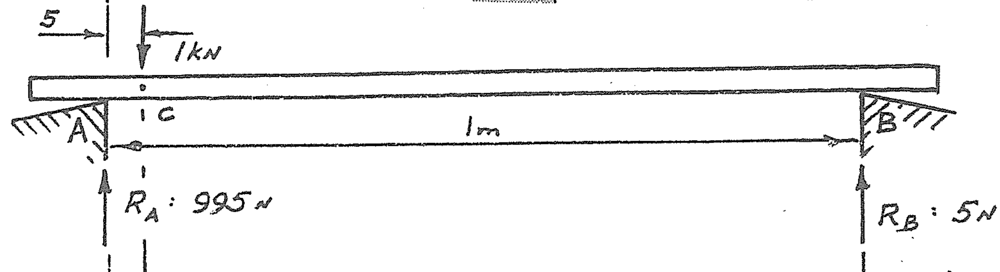
At A: 375×0.25

$= 93.75$ Nm

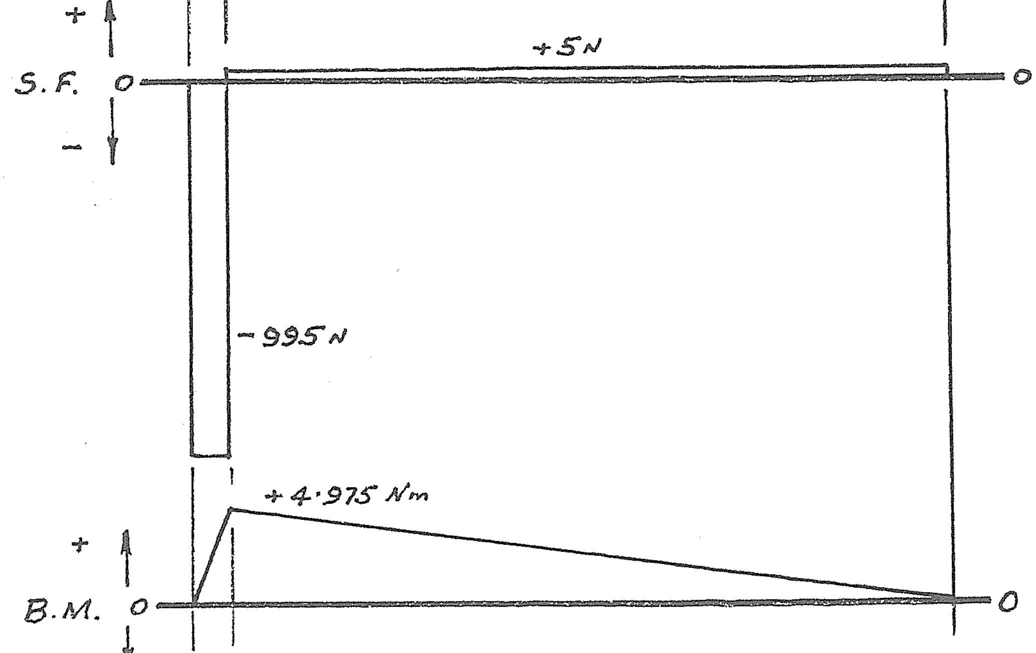
∴ B.M. at A = −93.75 Nm

17/15
A metal plate is sheared by a blade which exerts a force of 1 kN on it at *C*. The plate rests on fixed supports at *A* and *B* and may be treated as a simply supported beam. Sketch the shear force and bending moment diagrams for the plate.





Take moments to find R_A : 995 N ↑ and R_B : 5 N ↑



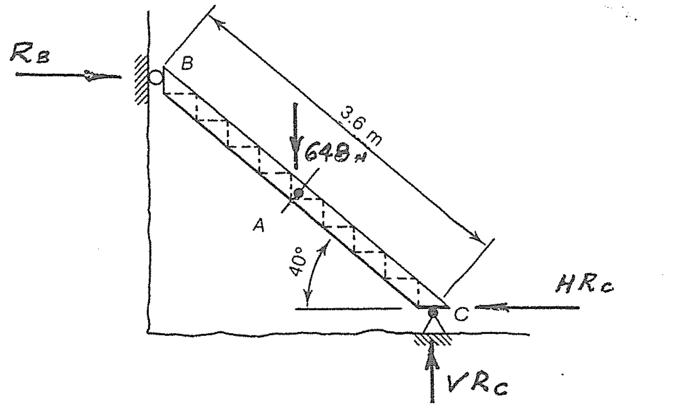
SHEAR FORCE

Left of Section	N	Rt. of Section	N
Left of A:	: 0	Left of A: 995↑ 5↑ 1000↓	: 0
Rt. of A: 995↑	: −995↓	Rt. of A: 1000↓ 5↑	: −995↑
Left of C: 995↑	: −995↓	Left of C: 1000↓ 5↑	: −995
Rt. of C: 995↑ 1000↓	: +5↑	Rt. of C: 5↑	: +5↓
Left of B: 995↑ 1000↓	: +5↑	Left of B: 5↑	: +5
Rt. of B: 995↑ 5↑ 1000↓	: 0	Rt of B:	: 0

BENDING MOMENT

Left of Section	Nm	Rt. of Section	Nm
At A:	: 0	At A: 1000×5 5×1000	= 0
At C: 995×5 : 4·975	: +4·975	At C: 5×995 = 4·975	= +4·975
At B: 995×1000 1000×995	: 0	At B:	: 0

17/16
The steel channel shown forms one side of a short flight of stairs. If the channel has a mass of 18 kg per metre, determine the axial force, shear force and bending moment at its centre section *A-A* due to its own weight-force. Assume that the beam has its centre of mass on the section *A-A*, the joint at *B* is a roller and the joint at *C* is a hinge.

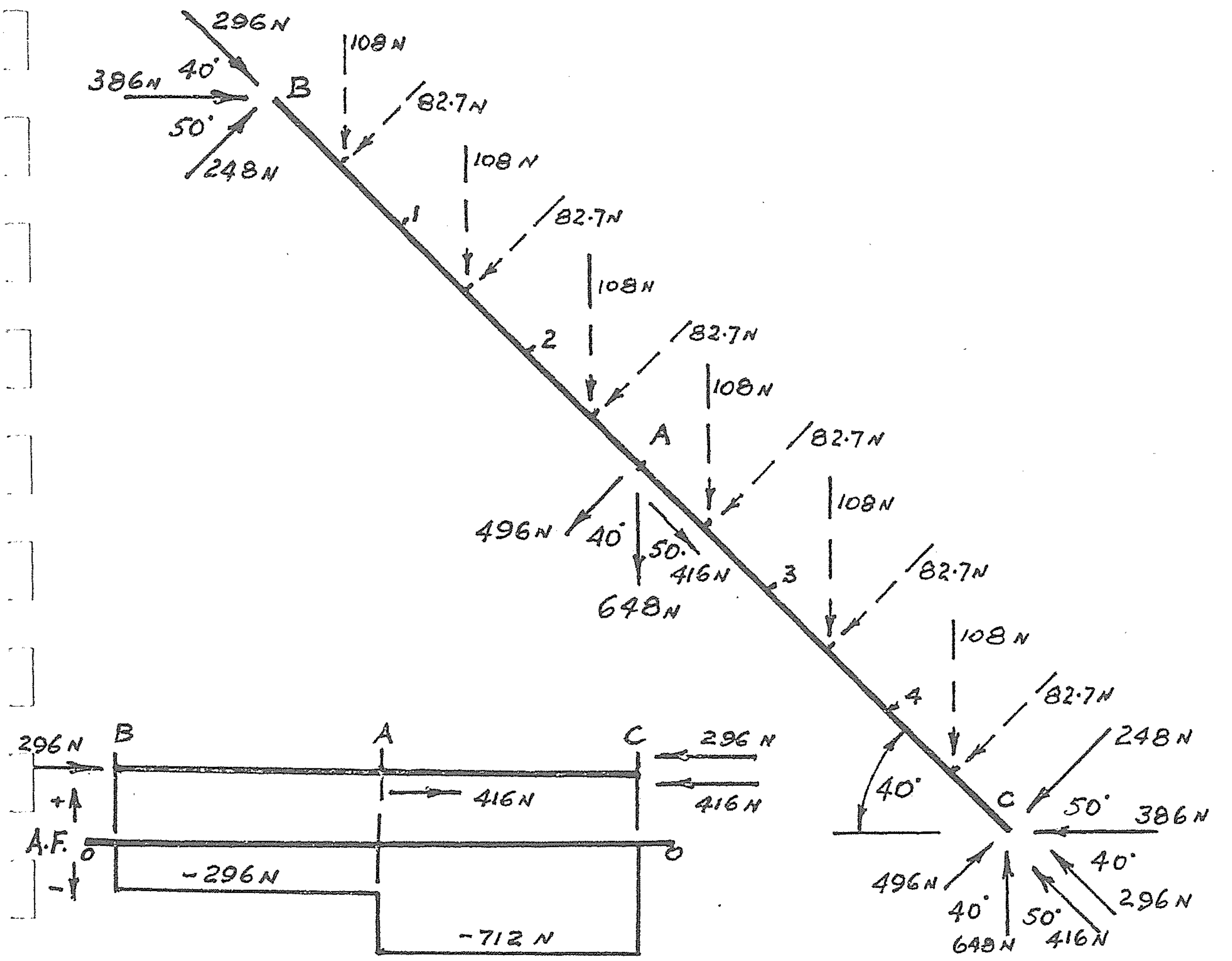


Take moments about C:

$$3.6 \sin 40^\circ \times R_B = 1.8 \cos 40^\circ \times 648$$

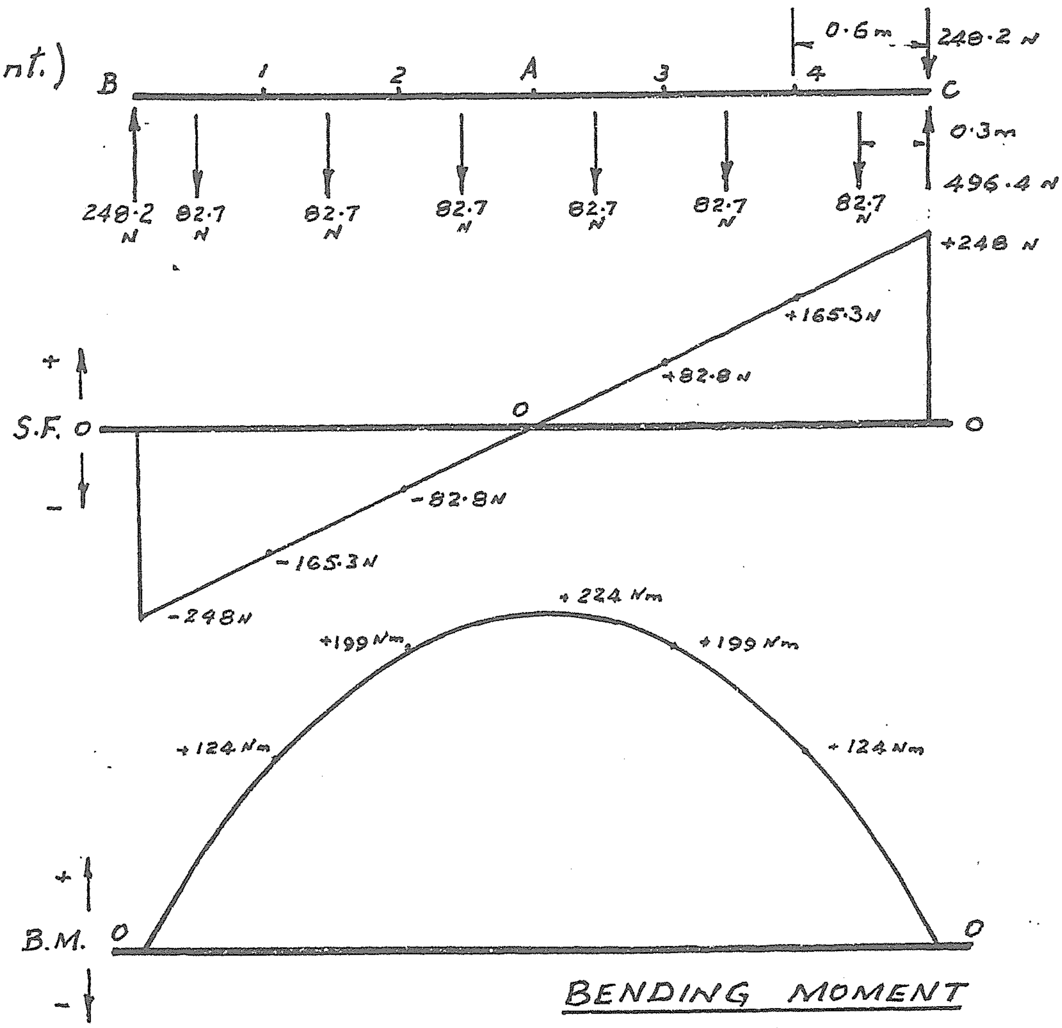
$$R_B = 386\ N$$

Resolve vert. & horiz. $HR_c = 386N$, $VR_c = 648\ N$



Left of Section	AXIAL FORCE N	Rt. of Section	N
Left of B:	$= 0$	Left of B: $\overrightarrow{296} + \overleftarrow{416} = \overrightarrow{712}$	
		$\overleftarrow{296} + \overleftarrow{416} = \overleftarrow{712}$	$= 0$
Rt. of B: $\overrightarrow{296}$	$= -\overleftarrow{296}$	Rt. of B: $\overrightarrow{416}$	
		$\overleftarrow{296} + \overleftarrow{416} = \overleftarrow{296}$	$= -\overrightarrow{296}$
Left of A: $\overrightarrow{296}$	$= -\overleftarrow{296}$	Left of A: $\overrightarrow{416}$	
		$\overleftarrow{296} + \overleftarrow{416} = \overleftarrow{296}$	$= -\overrightarrow{296}$
Rt. of A: $\overrightarrow{296} + \overrightarrow{416} = \overrightarrow{712}$	$= -\overleftarrow{712}$	Rt. of A: $\overleftarrow{296} + \overleftarrow{416} = \overleftarrow{712}$	$= -\overrightarrow{712}$
Left of C: $\overrightarrow{296} + \overrightarrow{416} = \overrightarrow{712}$	$= -\overleftarrow{712}$	Left of C: $\overleftarrow{296} + \overleftarrow{416} = \overleftarrow{712}$	$= -\overrightarrow{712}$
Rt. of C: $\overrightarrow{296} + \overrightarrow{416} = \overrightarrow{712}$		Rt. of C:	$= 0$
$\overleftarrow{296} + \overleftarrow{416} = \overleftarrow{712}$	$= 0$		

17/16 (cont.)



SHEAR FORCE

Left of Section

	N
Left of B:	: 0
Rt. of B: 248·2↑	: −248·2↓
At 1: 248·2↑ 82·7↓ : 165·5↑	: −165·5↓
At 2: 248·2↑ 2×82·7↓ : 82·8↑	: −82·8↓
At A: 248·2↑ 3×82·7↓	: 0
At 3: 248·2↑ 4×82·7↓ : 82·8↓	: +82·8↑
At 4: 248·2↑ 5×82·7↓ : 165·3↓	: +165·3↑
Left of C: 248·2↑ 6×82·7↓ : 248↓	: +248↑
At C: 248·2↑ 648 sin 50↑ : 745↑ 386 sin 40↓ 6×82·7↓ : 745↓	= 0

BENDING MOMENT

Rt. of Section

	Nm
At C:	0
At 4: 0·3×82·7 + 0·6×248·2 − 0·6×496·4	= +124
At 3: 0·3×82·7 + 0·9×82·7 + 1·2×248·2 −1·2×496·4	= +199
At A: 0·3×82·7 + 0·9×82·7 + 1·5×82·7 +1·8×248·2 −1·8×496·4	= +224
At 2: 0·3×82·7 + 0·9×82·7 + 1·5×82·7 +2·1×82·7 + 2·4×248·2 − 2·4×496·4	= +199
At 1: 0·3×82·7 + 0·9×82·7 + 1·5×82·7 +2·1×82·7 + 2·7×82·7 + 3×248·2 −3×496·4	= 124
At B: 0·3×82·7 + 0·9×82·7 + 1·5×82·7 +2·1×82·7 + 2·7×82·7 + 3·3×82·7 +3·6×248·2 − 3·6×496·4	= 0

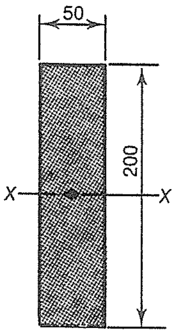
At Section A-A

Axial Force = 712 N (C)

Shear Force = 0

Bending Moment = +224 Nm

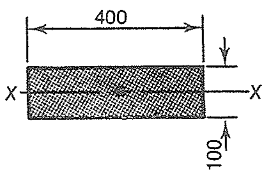
17/17
Calculate the second moments of area about the axis X-X of the sections shown in the diagram.



$$I_{xx} = \frac{bd^3}{12} \qquad \text{Where } b = 50 \;\&\; d = 200$$

$$= \frac{50 \times 200^3}{12}$$

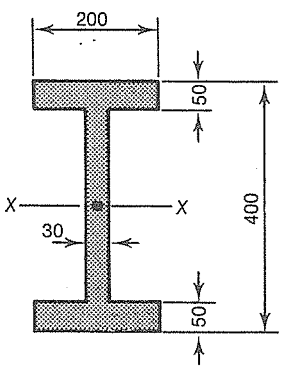
$$= \underline{33 \times 10^6 \text{ mm}^4}$$



$$I_{xx} = \frac{bd^3}{12} \qquad \text{Where } b = 400 \;\&\; d = 100$$

$$= \frac{400 \times 100^3}{12}$$

$$= \underline{33 \times 10^6 \text{ mm}^4}$$

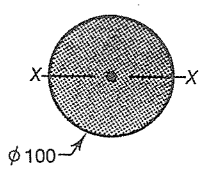


$$I_{xx} = \frac{BD^3 - bd^3}{12} \qquad \text{Where } B = 200,\; D = 400,\; b = 170 \;\&\; d = 300$$

$$= \frac{200 \times 400^3 - 170 \times 300^3}{12}$$

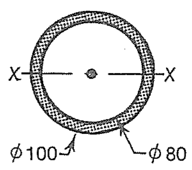
$$= 10.67 \times 10^8 - 3.83 \times 10^8$$

$$= \underline{6.84 \times 10^8 \text{ mm}^4}$$



$$I_{xx} = \frac{\pi D^4}{64} \qquad \text{Where } D = 100$$

$$= \underline{4.9 \times 10^6 \text{ mm}^4}$$



$$I_{xx} = \frac{\pi (D^4 - d^4)}{64} \qquad \text{Where } D = 100 \;\&\; d = 80$$

$$= \frac{\pi (1 \times 10^8 - 0.4096 \times 10^8)}{64}$$

$$= \underline{2.89 \times 10^6 \text{ mm}^4}$$

17/18

A length of 530 UB 94 is stacked horizontally across two other beams which are 4 metres apart. Considering the 4-metre length as a simply supported beam, determine

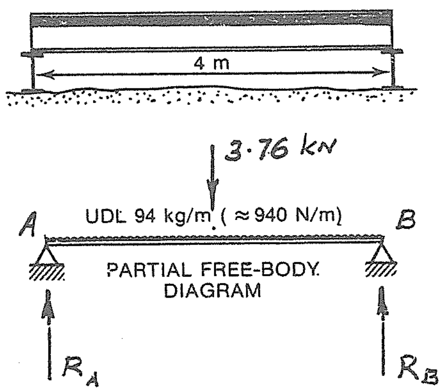
(i) the bending moment diagram for the beam;

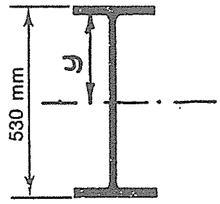
(ii) the maximum bending moment present and its position;

(iii) the maximum stress in the upper and lower flanges, if I_{XX} for this section is $554 \times 10^6\ mm^4$;

(iv) the radius of curvature at the midpoint of the span.

530 UB 94 has a mass of 94 kg per metre and a cross-section as shown in the diagram.

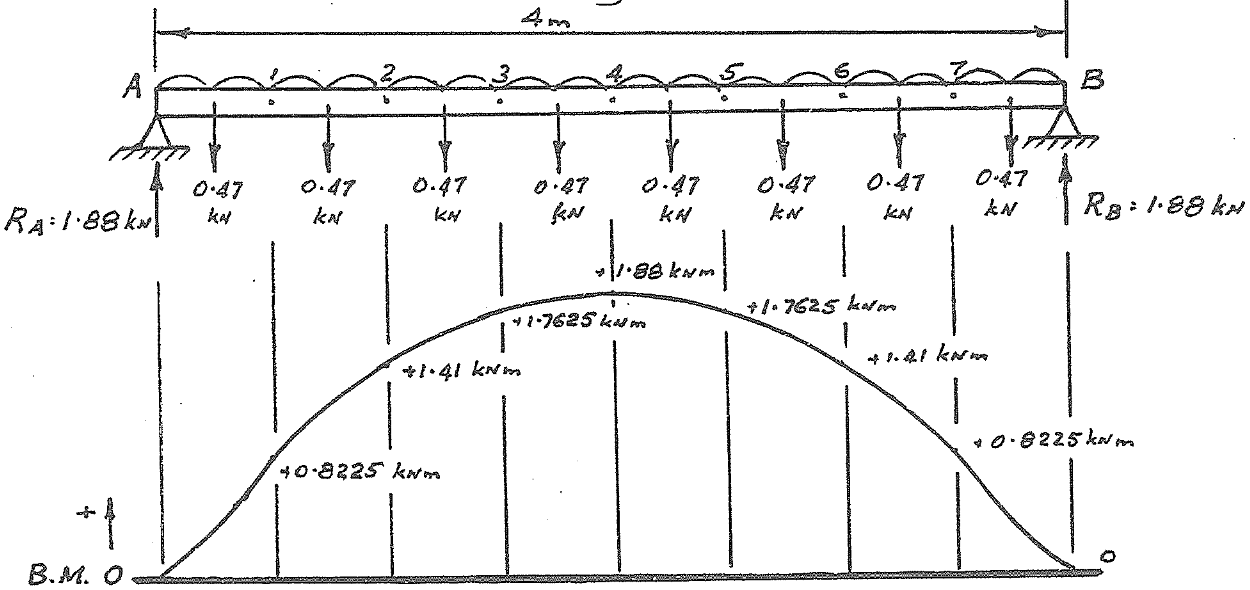




$E_{st.}$: 200GPa

Take moments and resolve to give values of 1·88 kN ↑ for R_A & R_B

(i)



BENDING MOMENTS — Left of Section kNm

At A : = 0

At 1 : $0{\cdot}25 \times 0{\cdot}47 - 0{\cdot}5 \times 1{\cdot}88$ = 0·8225 : +0·8225

At 2 : $0{\cdot}47(0{\cdot}25 + 0{\cdot}75) - 1 \times 1{\cdot}88$ = 1·41 : + 1·41

At 3 : $0{\cdot}47(0{\cdot}25 + 0{\cdot}75 + 1{\cdot}25) - 1{\cdot}5 \times 1{\cdot}88$: 1·7625 = + 1·7625

At 4 : $0{\cdot}47(0{\cdot}25 + 0{\cdot}75 + 1{\cdot}25 + 1{\cdot}75) - 2 \times 1{\cdot}88$: 1·88 : + 1·88

At 5 : $0{\cdot}47(0{\cdot}25 + 0{\cdot}75 + 1{\cdot}25 + 1{\cdot}75 + 2{\cdot}25) - 2{\cdot}5 \times 1{\cdot}88$ = 1·7625 : + 1·7625

At 6 : $0{\cdot}47(0{\cdot}25 + 0{\cdot}75 + 1{\cdot}25 + 1{\cdot}75 + 2{\cdot}25 + 2{\cdot}75) - 3 \times 1{\cdot}88$

= 1·41 : + 1·41

At 7 : $0{\cdot}47(0{\cdot}25 + 0{\cdot}75 + 1{\cdot}25 + 1{\cdot}75 + 2{\cdot}25 + 2{\cdot}75 + 3{\cdot}25)$

$- 3{\cdot}5 \times 1{\cdot}88$ = 0·8225 : +0·8225

At B : $0{\cdot}47(0{\cdot}25 + 0{\cdot}75 + 1{\cdot}25 + 1{\cdot}75 + 2{\cdot}25 + 2{\cdot}75 + 3{\cdot}25 + 3{\cdot}75)$

$- 4 \times 1{\cdot}88$: 0 = 0

(ii) Max. B.M. = + 1·88 kNm at midpoint

17/18 (cont.)

(iii) $$\frac{M}{I} = \frac{f}{y}$$

Where $M = 1 \cdot 88\ kNm = 1 \cdot 88 \times 10^3\ Nm$

$I = 554 \times 10^6\ mm^4$

$= \dfrac{554 \times 10^6}{10^{12}}\ m^4$

$(1 m^4 = 10^{12} mm^4)$

f = Stress - Pa

$y = 265\ mm$

$= 0 \cdot 265\ m$

$$f = \frac{My}{I}$$

$$= \frac{1 \cdot 88 \times 10^3 \times 0 \cdot 265 \times 10^{12}}{554 \times 10^6}$$

$$= 0 \cdot 899 \times 10^6\ Pa$$

or $\underline{0 \cdot 899\ MPa}$

(iv) $$\frac{M}{I} = \frac{E}{r}$$

Where $M = 1 \cdot 88 \times 10^3\ Nm$

$I = \dfrac{554 \times 10^6}{10^{12}}\ m^4$

$E = 200 \times 10^9\ Pa$

r = radius in m

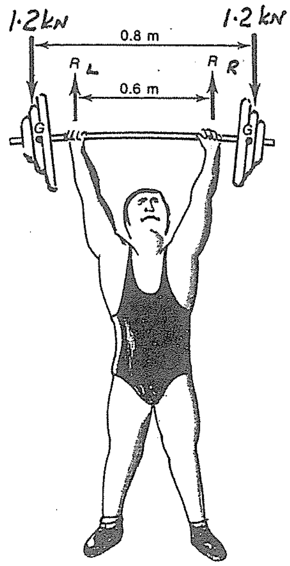
$$r = \frac{EI}{M}$$

$$= \frac{200 \times 10^9 \times 554 \times 10^6}{10^{12} \times 1 \cdot 88 \times 10^3}$$

$$= 58936\ m$$

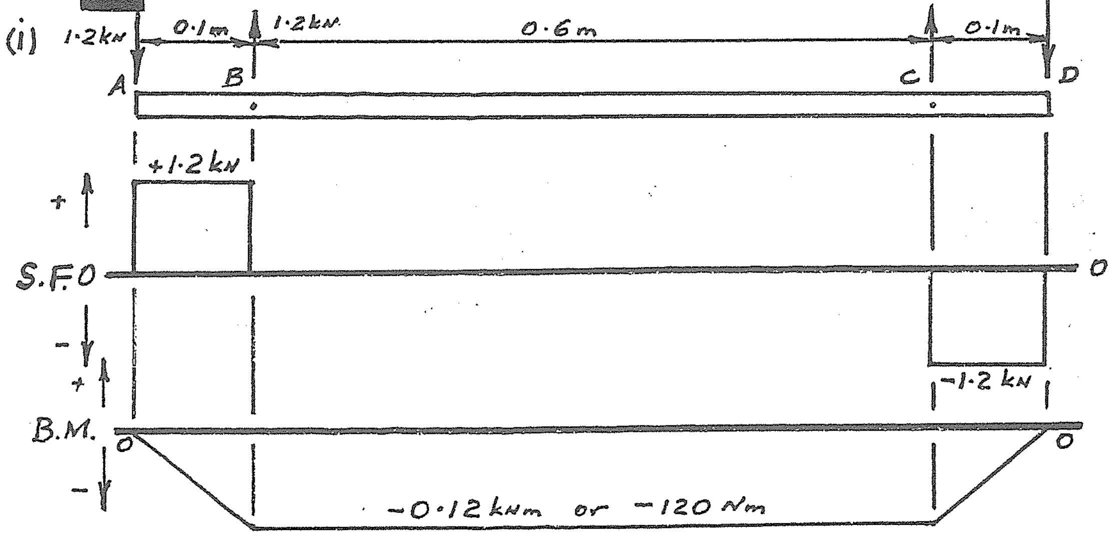
$$= \underline{58 \cdot 9\ km}$$

17/19
A weightlifter who has just completed his lift holds the bar and weights directly above his head. Neglecting the mass of the bar, determine the distribution of shear force and bending moment along the bar, given that the 120-kg load at each end of the bar has its centre of mass at G. What is the radius of curvature of the bar at the weightlifter's hands if the bar has a diameter of 36 mm and E for the bar material is 200 GPa?



$R_L = R_R = 1{\cdot}2\,kN \uparrow$

(i)



SHEAR FORCE

Left of Section — kN

At A : $= 0$

Rt. of A : $1{\cdot}2\downarrow$ $= +1{\cdot}2\uparrow$

Left of B: $1{\cdot}2\downarrow$ $= +1{\cdot}2\uparrow$

Rt. of B : $1{\cdot}2\downarrow\ 1{\cdot}2\uparrow$ $= 0$

Left of C : $1{\cdot}2\downarrow\ 1{\cdot}2\uparrow$ $= 0$

Rt. of C : $1{\cdot}2\downarrow\ 1{\cdot}2\uparrow\ 1{\cdot}2\uparrow = 1{\cdot}2\uparrow = -1{\cdot}2\downarrow$

Left of D : $1{\cdot}2\downarrow\ 1{\cdot}2\uparrow\ 1{\cdot}2\uparrow = 1{\cdot}2\uparrow = -1{\cdot}2\downarrow$

At D : $1{\cdot}2\downarrow\ 1{\cdot}2\uparrow\ 1{\cdot}2\uparrow\ 1{\cdot}2\downarrow = 0$

BENDING MOMENT

Rt. of Section — kNm

At A : $0{\cdot}1\times1{\cdot}2 + 0{\cdot}7\times1{\cdot}2 - 0{\cdot}8\times1{\cdot}2 = 0$

At B : $0{\cdot}6\times1{\cdot}2 - 0{\cdot}7\times1{\cdot}2 = 0{\cdot}12 = -0{\cdot}12$

At C : $0{\cdot}1\times1{\cdot}2 = 0{\cdot}12 = -0{\cdot}12$

At D : $= 0$

(ii) $\dfrac{M}{I} = \dfrac{E}{r}$

Where $M = 120$ Nm

$E = 200\times10^9$ Pa

$I = \dfrac{\pi D^4}{64} = \dfrac{\pi\times0{\cdot}036^4}{64}\ m^4$

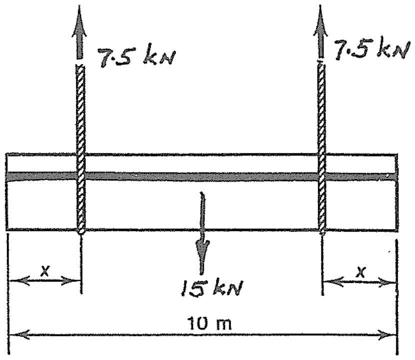
$r = \dfrac{EI}{M}$

$= \dfrac{200\times10^9\times\pi\times0{\cdot}036^4}{120\times64}$

$= \underline{137{\cdot}4\ m}$

r = rad. in m.

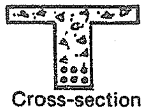
17/20
A prestressed concrete deck beam for a bridge is being lifted into position as shown. It has not been designed to accept a negative bending moment greater than 2 kN m. If the beam has a mass of 150 kg/m, at what maximum distance x from each end of the beam can the slings be placed?



Beam mass = 1500 kg

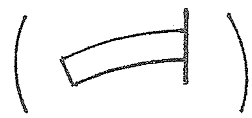
Beam weight : 15 kN

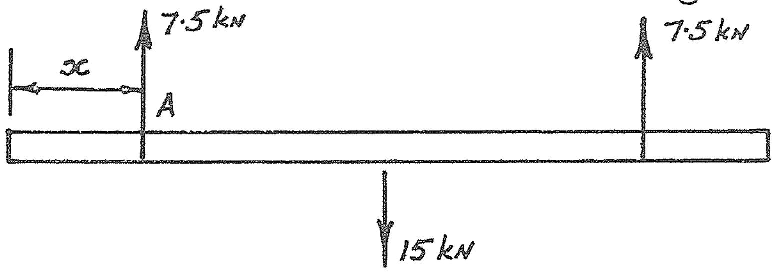
and Sling tensions will be 7.5 kN ea.



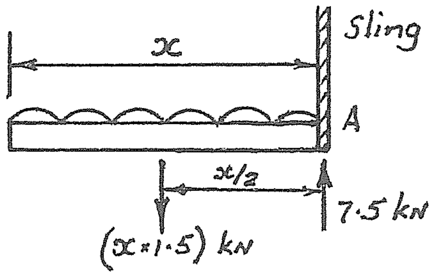
Cross-section of beam

Max. Negative B.M. will occur at the slings when the beam tends to "hog"





Consider the left hand side of the sling at A



At A) B.M. $= -x \times 1.5 \times x/2$ (Left of section)

Max. Allowable B.M. : 2 kNm

$$x \times 1.5 \times x/2 = 2$$

$$\frac{1.5x^2}{2} = 2$$

$$1.5x^2 = 4$$

$$\underline{x = 1.63\ m}$$

17/21

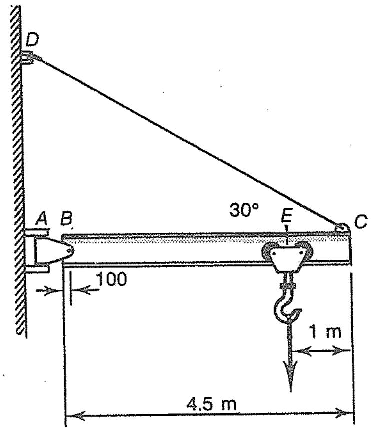
A small jib crane in a foundry is designed so that its beam can swing in a horizontal arc about the pivot at A. For the position shown, determine

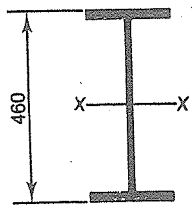
(i) the reaction at the pin B, in terms of its horizontal and vertical components, and the tension in the cable CD, given that the total load on the crane is 1 tonne and the beam has a mass of 82 kg/m.

(ii) the shear force and bending moment present in the beam at the section E (midway between the trolley wheels of the monorail hoist); and

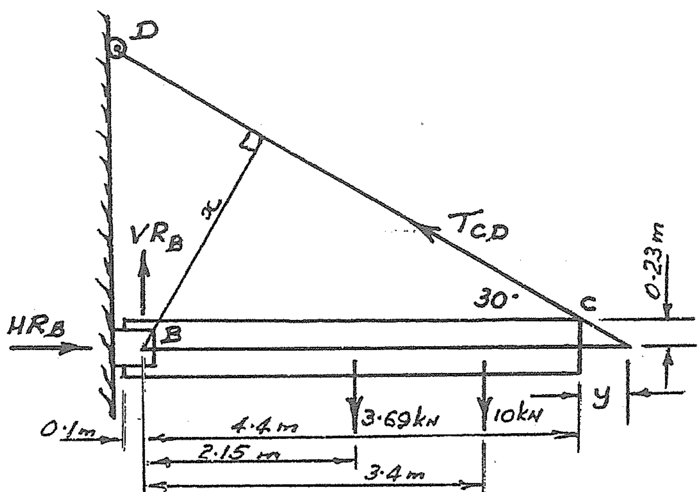
(iii) the maximum stress in the upper and lower flanges at the section E.

The beam is a 460 UB 82; its cross-section is as shown in the diagram and I_{XX} for this section is 371×10^6 mm^4.





Section of
460 UB 82



$$\tan 30^\circ : \frac{0.23}{y}$$

$$\therefore y : 0.398 \text{ m}$$

$$x : 4.798 \sin 30^\circ$$

$$: 2.399 \text{ m}$$

(i) Take moments about B

$$T_{CD} \times 2.399 : 3.69 \times 2.15 + 10 \times 3.4$$

$$T_{CD} : \underline{17.48 \text{ kN}}$$

Resolve vert.

$$VR_B + 17.48 \sin 30 : 10 + 3.69$$

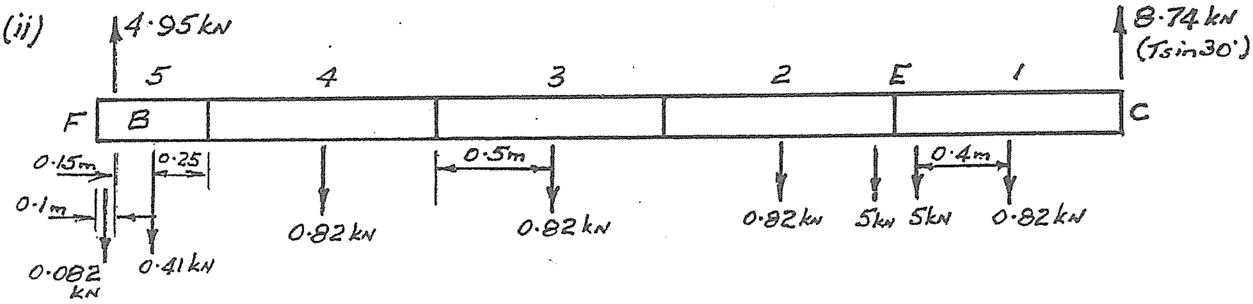
$$VR_B : \underline{4.95 \text{ kN}} \uparrow$$

Resolve horiz.

$$HR_B = 17.48 \cos 30^\circ$$

$$: \underline{15.14 \text{ kN}} \rightarrow$$

(ii)



Assume the wheel centres are 0.2 m apart and that the 10 kN load is equally divided, 5 kN at each wheel.

17/21 (ii) (cont.)

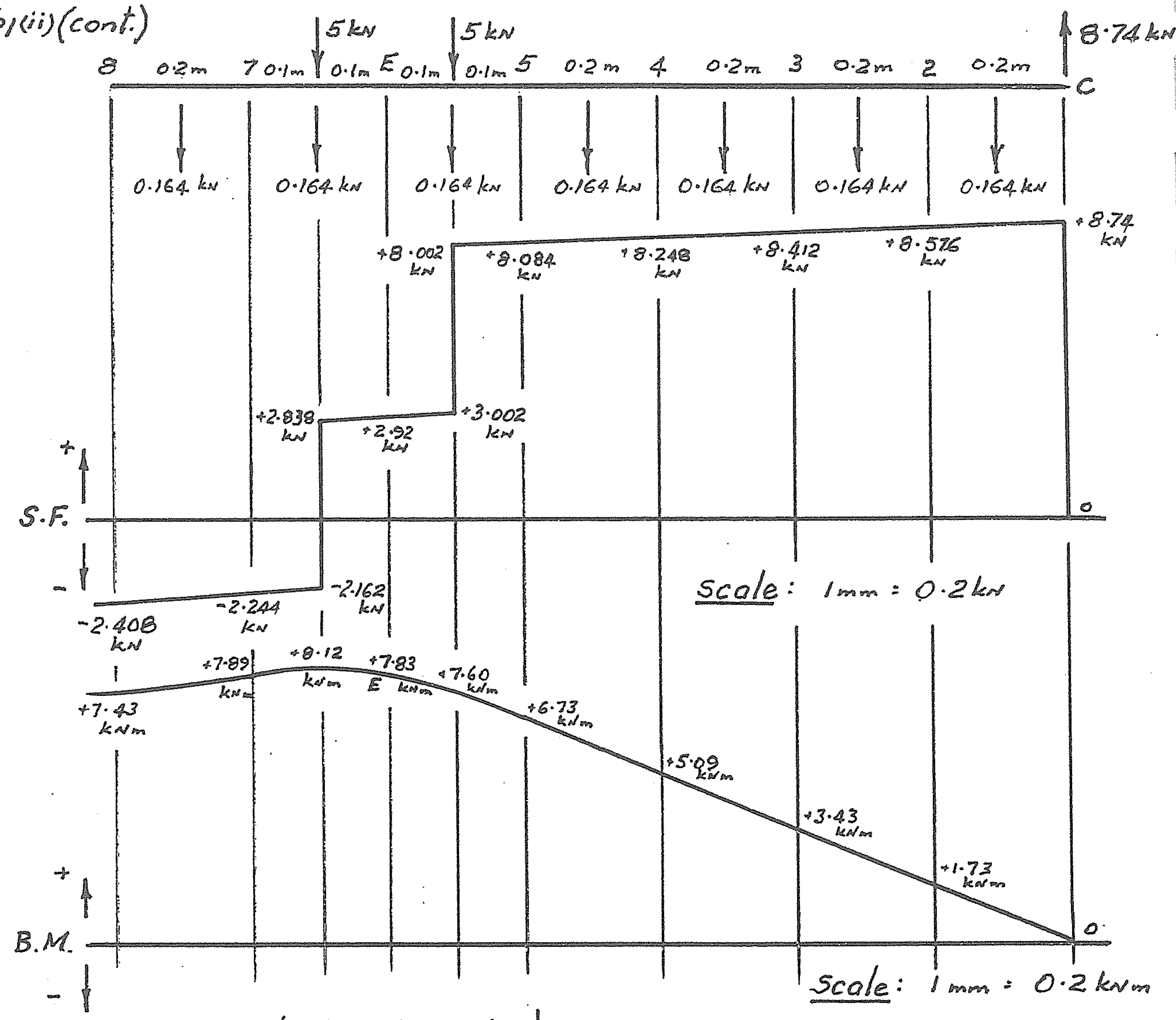
For S.F. at E
Rt. of Section

	kN
Rt. of C :	: 0
At C : $8.74\uparrow$	: $+8.74\downarrow$
At 2 : $8.74\uparrow,\ 0.164\downarrow = 8.576\uparrow$	: $+8.576\downarrow$
At 3 : $8.74\uparrow,\ 2\times0.164\downarrow = 8.412\uparrow$	: $+8.412\downarrow$
At 4 : $8.74\uparrow,\ 3\times0.164\downarrow = 8.248\uparrow$	: $+8.248\downarrow$
At 5 : $8.74\uparrow,\ 4\times0.164\downarrow = 8.084\uparrow$	: $+8.084\downarrow$
Rt. of 5kN : $8.74\uparrow,\ 4\times0.164\downarrow,\ 0.082\downarrow = 8.002\uparrow$	: $+8.002\downarrow$
Left of 5kN : $8.74\uparrow,\ 4\times0.164\downarrow,\ 0.082\downarrow,\ 5 = 3.002\uparrow$	: $+3.002\downarrow$
At E : $8.74\uparrow,\ 5\times0.164\downarrow,\ 5\downarrow = 2.92\uparrow$	: $+2.92\downarrow$
Rt. of 5kN : $8.74\uparrow,\ 5\times0.164\downarrow,\ 0.082\downarrow,\ 5\downarrow = 2.838\uparrow$	: $+2.838\downarrow$
Left of 5kN : $8.74\uparrow,\ 5\times0.164\downarrow,\ 0.082\downarrow,\ 5\downarrow,\ 5\downarrow = 2.162\downarrow$	: $-2.162\uparrow$
At 7 : $8.74\uparrow,\ 6\times0.164\downarrow,\ 5\downarrow,\ 5\downarrow = 2.244\downarrow$	: $-2.244\uparrow$
At 8 : $8.74\uparrow,\ 7\times0.164\downarrow,\ 5\downarrow,\ 5\downarrow = 2.408\downarrow$	: $-2.408\uparrow$

For B.M. at E
Rt. of Section

	kNm
At C :	: 0
At 2 : $\overset{\curvearrowleft}{8.74\times0.2},\ \overset{\curvearrowright}{0.164\times0.1} = \overset{\curvearrowleft}{1.7316}$	: $+\overset{\curvearrowright}{1.7316}$
At 3 : $\overset{\curvearrowleft}{8.74\times0.4},\ 0.164\overset{\curvearrowright}{(0.1+0.3)} = \overset{\curvearrowleft}{3.43}$	: $+\overset{\curvearrowright}{3.43}$
At 4 : $\overset{\curvearrowleft}{8.74\times0.6},\ 0.164\overset{\curvearrowright}{(0.1+0.3+0.5)} = \overset{\curvearrowleft}{5.0964}$	: $+\overset{\curvearrowright}{5.0964}$
At 5 : $\overset{\curvearrowleft}{8.74\times0.8},\ 0.164\overset{\curvearrowright}{(0.1+0.3+0.5+0.7)} = \overset{\curvearrowleft}{6.7296}$	: $+\overset{\curvearrowright}{6.7296}$
At E : $\overset{\curvearrowleft}{8.74\times1},\ 0.164\overset{\curvearrowright}{(0.1+0.3+0.5+0.7+0.9)},\ \overset{\curvearrowright}{5\times0.1} = \overset{\curvearrowleft}{7.83}$	: $+\overset{\curvearrowright}{7.83}$
At 5kN : $\overset{\curvearrowleft}{8.74\times1.1},\ 0.164\overset{\curvearrowright}{(0.2+0.4+0.6+0.8+1)},\ \overset{\curvearrowright}{5\times0.2} = \overset{\curvearrowleft}{8.122}$	: $+\overset{\curvearrowright}{8.122}$
At 7 : $\overset{\curvearrowleft}{8.74\times1.2},\ 0.164\overset{\curvearrowright}{(0.1+0.3+0.5+0.7+0.9+1.1)},\ \overset{\curvearrowright}{5\times0.1},\ \overset{\curvearrowright}{5\times0.3} = \overset{\curvearrowleft}{7.8976}$	: $+\overset{\curvearrowright}{7.8976}$
At 8 : $\overset{\curvearrowleft}{8.74\times1.4},\ 0.164\overset{\curvearrowright}{(0.1+0.3+0.5+0.7+0.9+1.1+1.3)},\ \overset{\curvearrowright}{5\times0.3},\ \overset{\curvearrowright}{5\times0.5} = \overset{\curvearrowleft}{7.4324}$	: $+\overset{\curvearrowright}{7.4324}$

17/21(ii)(cont.)



S.F. at E = 2.92 kN ↓

B.M. at E = 7.83 kNm clockwise

(iii)

$$1\,mm^4 = 10^{-12}\,m^4$$

$$\therefore\ I = 371 \times 10^6\,mm^4 = 371 \times 10^{-6}\,m^4$$

$$y = 230 \times 10^{-3}\,m$$

$$M = 7.83 \times 10^3\,Nm$$

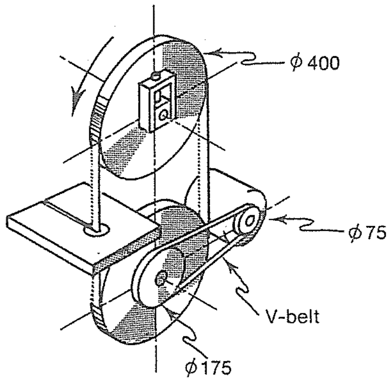
$$\sigma = ?\ Pa$$

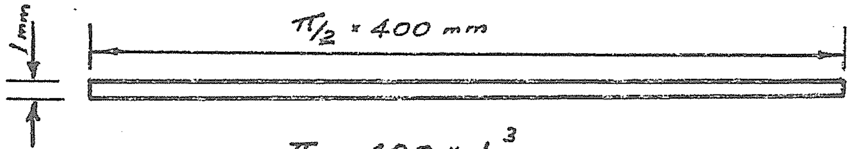
$$\sigma = \frac{y \times M}{I}$$

$$= \frac{230 \times 10^{-3} \times 7.83 \times 10^3}{371 \times 10^{-6}}\,Pa$$

or **4.85 MPa**

17/22
During its operation the 10-mm wide bandsaw blade is wrapped around the 400-mm diameter driving wheel as shown. Given that the blade thickness is 1 mm, determine the maximum stress induced in the blade by this bending operation. $E_{\text{blade}} = 200$ GPa.





$$I : \frac{\pi \times 400 \times 1^3}{2 \times 12 \times 10^{12}} \; m^4$$

$M :$? Nm

$E : 200 \times 10^9$ Pa

$r : 200 \times 10^{-3}$ m

$y : 0{\cdot}5 \times 10^{-3}$ m

$$\frac{M}{I} : \frac{E}{r}$$

$$M : \frac{E \times I}{r}$$

$$: \frac{200 \times 10^9 \times \pi \times 400 \times 1^3 \times 10^3}{2 \times 12 \times 10^{12} \times 200}$$

$: 52{\cdot}36$ Nm

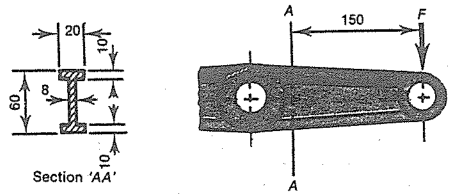
$$\frac{M}{I} : \frac{\sigma}{y}$$

$$\sigma : \frac{M \times y}{I}$$

$$: \frac{52{\cdot}36 \times 0{\cdot}5 \times 12 \times 10^{12}}{10^6 \times 10^3 \times \pi \times 400}$$

$: \underline{\underline{250 \text{ MPa}}}$

17/23
A rocker arm used in a steam engine has a I-section whose dimensions at the section *A-A* approximate to those given in the diagram. Determine the load F at the end of this arm which causes a maximum tensile stress of 10 MPa to occur at this section.



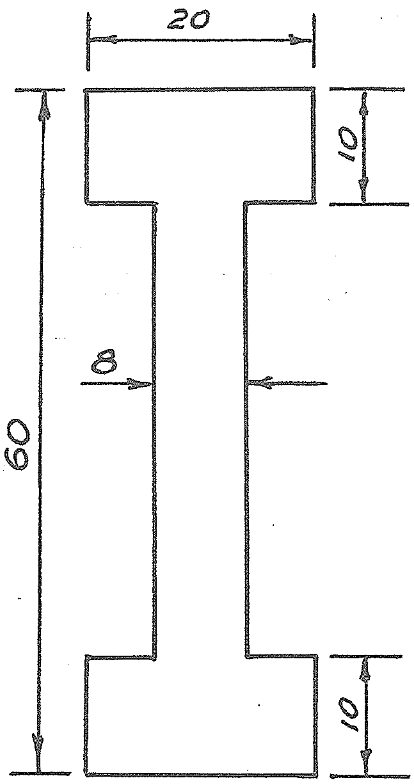
Section 'AA'

$$I_{AA} = \frac{BD^3 - bd^3}{12}$$

$$= \frac{20 \times 60^3 - 12 \times 40^3}{12}$$

$$= 360000 - 64000$$

$$= 296 \times 10^3 \text{ mm}^4$$



Section 'AA'

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$M = \frac{\sigma \times I}{y}$$

$$= \frac{10 \times 296 \times 10^3}{30}$$

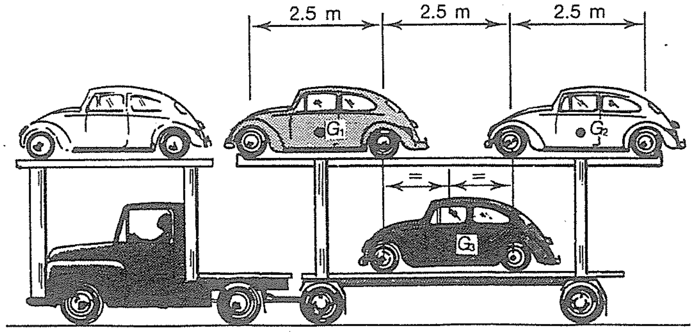
$$= 98666 \cdot 67 \text{ Nmm}$$

$$F \times 150 = M$$

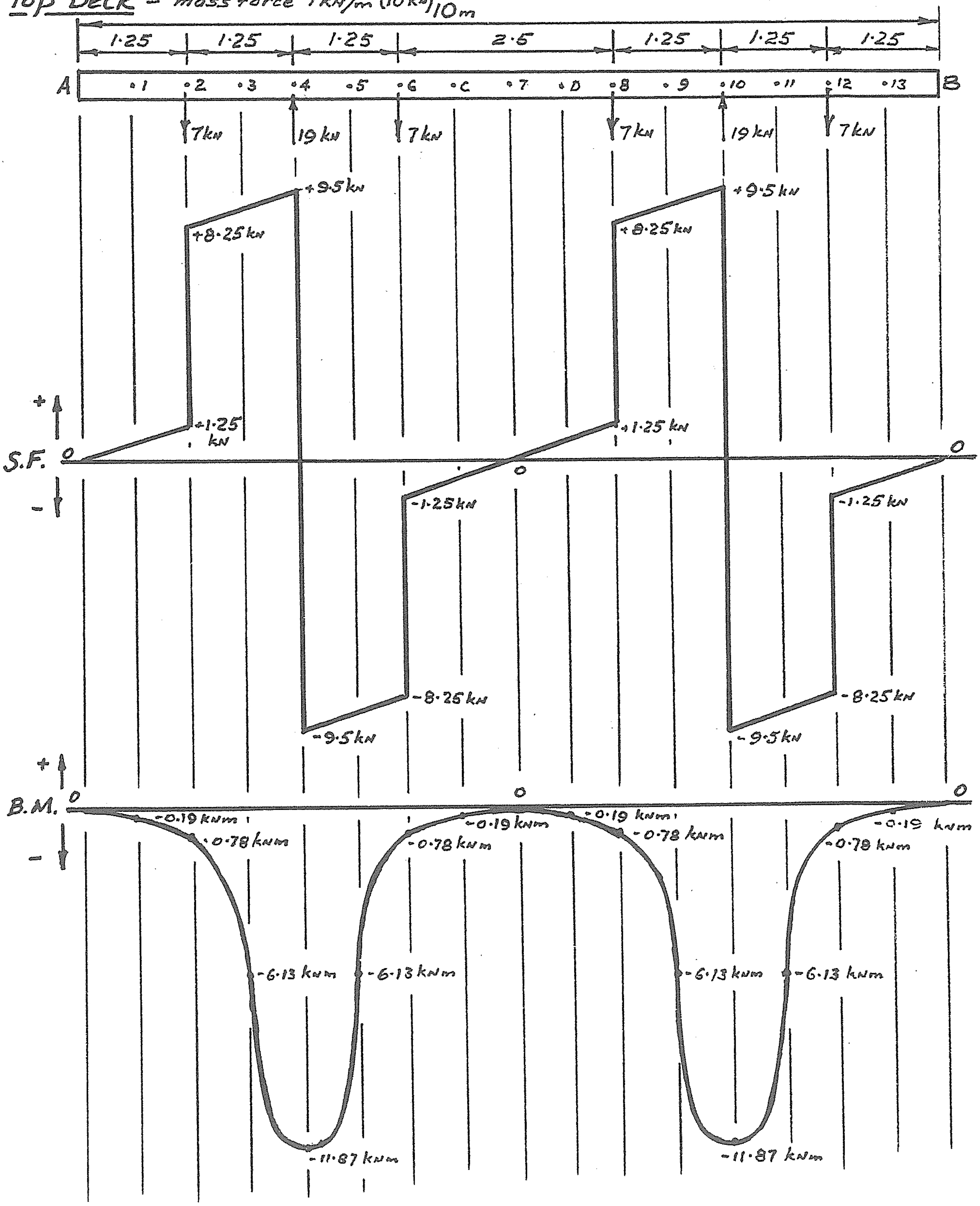
$$F = \underline{657 \cdot 78 \text{ N}}$$

17/24
The trailer of a car transporter carries three identical cars whose centres of mass are shown at G_1, G_2 and G_3. Each car has a mass of 1.4 tonnes. The mass of each deck is a uniform 100 kg/m. The overall length of the upper deck is 10 metres and the lower deck 6 metres.

Construct diagrams showing the distribution of shear force and bending moment in both the upper and the lower decks of the trailer. Ignore the mass of the pillars.



Top Deck – mass force 1 kN/m (10 kN) 10 m



17/24 (cont.)

Top Deck

S.F. - Left of Section

kN

At A : $= 0$

Left of 2 : $1{\cdot}25\downarrow$ $= +1{\cdot}25$

Rt. of 2 : $1{\cdot}25\downarrow + 7\downarrow = 8{\cdot}25\downarrow$ $= +8{\cdot}25$

Left of 4 : $1{\cdot}25\downarrow + 7\downarrow + 1{\cdot}25\downarrow = 9{\cdot}5\downarrow$ $= +9{\cdot}5$

Rt. of 4 : $1{\cdot}25\downarrow + 7\downarrow + 1{\cdot}25\downarrow - 19\uparrow = 9{\cdot}5\uparrow$ $= -9{\cdot}5$

Left of 6 : $1{\cdot}25\downarrow + 7\downarrow + 1{\cdot}25\downarrow - 19\uparrow + 1{\cdot}25\downarrow = 8{\cdot}25\uparrow$ $= -8{\cdot}25$

Rt. of 6 : $1{\cdot}25\downarrow + 7\downarrow + 1{\cdot}25\downarrow - 19\uparrow + 1{\cdot}25\downarrow + 7\downarrow = 1{\cdot}25\uparrow$ $= -1{\cdot}25$

At 7 : $1{\cdot}25\downarrow + 7\downarrow + 1{\cdot}25\downarrow - 19\uparrow + 1{\cdot}25\downarrow + 7\downarrow + 1{\cdot}25\downarrow = 0$ $= 0$

Left of 8 : $1{\cdot}25\downarrow + 7\downarrow + 1{\cdot}25\downarrow - 19\uparrow + 1{\cdot}25\downarrow + 7\downarrow + 1{\cdot}25\downarrow + 1{\cdot}25\downarrow = 1{\cdot}25\downarrow$ $: +1{\cdot}25$

Rt. of 8 : $1{\cdot}25\downarrow + 7\downarrow + 1{\cdot}25\downarrow - 19\uparrow + 1{\cdot}25\downarrow + 7\downarrow + 1{\cdot}25\downarrow + 1{\cdot}25\downarrow + 7\downarrow$
$= 8{\cdot}25\downarrow$ $= +8{\cdot}25$

Left of 10 : $1{\cdot}25 + 7\downarrow + 1{\cdot}25\downarrow - 19\uparrow + 1{\cdot}25\downarrow + 7\downarrow + 1{\cdot}25\downarrow + 1{\cdot}25\downarrow + 7\downarrow$
$+ 1{\cdot}25\downarrow = 9{\cdot}5\downarrow$ $= +9{\cdot}5$

Rt. of 10 : $1{\cdot}25 + 7\downarrow + 1{\cdot}25\downarrow - 19\uparrow + 1{\cdot}25\downarrow + 7\downarrow + 1{\cdot}25\downarrow + 1{\cdot}25\downarrow + 7\downarrow$
$+ 1{\cdot}25\downarrow - 19\uparrow = 9{\cdot}5\uparrow$ $= -9{\cdot}5$

Left of 12 : $1{\cdot}25 + 7\downarrow + 1{\cdot}25\downarrow - 19\uparrow + 1{\cdot}25\downarrow + 7\downarrow + 1{\cdot}25\downarrow + 1{\cdot}25\downarrow + 7\downarrow$
$+ 1{\cdot}25\downarrow - 19\uparrow + 1{\cdot}25\downarrow = 8{\cdot}25\uparrow$ $= -8{\cdot}25$

Rt. of 12 : $1{\cdot}25 + 7\downarrow + 1{\cdot}25\downarrow - 19\uparrow + 1{\cdot}25\downarrow + 7\downarrow + 1{\cdot}25\downarrow + 1{\cdot}25\downarrow + 7\downarrow$
$+ 1{\cdot}25\downarrow - 19\uparrow + 1{\cdot}25\downarrow + 7\downarrow = 1{\cdot}25\uparrow$ $= -1{\cdot}25$

At B : $1{\cdot}25 + 7\downarrow + 1{\cdot}25\downarrow - 19\uparrow + 1{\cdot}25\downarrow + 7\downarrow + 1{\cdot}25\downarrow + 1{\cdot}25\downarrow + 7\downarrow$
$+ 1{\cdot}25\downarrow - 19\uparrow + 1{\cdot}25\downarrow + 7\downarrow + 1{\cdot}25\downarrow = 0$ $= 0$

17/24 (cont.)

Top Deck

B.M. - Right of Section kNm

At B : = 0

At 13 : $0.625 \times 0.312 = 0.195$ $= -0.195$

At 12 : $1.25 \times 0.625 = 0.781$ $= -0.781$

At 11 : $7 \times 0.625 + 1.875 \times 0.936 = 6.134$ $= -6.134$

At 10 : $7 \times 1.25 + 2.5 \times 1.25 = 11.875$ $= -11.875$

At 9 : $-19 \times 0.625 + 7 \times 1.875 + 3.125 \times 1.563 = 6.134$ $= -6.134$

At 8 : $-19 \times 1.25 + 7 \times 2.5 + 3.75 \times 1.875 = 0.781$ $= -0.781$

At D : $7 \times 0.625 - 19 \times 1.875 + 7 \times 3.125 + 4.375 \times 2.1875 = 0.195$ $= -0.195$

At 7 : $7 \times 1.25 - 19 \times 2.5 + 7 \times 3.75 + 5 \times 2.5 = 0$ $= 0$

At C : $7 \times 1.875 - 19 \times 3.125 + 7 \times 4.375 + 5.625 \times 2.8125 = 0.195$ $= -0.195$

At 6 : $7 \times 2.5 - 19 \times 3.75 + 7 \times 5 + 6.25 \times 3.125 = 0.781$ $= -0.781$

At 5 : $7 \times 0.625 + 7 \times 3.125 - 19 \times 4.375 + 7 \times 5.625$
$+ 6.875 \times 3.438 = 6.134$ $= -6.134$

At 4 : $7 \times 1.25 + 7 \times 3.75 - 19 \times 5 + 7 \times 6.25 + 7.5 \times 3.75$
$= 11.875$ $= -11.875$

At 3 : $-19 \times 0.625 + 7 \times 1.875 + 7 \times 4.375 - 19 \times 5.625 + 7 \times 6.875$
$+ 8.125 \times 4.063 = 6.134$ $= -6.134$

At 2 : $-19 \times 1.25 + 7 \times 2.5 + 7 \times 5 - 19 \times 6.25 + 7 \times 7.5$
$+ 8.75 \times 4.375 = 0.781$ $= -0.781$

At 1 : $7 \times 0.625 - 19 \times 1.875 + 7 \times 3.125 + 7 \times 5.625 - 19 \times 6.875$
$+ 7 \times 8.125 + 9.375 \times 4.688 = 0.195$ $= -0.195$

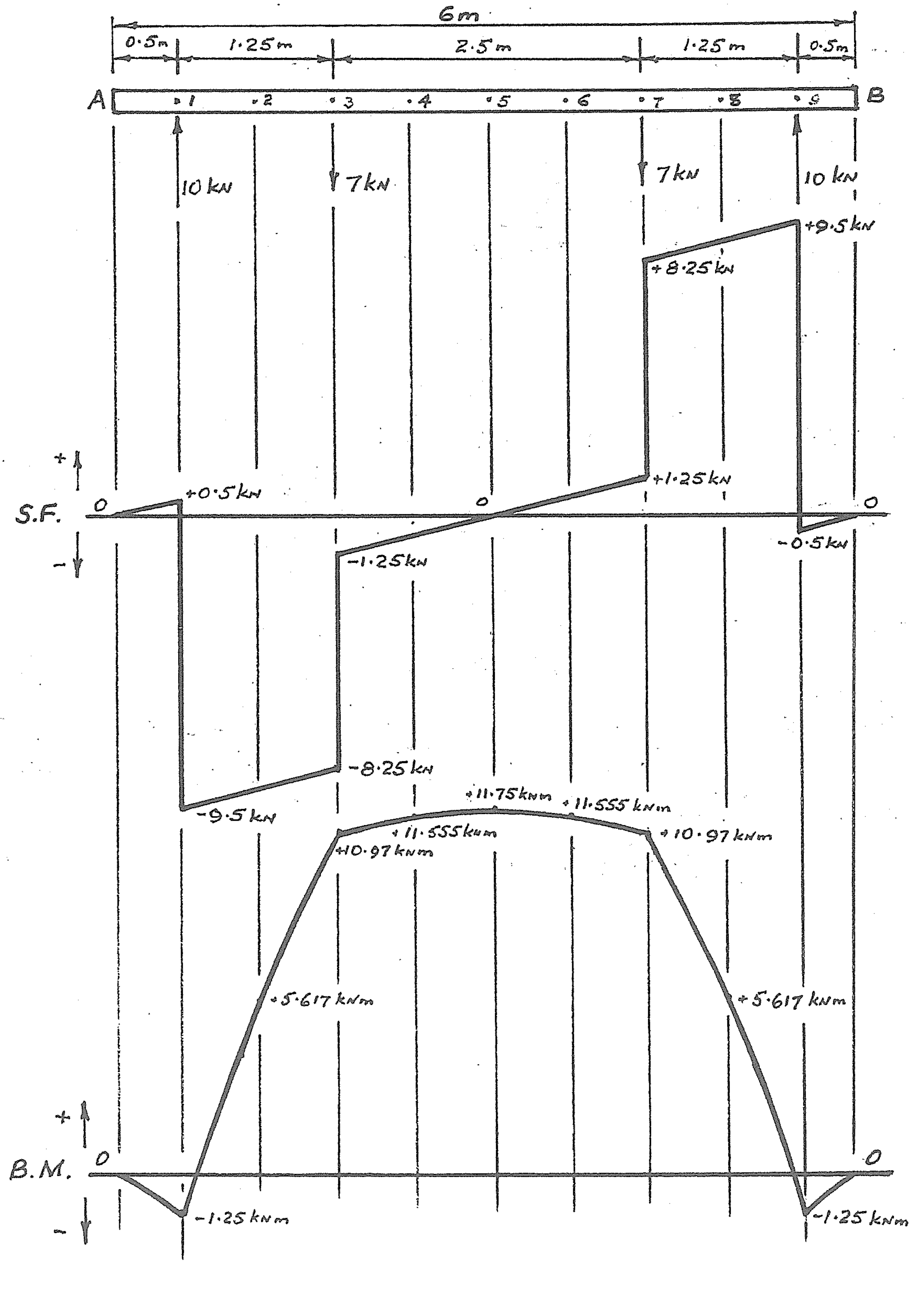
At A : $7 \times 1.25 - 19 \times 2.5 + 7 \times 3.75 + 7 \times 6.25 - 19 \times 7.5$
$+ 7 \times 8.75 + 10 \times 5 = 0$ $= 0$

17/24 (cont.)

Bottom Deck - mass force 1 kN/m : 6 kN

Note: The mass force of 38 kN (top deck + 2 cars) acting down at points 1 & 4 is cancelled out by equal and opposite reactive forces offered by the bottom deck.

∴ Nett reactive forces on bottom deck equal 10 kN↑ at each of points 1 & 4.



17/24 (cont.) **Bottom Deck**

S. F. - Left of Section kN

At A :	: 0
Left of 1 : $0.5\downarrow$	: $+0.5\uparrow$
Rt. of 1 : $0.5\downarrow - 10\uparrow = 9.5\uparrow$	: $-9.5\downarrow$
Left of 3 : $0.5\downarrow - 10\uparrow + 1.25\downarrow = 8.25\uparrow$	: $-8.25\downarrow$
Rt. of 3 : $0.5\downarrow - 10\uparrow + 1.25\downarrow + 7\downarrow = 1.25\uparrow$	: $-1.25\downarrow$
At 5 : $0.5\downarrow - 10\uparrow + 1.25\downarrow + 7\downarrow + 1.25\downarrow = 0$	: 0
Left of 7 : $0.5\downarrow - 10\uparrow + 1.25\downarrow + 7\downarrow + 2.5\downarrow = 1.25\downarrow$	: $+1.25\uparrow$
Rt. of 7 : $0.5\downarrow - 10\uparrow + 1.25\downarrow + 7\downarrow + 2.5\downarrow + 7\downarrow = 8.25\downarrow$	: $+8.25\uparrow$
Left of 9 : $0.5\downarrow - 10\uparrow + 1.25\downarrow + 7\downarrow + 2.5\downarrow + 7\downarrow + 1.25\downarrow = 9.5\downarrow$	: $+9.5\uparrow$
Rt. of 9 : $0.5\downarrow - 10\uparrow + 1.25\downarrow + 7\downarrow + 2.5\downarrow + 7\downarrow + 1.25\downarrow - 10\uparrow = 0.5\uparrow$	: $-0.5\downarrow$
At B : $0.5\downarrow - 10\uparrow + 1.25\downarrow + 7\downarrow + 2.5\downarrow + 7\downarrow + 1.25\downarrow - 10\uparrow + 0.5\downarrow = 0$	: 0

B.M. - Rt. of Section kNm

At B :	: 0
At 9 : $0.5 \times 0.25 = 0.125$	: -0.125
At 8 : $-10 \times 0.625 + 1.125 \times 0.563 = 5.617$	: $+5.617$
At 7 : $-10 \times 1.25 + 1.75 \times 0.875 = 10.97$	: $+10.97$
At 6 : $7 \times 0.625 - 10 \times 1.875 + 2.375 \times 1.1875 = 11.555$	: $+11.555$
At 5 : $7 \times 1.25 - 10 \times 2.5 + 3 \times 1.5 = 11.75$	: $+11.75$
At 4 : $7 \times 1.875 - 10 \times 3.125 + 3.625 \times 1.8125 = 11.555$	: $+11.555$
At 3 : $7 \times 2.5 - 10 \times 3.75 + 4.25 \times 2.125 = 10.97$	: $+10.97$
At 2 : $7 \times 0.625 + 7 \times 3.125 - 10 \times 4.375 + 4.875 \times 2.4375 = 5.617$	: $+5.617$
At 1 : $7 \times 1.25 + 7 \times 3.75 - 10 \times 5 + 5.5 \times 2.75 = 0.125$	: -0.125
At A : $-10 \times 0.5 + 7 \times 1.75 + 7 \times 4.25 - 10 \times 5.5 + 6 \times 3 = 0$	: 0

18

Principles of Machines

$$\text{mechanical advantage} = \frac{\text{load}}{\text{effort}}$$

$$\text{velocity ratio} = \frac{\text{distance moved by EFFORT}}{\text{distance moved by LOAD}}$$

$$\%\ \text{efficiency},\ \eta = \frac{\text{MA}}{\text{VR}} \times \frac{100}{1}$$

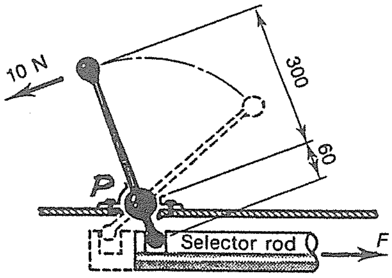
PRINCIPLES OF MACHINES

Levers: *First Order Levers; Second Order Levers; Third Order Levers. Mechanical Advantage. Equilibrium of Levers. Typical Levers.* The Wheel. Efficiency. Pulleys: *The Single Fixed Pulley; The Single Movable Pulley; Multiple Movable Pulleys. The Weston Differential Pulley Block. Pulleys and Belts.* Chain Drives. The Inclined Plane. *The Archimedean Screw. The Worm and Worm Wheel. Screw Threads.* The Hydraulic Ram.

$g = 10\ m/s^2$

18/1

Determine the force, *F*, in the selector rod when an effort of 10 N is applied to the gear lever shown. What is the mechanical advantage?



Take moments about P: $F \times 60 = 10 \times 300$

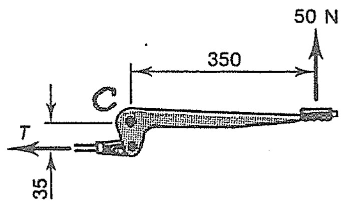
$F = \underline{50\ N}$

$M.A. = L/E$

$= 50/10 = \underline{5:1}$

18/2

What is the tension, *T*, in the handbrake cable when a force of 50 N is applied to the handle? What is the mechanical advantage?



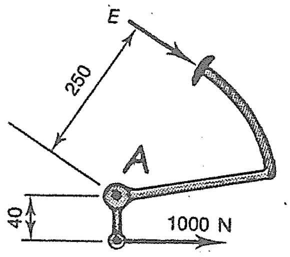
Take moments about C: $T \times 35 = 50 \times 350$

$T = \underline{500\ N}$

$M.A. = L/E = 500/50 = \underline{10:1}$

18/3

Determine the effort, *E*, on the clutch pedal shown needed to obtain a force of 1000 N in the linkage. What is the velocity ratio?



Take moments about A: $E \times 250 = 1000 \times 40$

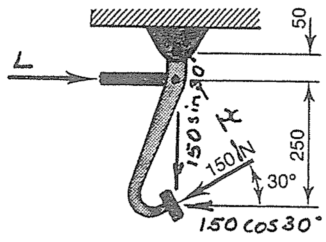
$E = \underline{160\ N}$

For 100% Efficiency $V.R. = M.A.$

$M.A. = L/E = 1000/160 = 6{\cdot}25:1$

$\therefore V.R. = \underline{6{\cdot}25:1}$

18/4
The pendant brake pedal shown has a force of 150 N applied at an angle of 30°. What is the load in the push-rod? What is the MA of this lever?



$x = 300 \sin 60°$

Take moments about P

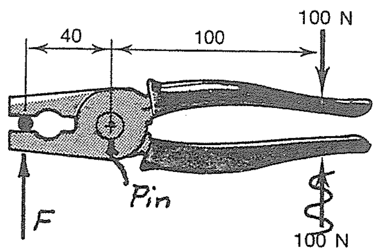
$L \times 50 = 150 \times 300 \sin 60°$

$L = \underline{779.4\ N}$

Effective effort $= 150 \cos 30° = 129.9\ N$

$M.A. = L/E = 779.9/129.9 = \underline{6:1}$

18/5
Determine the gripping force on the rod if an effort of 100 N is applied to the handles of the pliers.



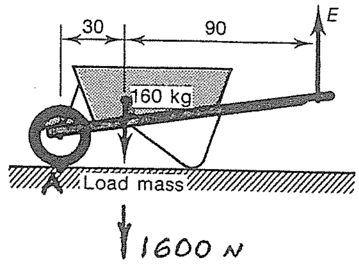
Consider the single handle/jaw as shown.

Take moments about the pin

$F \times 40 = 100 \times 100$

$F = \underline{250\ N}$

18/6
What effort, *E*, will be needed to support the barrow shown?

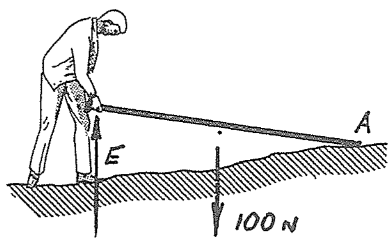


Take moments about A

$E \times 120 = 1600 \times 30$

$E = \underline{400\ N}$

18/7
A crowbar has a mass of 10 kg. What effort will be required to just support one end of it?



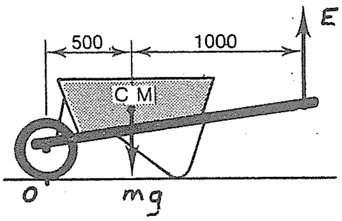
Let the length of the bar be "2ℓ"

Take moments about A

$E \times 2\ell = 100 \times \ell$

$E = \underline{50\ N}$

18/8
The barrow and its load have their combined centre of mass located as shown. Determine the mechanical advantage of the wheelbarrow as a lever, and discuss the convenience of the wheel.



Take moments about O

$E \times 1500 = mg \times 500$ Let wt. force be mg N

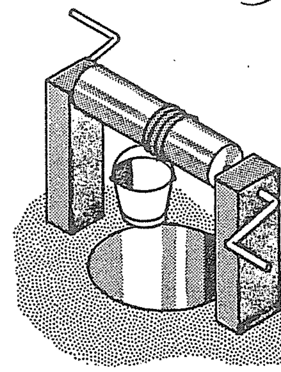
$E = mg/3$

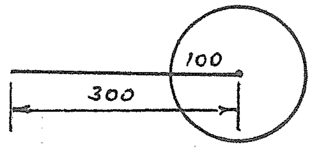
$M.A. = L/E = \frac{mg \times 3}{mg} = \underline{3:1}$

Rolling friction is less than sliding friction.

18/9
The windlass has a 200-mm winding drum and crank handles operating in a 600-mm diameter path. Determine the velocity ratio of this machine.

Approximately half of the input effort is used up overcoming friction, bending the rope around the drum, and raising the mass of the bucket. Considering these losses, suggest a suitable bucket size, if the effort on the handle is not to exceed 160 N.





$V.R. = \frac{\text{Distance moved by } E}{\text{Distance moved by } L}$

$= \frac{\pi \times 600}{\pi \times 200}$

$= \underline{3:1}$

$\eta = M.A./V.R.$

$\frac{50}{100} = \frac{M.A.}{3}$

$M.A. = 1{\cdot}5 : 1$

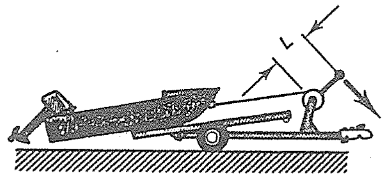
$M.A. = L/E$

$1{\cdot}5 = L/160$

$L = 240\,N$ or 24 kg of water

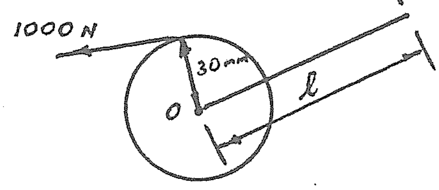
Assuming 1 kg of water occupies 1 litre, a <u>24 litre bucket</u> would be suitable

18/10
Design a suitable length, L, of the crank arm for the boat trailer winch if the winding drum plus the cable averages 60 mm diameter and the tension in the cable does not exceed 1000 N. (Estimate that frictional and other losses will use about 10% of the input effort.) of 250 N



or $250 \times \frac{90}{100}$
$= 225\,N$

$250 - 10\%$
$= 225\,N$

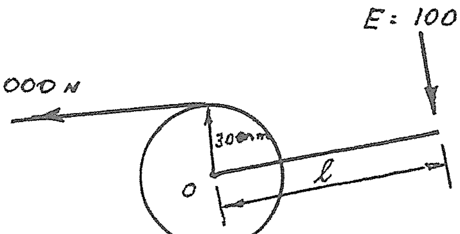


Take moments about O

$225 \times \ell = 1000 \times 30$

$\ell = \underline{133{\cdot}3 \text{ mm}}$

18/11
The efficiency of the boat trailer winch (shown in Problem 18/10) is found by experiment to be only 40%. What length of crank arm is now needed? Is this length practicable? What other design modifications could be made to have a similar effect?



Effective effort $E = \dfrac{250 \times 40}{100} = 100$ N — or $250 - \left(\frac{60}{100} \times 2\ldots\right)$ 40% LOS...

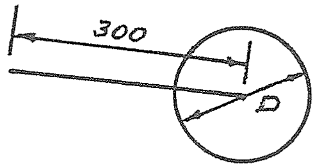
Take moments about O

$100 \times \ell = 1000 \times 30$

$\ell = \underline{300 \text{ mm}}$

This is a little long, will foul the trailer parts, could bend the handle arm etc. Use gear ratio or reduce drum diameter.

18/12
The windlass shown in Problem 18/9 is updated by addition of a light nylon cord and ball-race bearings and achieves an efficiency of 90%. The crank now feels far too easy to wind and the effort seems wasted. What new combination of drum diameter and bucket size can you suggest so that the previous effort on the crank is required?



<u>Larger Drum dia.</u>

$\eta = M.A./V.R.$

$90/100 = 1.5/V.R.$

$V.R. = 5:3$

$V.R. = \dfrac{\text{Effort dist.}}{\text{Load dist.}}$

$\dfrac{5}{3} = \dfrac{\pi \times 600}{\pi \times D}$ $\therefore D = \underline{360 \text{ mm}}$

<u>Same size Drum</u>

$V.R. = 3:1$

$90/100 = M.A/3$ Efficiency

$\therefore M.A. = 2.7:1$

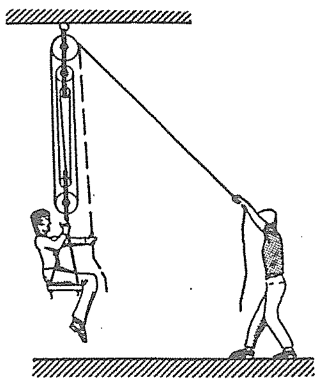
$M.A. = L/E$

$2.7 = L/160$

$L = 432$ N or 43 kg

Thus a <u>43 litre bucket.</u>

18/13
From a strong-point in the classroom ceiling, a group is experimenting with a block and tackle. The boys are sitting in a "bosun's chair" and being hauled up and down, when Tom discovers it is easier to pull himself up, than to pull up Dick, who has the same mass. Why is it easier?

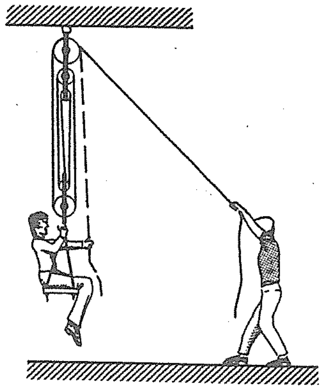


When the person in the chair raises himself there are 6 ropes supporting the load instead of 5

$\therefore$ Easier since <u>E = 1/6 of load instead of 1/5 of load.</u>

18/14

Tom has a mass of 50 kg. By pulling the rope with a spring balance, Harry discovers that it needs a force of approximately 60 N to just support Tom (that is, prevent "overhaul") and a force of about 170 N to slowly raise him further. If the system has a velocity ratio of 5 : 1, what is its mechanical advantage? What is its efficiency? How could this be improved?



M.A. to raise Tom : 500/170

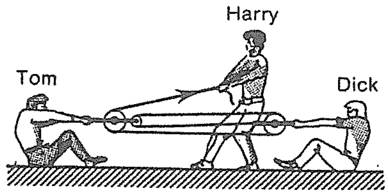
: $\underline{2 \cdot 94 : 1}$ (3:1)

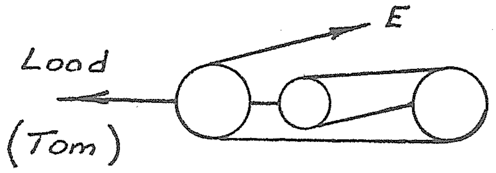
η : $3{:}1 / 5{:}1$ = $\underline{60\%}$

Use larger pulleys, better bearings, lighter rope.

18/15

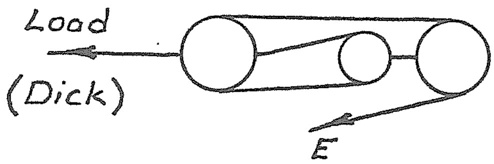
Tom and Dick have the same mass, and hold onto a block each, with the rope stretched out along the floor. Harry pulls the free end of the rope. Which boy will slide along the floor? Why? How much rope will Harry have pulled when that boy has slid one metre if there is a single and a double sheave rigged as shown?





4 ropes pulling load

∴ M.A. = 4:1



3 ropes pulling load

∴ M.A. = 3:1

∴ <u>Tom will slide</u>

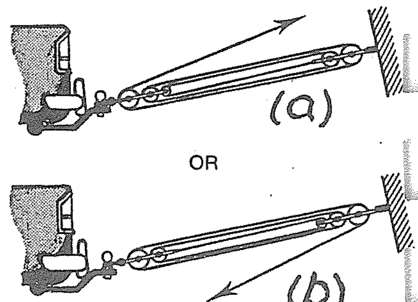
For 100% efficiency M.A. = V.R. = 4:1

$$V.R. = \frac{\text{Distance moved by } E}{\text{Distance moved by } L}$$

∴ <u>Effort moves 4 m</u>

18/16

Your car will not start, and you need to haul it uphill into the garage, to be repaired. You have a suitable anchor point in the garage and a block and tackle with five sheaves. Which block will you hook onto the car, the three-sheave block, or the two-sheave block? Why?

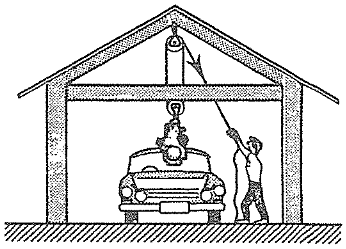


Arrangement (a): 6 ropes pulling load ∴ M.A. = 6 : 1

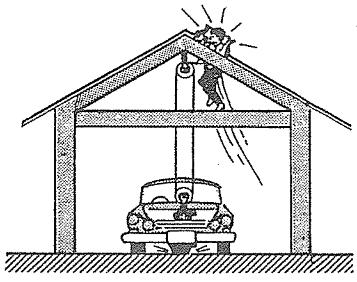
Arrangement (b): 5 ropes pulling load ∴ M.A. = 5:1

∴ The 3 sheave block should be attached to the car to gain higher M.A. (arrangement (a))

18/17
The car engine has a mass of 150 kg and needs to be lifted out for repair. Your block and tackle has five sheaves, and is slung from the ridge of the garage roof. It is about 60% efficient. Will your 60-kg mass be heavy enough to lift it out alone, or will it "overhaul" (that is, lift you)?



OR



The 5 sheave block and tackle is suspended with 3 sheaves at the top $\therefore$ M.A. $= 5:1$

With an efficiency of 60% the effective M.A. $= 60\%$ of $5:1$

$= 3:1$

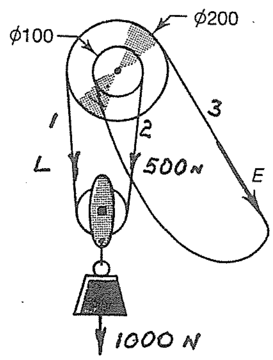
Mass force of engine $= 1500$ N

Mass force of worker $= 600$ N

$\therefore$ A M.A. of $3:1$ provides:

1800 N $\therefore$ worker can lift the engine

18/18
Determine the effort required to (a) support; (b) raise the load of 100 kg if the Weston Differential rope block shown is only 50% efficient.



Ratio of $\dfrac{\text{Large Pulley}}{\text{Small Pulley}} = \dfrac{2}{1}$

When effort moves 2 links on large pulley the small pulley moves 1 link and the load is raised ½ link.

$\therefore$ V.R. $= 2 : \frac{1}{2}$ or $4:1$ and for 50% effic. M.A. $= 2:1$

(a) Effort in chain 3 must balance load force in chain 1 only, since in Weston Diff. system the force in chain 2 (500N) is supported by links in the small pulley

For equilibrium $L \times 100 = 500 \times 50$

$\therefore L = 250$ N

M.A. $= L/E$ $\therefore$ Effort $=$ 125 N or more

(b) To raise load:

M.A. $= 2:1$

$L = 1000$ N

$E = ?$

M.A. $= L/E$

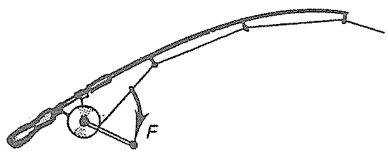
$E = L/\text{M.A}$

$= 1000/2$

$=$ 500 N or more

Problems 19 & 20 are experimental and Problem 21 is a worked example

18/22
Fred is carefully reeling in his catch on his 50-mm diameter reel and the fish continues to pull at 250-N force. If the length of the handle is 100 mm, what force must delighted Fred exert to secure his catch?

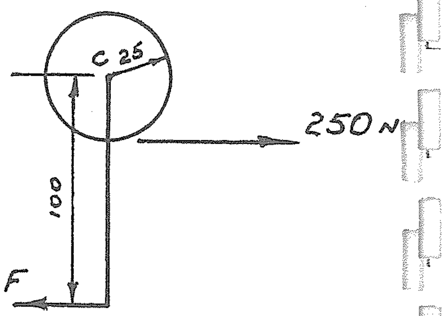


Take moments about C

$$F \times 100 = 250 \times 25$$

$$F = \frac{250 \times 25}{100}$$

$$= \underline{62 \cdot 5 \text{ N or greater}}$$



18/23
In a test on a lifting machine, it was found that an effort of 120 N was required to lift a load of 300 kg. The effort moved through a distance of 10 m while the load moved 300 mm. Determine
(i) the distance moved by the effort while the load is moving 1 m;
(ii) the work done (output) by the machine while the load is moving 1 m;
(iii) the efficiency per cent of the machine when the load is 300 kg;
(iv) the effort required to lift the 300 kg load, if the machine were perfect.

$$M.A. = L/E$$

$$= \frac{300 \times 10}{120}$$

$$= 25 : 1$$

$$V.R. = \frac{\text{Effort dist.}}{\text{Load dist.}}$$

$$= \frac{10 \times 10^3}{300}$$

$$= 33 \cdot 3 : 1$$

(i) For 1 m of Load movement
Effort moves 33·3 m.

(ii) Work done $= F \times S$

$$= \text{Effort} \times V.R.$$

$$= \frac{120 \times 33 \cdot 3}{10^3}$$

$$= \underline{4 \text{ kNm}}$$

(iii) $$\text{Efficiency} = \frac{M.A.}{V.R.} \times 100$$

$$= \frac{25 \times 100}{33 \cdot 3}$$

$$= \underline{75\%}$$

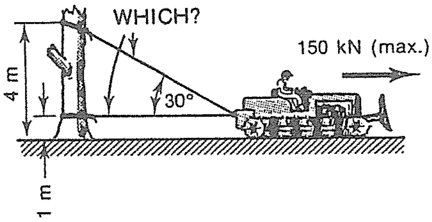
(iv) For 100% Effic. M.A. = V.R. = 33·3 : 1

$$M.A. = L/E$$

$$33 \cdot 3 = \frac{300 \times 10}{E}$$

$$\therefore E = \underline{90 \text{ N}}$$

18/24
Fred's D4 Caterpillar tractor (bulldozer) has a maximum drawbar pull of 150 kN near stalling point, under full engine power. Fred is about to use it to remove a stump, and is undecided where he should attach his cable to the stump in order to get the maximum bending moment at the base of the stump. Explain to Fred where he should attach it, and why.



When attached 1 m from ground B.M = 150 kNm

When attached 4 m from ground:

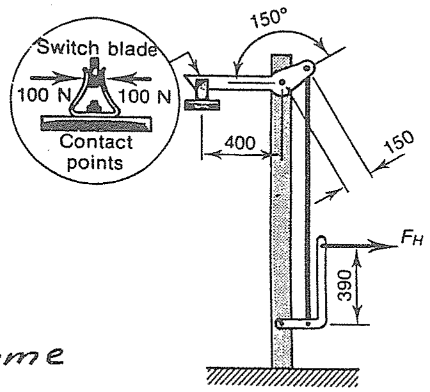
Effective Horiz. force = $150 \cos 30^\circ$

$\therefore$ B.M. = $150 \cos 30^\circ \times 4$.

= 519.6 kNm

$\therefore$ <u>Attach at 4 m level</u>

18/25
A remote lever-actuated electrical circuit breaker at the top of a pole is shown. Given that the normal force between the blade and the switch contacts is 100 N, determine the magnitude of the force F_H that must be applied to the handle when operating the switch, if the coefficient of static friction present between the switch contacts and the blade is 0.5.



Friction force to be overcome

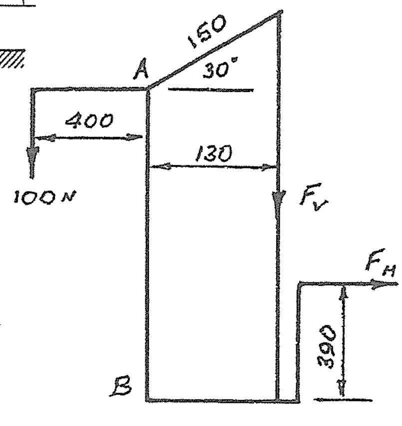
$= 100 \times 0.5 \times 2$

$= 100$ N

Take moments about A

$F_V \times 130 = 100 \times 400$

$F_V = 308$ N

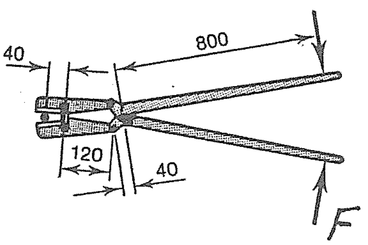


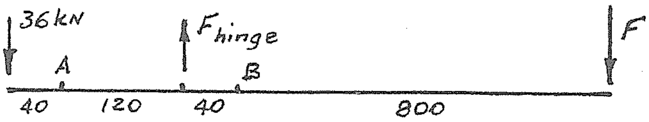
Take moments about B

$F_V \times 130 = F_H \times 390$

$\therefore F_H = \underline{103 \text{ N}}$

18/26
If a 14-mm steel bar requires a force of 36 kN to cut it with the bolt-cutters shown, what force does the operator have to exert on the handles?





Take moments about A

$F_{hinge} \times 120 = 36 \times 40$

$F_{hinge} = 12$ kN

Take moments about B

$F \times 800 = 12 \times 40$

$F = \underline{0.6 \text{ kN}}$

18/27
A set of pulley blocks is used to lift a load of 200 kg.
(i) If the effort required is 300 N and the velocity ratio of the pulley blocks is 8 to 1, what is the efficiency of the equipment?
(ii) Explain briefly why the efficiency is less than 100 per cent.

Load : 200×10 N
Effort : 300 N

(i) $M.A. = L/E = \frac{2000}{300} = 6.67 : 1$

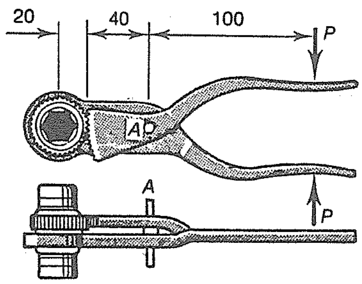
$$\eta = \frac{M.A}{V.R.} = \frac{6.67 \times 100}{8} = \underline{83.3\%}$$

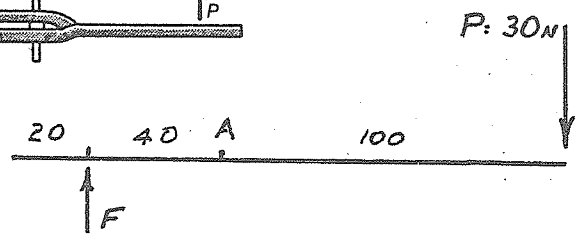
(ii) Losses in friction, extension of ropes, slippage etc

18/28
A socket spanner is designed for use by a crewman of a spacecraft where he has no platform against which to push.

The pin A fits into a hole in the spacecraft near the bolt to be turned. By squeezing the handles of the tool, the bolt turns. One side of the tool is used for tightening and the opposite side for loosening a bolt. The reaction against the pin A provides the "anti-torque" characteristic of the tool.

For a gripping force of $P = 30$ N, determine the torque transmitted to the bolt.





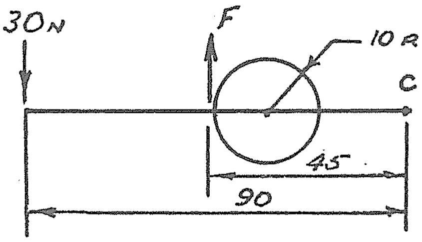
Take moments about A

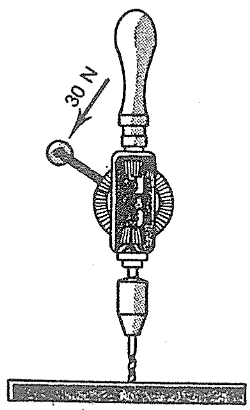
$$F \times 40 = 30 \times 100$$

$$F = \frac{30 \times 100}{40}$$

$$\text{Torque} = F \times \text{rad.} = \frac{30 \times 100 \times 20}{40 \times 1000} = \underline{1.5\ Nm}$$

18/29
Determine the theoretical torque available to drive the drill bit with a force of 30 N on the handle. The pitch circle diameter of the hand wheel is 90 mm and that of the pinion is 20 mm. The crank is 90 mm long.





Take moments about C

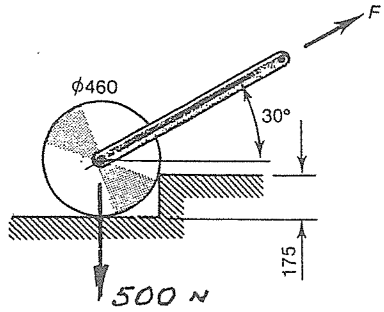
$$F \times 45 = 30 \times 90$$

$$F = \frac{30 \times 90}{45} = 60N$$

F becomes the force driving the pinion

$$\therefore \text{Torque} = F \times \text{rad.} = \frac{60 \times 10}{1000} = \underline{0.6\ Nm}$$

18/30
A garden roller of mass 50 kg is to be moved up the step. Is it easier to lift it vertically, or to drag it up the step using the handle inclined as shown?



Force needed to lift the roller $= 500\ N$

$\sin\theta = 55/230$

$\theta = 13{\cdot}8^\circ$

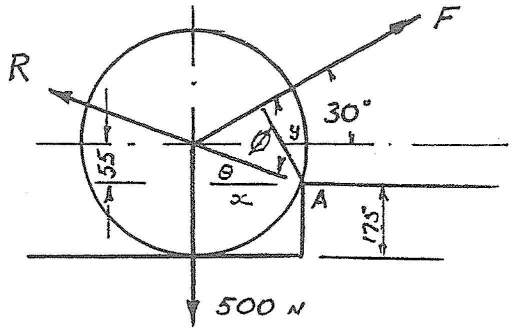
$\cos 13{\cdot}8^\circ = x/230$

$x = 223\ mm$

$\phi = (30 + 13{\cdot}8) = 43{\cdot}8^\circ$

$\sin\phi = y/230$

$y = 159\ mm$



Take moments about A

$F \times 159 = 500 \times 223$

$F = 701\ N$

$\therefore$ Easier to lift the roller.

18/31
For a given lifting machine the effort moves 10 times as fast as the load. If an effort of 200 N lifts a load of 160 kg, find the mechanical advantage and the efficiency per cent.

$V.R. = 10 : 1$

$M.A. = L/E$

$= \dfrac{160 \times 10}{200}$

$= 8 : 1$

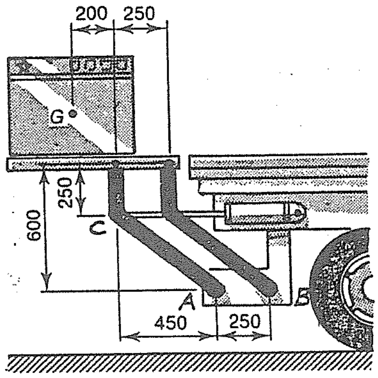
$\eta = \dfrac{M.A.}{V.R} \times 100$

$= \dfrac{8}{10} \times 100$

$= 80\%$

18/32
A power operated loading platform for the back of a truck is shown in the diagram. The position of the platform is controlled by the hydraulic cylinder which pulls at C. The links are pivoted to the truck frame at A, B.

Determine the force P supplied by the cylinder in order to support the platform in the position shown. The mass of the platform and links may be neglected compared with that of the 200 kg washing machine on it.



$$\tan\theta = \frac{350}{450}$$

$$= 0.7777$$

$$\therefore \theta = 37.87^\circ$$

$\sin 37.87 = \frac{x}{250}$. 153.46

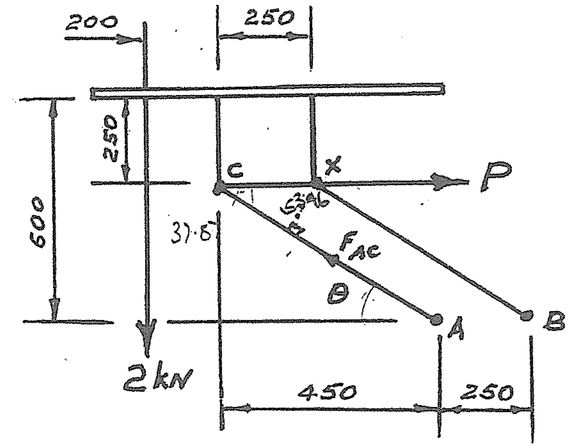
$F_{AC} \times 153.46 = 2000 \times 0.450$.

$F_{AC} = 586.47$.

Take moments about x

$$F_{AC} \times 250 \sin 37.87^\circ = 450 \times 2$$

$$F_{AC} = \frac{45 \times 2}{25 \sin 37.87^\circ}$$



Resolve horiz.

$$P = F_{AC} \cos 37.87^\circ$$

$$= \frac{45 \times 2 \times \cos 37.87^\circ}{25 \sin 37.87^\circ}$$

$$= \frac{71.0465}{15.3468}$$

$$= \underline{4.63 \text{ kN}}$$

18/33
What load may be lifted by an effort of 40 N applied to a lifting machine, if the velocity ratio is 80, and the efficiency is 60 per cent?

$$\eta = \frac{M.A.}{V.R.}$$

$$\therefore M.A. = \eta \times V.R.$$

$$= \frac{60 \times 80}{100}$$

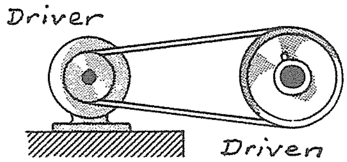
$$= 48 : 1$$

$$M.A. = L/E$$

$$L = 48 \times 40$$

$$= \underline{1920 \text{ N}}$$

18/34
An electric motor drives a steel shaft by means of a belt drive. The pulley fitted to the motor shaft is 100 mm diameter, while the one fitted to the driven shaft is 200 mm diameter. If it is assumed that there is no belt slip and the speed of the electric motor is 1450 revolutions per minute, determine
(i) the linear speed of the belt;
(ii) the angular speed of the driven shaft.



(i) Distance travelled by belt in 1 minute $= \frac{\pi D \times 1450}{1000}$ m

or $\frac{\pi \times 100 \times 1450}{1000 \times 60}$ m/s

$= \underline{7{\cdot}59 \text{ m/s}}$

(ii) $\frac{\text{Revs. of Driven Pulley}}{\text{Revs. of Driver Pulley}} = \frac{100}{200}$

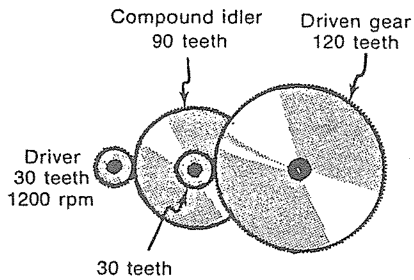
$\therefore$ Revs. of Driven Pulley $= \frac{100 \times 1450}{200}$

$= \underline{725 \text{ R.P.M.}}$

or $\frac{2\pi \times 725}{60}$ rad./s

$= \underline{75{\cdot}9 \text{ rad./s}}$

18/35
For the compound gear train shown in the diagram determine the speed of the driven gear and the overall velocity ratio.



$\frac{\text{Revs. of 90 tooth idler}}{\text{Revs. of 30 tooth driver}} = \frac{30}{90}$

$\therefore$ Idler revolves at $\frac{30}{90} \times 1200$

$= \underline{400 \text{ R.P.M}}$

The 30 tooth wheel also revolves at 400 R.P.M.

$\frac{\text{Revs. of 120 tooth Driven Gear}}{\text{Revs. of 30 tooth wheel}} = \frac{30}{120}$

$\therefore$ Revs. of Driven Gear $= \frac{30 \times 400}{120} = \underline{100 \text{ R.P.M.}}$

V.R. = inverse ratio of circumferences

$= \frac{90 \times 120}{30 \times 30} = \underline{12:1}$

18/36
A rope pulley block and tackle has a velocity ratio of 5 : 1. When lifting a load of 200 kg its efficiency is 80 per cent. What effort would be necessary to lift the 200-kg load?

$\eta = M.A./V.R.$

$M.A = \dfrac{80 \times 5}{100}$

$= 4 : 1$

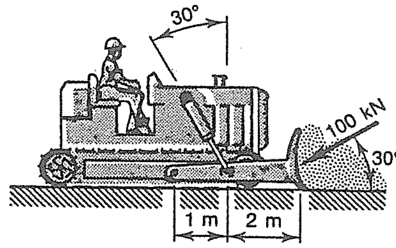
$M.A = L/E$

$E = L/M.A.$

$= \dfrac{200 \times 10}{4}$

$= \underline{500 \text{ N}}$

18/37
A crawler tractor with a dozing blade (a bulldozer) is pushing a mound of earth with a force of 100 kN.
(i) Determine the force in each of the pair of hydraulic piston rods which hold the blade in position.
(ii) Is it a tensile force or a compressive force?
(iii) What is the pressure in the hydraulic lines, if each cylinder has a cross-sectional area of 8000 mm^2?

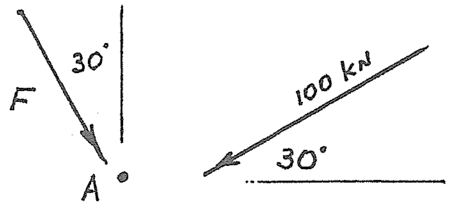


(i) Resolve forces horiz.

$F \cos 60° = 100 \cos 30°$

$F = 173{\cdot}2$ kN total

or $\underline{86{\cdot}6 \text{ kN each side}}$

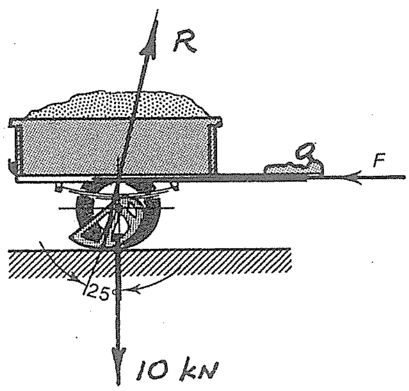


(ii) The 100 kN force produces a tensile force through the pin at A on the piston rod

(iii) Pressure $= \dfrac{86{\cdot}6 \times 1000}{8000}$

$= \underline{10{\cdot}8 \text{ MPa}}$

18/38
The trailer plus load has a mass of 1 tonne and is equipped with two jacking cams. By lowering the cams and reversing the trailer the load can be removed from the wheels. What minimum horizontal force F must be exerted on the drawbar in order to just lift the loaded trailer off its wheels?



For equilibrium the 3 forces must be concurrent

Resolve vert.

$R \cos 25° = 10$

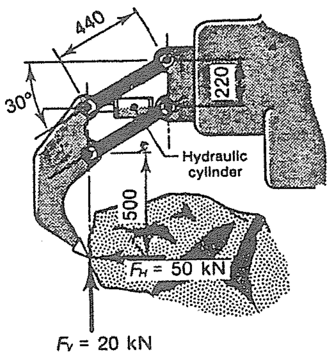
$R = 11{\cdot}034$ kN

Resolve horiz.

$F = R \sin 25$

$= \underline{4{\cdot}66 \text{ kN}}$

18/39
A crawler tractor with a single ripper attached is excavating a building site. When removing a large rock, the tine point of the ripper is acted upon by a force having vertical and horizontal components 20 kN and 50 kN. Determine the forces present in the hydraulic cylinder and in the top and bottom links for this system of loading.



Resolve vert.

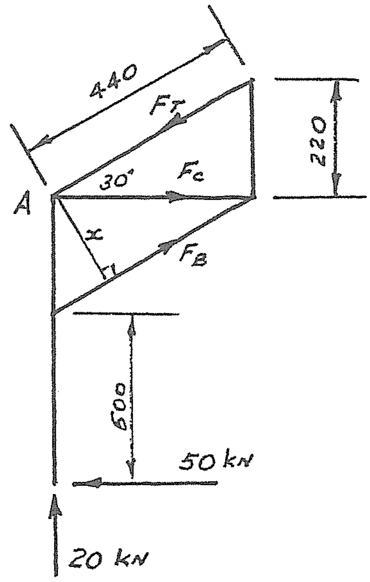
$F_T \sin 30^\circ - F_B \sin 30 = 20$ ------- (1)

Resolve horiz.

$F_T \cos 30^\circ - F_B \cos 30^\circ - F_c = -50$ ----- (2)

Take moments about A

$F_B \times x - 50 \times 720 = 0$ ---------- (3)



Simplifying (1), (2) & (3)

$0.5 F_T - 0.5 F_B = 20$

$0.866 F_T - 0.866 F_B - F_c = -50$

$190.5 F_B = 36000$

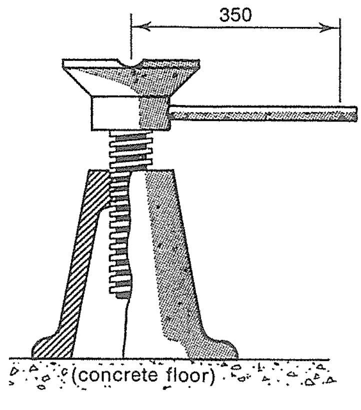
$\therefore F_B = \underline{188.98 \text{ kN}}$

Subst. in (1) $F_T = \underline{228.98 \text{ kN}}$

Subst. in (2) $F_c = \underline{84.63 \text{ kN}}$

By inspection link F_B is in Tension, link F_T is in Compression and piston rod F_c is in Tension.

18/40
A screwjack has a screw with a pitch of 5 mm and is operated by a tommy bar of 350 mm in effective radius.
(i) What is the velocity ratio?
(ii) If the efficiency is 50 per cent, what is the effort required to lift 1 tonne?
(iii) What is the mechanical advantage?



(i) $V.R. = \dfrac{\text{Dist. moved by } E}{\text{Dist. moved by } L}$

$= \dfrac{2\pi \times 350}{5}$

$= \underline{439.8 : 1}$

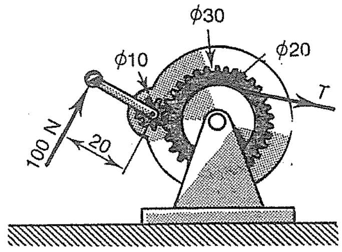
(ii) $\eta = M.A./V.R.$

$\dfrac{50}{100} = \dfrac{M.A.}{440} \quad \therefore M.A. = \underline{220:1}$

Effort $= \dfrac{L}{M.A.} = \dfrac{10000}{220} = \underline{45.5 \text{ N}}$

(iii) $M.A. = \underline{220:1}$

18/41
A hand winch has the dimensions shown in the diagram. It is 60% efficient. Determine the tension, T, in the cable when there is a force of 100 N on the handle.



Dist. moved by E in 1 rev. of $\phi 10$ wheel $= 2\pi \times 20$

1 rev. of $\phi 10$ wheel produces $1/3$ rev. of $\phi 30 = \frac{\pi \times 30}{3}$

$1/3$ rev. of $\phi 30$ is $1/3$ rev. of $\phi 20$.

$\therefore$ Dist. moved by L (T) $= \frac{\pi \times 20}{3}$ mm

$$V.R. = \frac{\text{Effort Dist.}}{\text{Load Dist.}} = \frac{2\pi \times 20 \times 3}{\pi \times 20} = 6:1$$

$$\eta = \frac{M.A.}{V.R.}$$

$$M.A. = \frac{60 \times 6}{100}$$

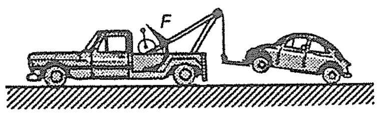
$$= 3{\cdot}6 : 1$$

$$M.A. = L/E$$

$$L = 3{\cdot}6 \times 100$$

i.e. $\underline{T = 360\ N}$

18/42
Fred has his first car accident and has to have his car towed away. The tow-truck driver's hands on the winch handle travel 40 m while he is winding up the crane cable and raising the car 500 mm. Of the work done, 35% is wasted due to friction. The crane cable is supporting a load of 0.7 tonne. What is the force exerted by the tow-truck driver's hand?



$$V.R. = \frac{40 \times 1000}{500}$$

$$= 80 : 1$$

$$\eta = M.A./V.R.$$

$$M.A. = \frac{65 \times 80}{100}$$

$$= 52 : 1$$

$$M.A. = L/E$$

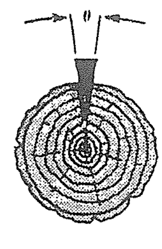
$$E = L/M.A.$$

$$= \frac{700 \times 10}{52}$$

$$= \underline{135\ N}$$

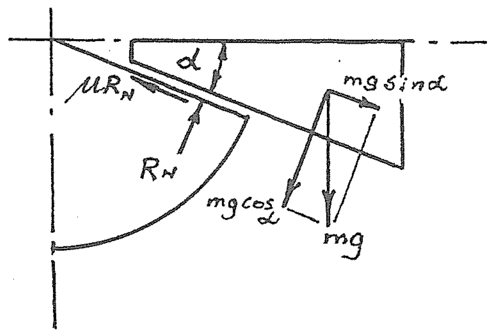
Effort reqd. not less than 135 N

18/43
A row of wedges is used to split a hardwood log. If the coefficient of friction between wood and wedge is 0.15, calculate the maximum included angle θ for which the wedges are self-locking in the log.



There are 2 approaches to this problem, depending on the interpretation of the term : "self-locking"

1. If it means the wedge will remain firmly jammed in the log under its own weight-force.



Resolve forces ⊥ to slope.

$$R_N = mg\cos\alpha$$

Resolve forces // to slope.

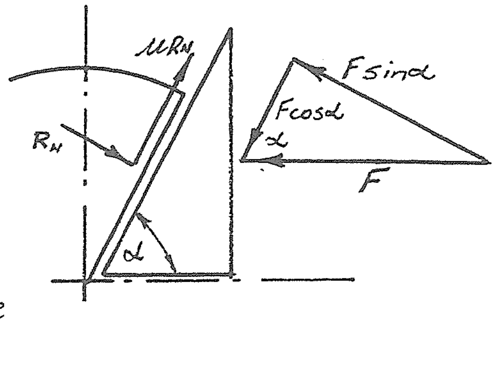
$$mg\sin\alpha = \mu R_N$$

$$\cancel{mg}\sin\alpha = 0.15 \times \cancel{mg}\cos\alpha$$

$$\frac{\sin\alpha}{\cos\alpha} = 0.15 = \tan\alpha$$

$$\therefore \alpha = 8.53^\circ \text{ or } \theta = \underline{17.06^\circ}$$

2. A more interesting interpretation of "self-locking" could be the angle where the wedge cannot be driven into the log, no matter what force is applied.



Consider force F applied to half of the wedge:

Resolve forces ⊥ to the wedge

$$R_N = F\sin\alpha$$

Resolve forces // to the wedge

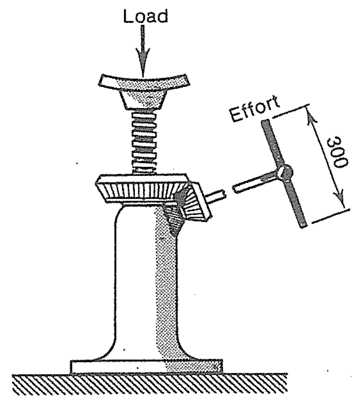
$$F\cos\alpha = \mu R_N$$

$$\cancel{F}\cos\alpha = \mu \times \cancel{F}\sin\alpha$$

$$\frac{\cos\alpha}{\sin\alpha} = 0.15 = \cot\alpha.$$

$$\therefore \alpha = 81.5^\circ \text{ or } \theta = \underline{163^\circ}$$

18/44
In the car-jack shown in the diagram, the bevel gears have a velocity ratio of 3 : 1 and the screw thread has a pitch of 7 mm. Its efficiency is found to be 20%. Find the approximate effort required to raise a 500-kg load.



Effort Dist. in 1 rev. $= \pi \times 300$

$\therefore$ Large bevel gear dist. $= \dfrac{\pi \times 300}{3}$

and with a pitch of 7mm Load dist. $= 7/3$

$$V.R. = \frac{\pi \times 300 \times 3}{7}$$

$$= 403.9 : 1$$

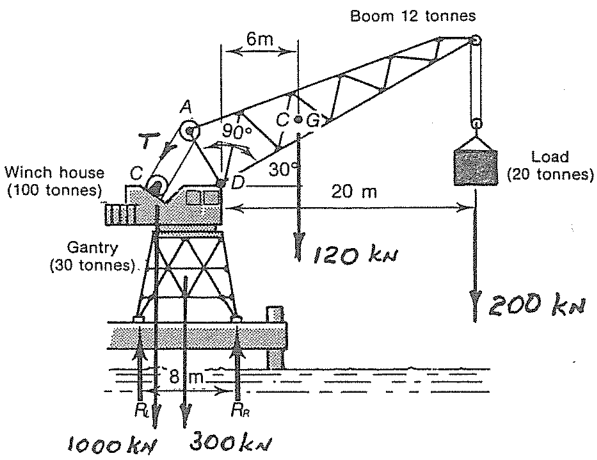
$\eta = M.A./V.R.$ $\qquad$ $M.A. = L/E$

$$M.A. = \frac{20 \times 403.9}{100} \qquad E = \frac{500 \times 10}{80.8}$$

$$= 80.8 : 1 \qquad = \underline{61.88\ N}$$

18/45
The main features of a travelling luffing crane are shown in the diagram. The distances AC, AD and CD are each 5 metres. The position of the boom is controlled by the winch at C winding a greasy wire rope which operates through a series of pulleys, providing a velocity ratio of 12 : 1. The winding system is 80% efficient. The centre of mass of the 100-tonne turntable and winchhouse is near the winding drum below the point C and the gantry is symmetrical.

Determine the tension in the cable at the winding drum and the force on the wheels near the edge of the wharf when the 20-tonne load is being raised at uniform velocity.



Take moments about D

$$T \times 5\sin 60° = 120 \times 6 + 200 \times 20$$

$$T = \frac{720 + 4000}{4.33} \qquad \eta = M.A./V.R.$$

$$= 1090\ kN \qquad M.A. = \frac{80 \times 12}{100} = 9.6 : 1$$

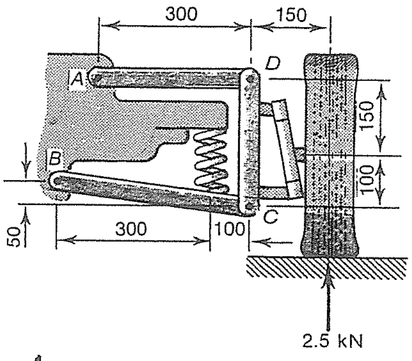
With a M.A. of 9.6 : 1, Actual $T = \dfrac{1090}{9.6} = \underline{113.5\ kN}$

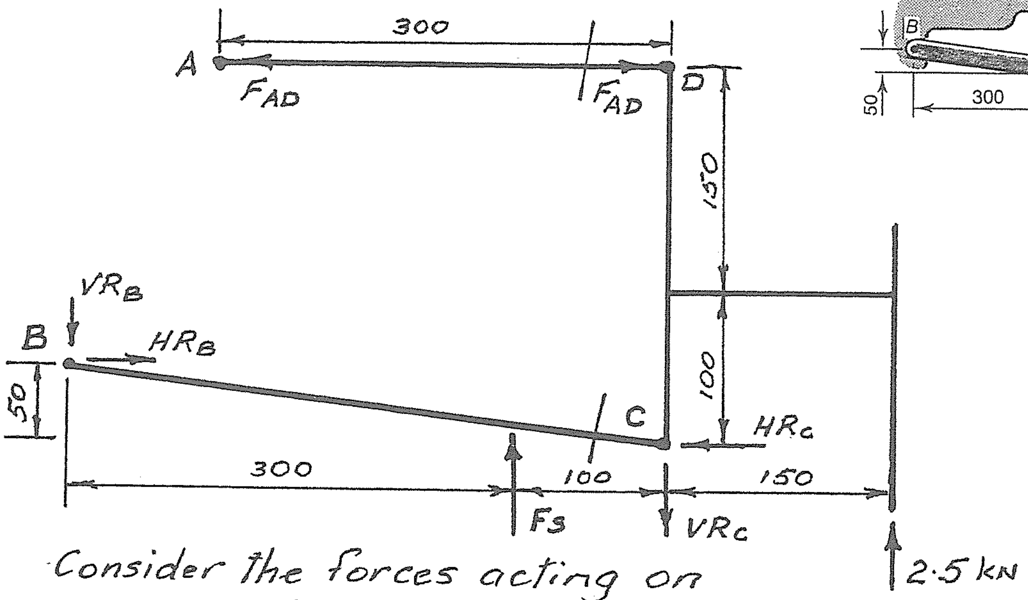
Take moments about R_L

$$8R_R = 1000 \times 1.5 + 300 \times 4 + 120 \times 12.5 + 200 \times 26.5$$

$$R_R = \underline{1187.5\ kN}$$

18/46
An automobile front-wheel assembly supports 250 kg. Determine the force exerted by the spring and the shear force in the swivel pins at A and B.





Consider the forces acting on member CD and the wheel.

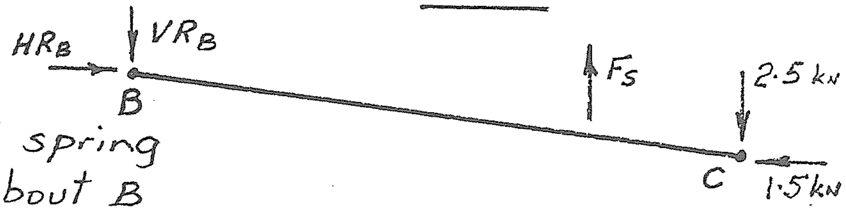
Take moments about C

$2.5 \times 150 = F_{AD} \times 250 \qquad \therefore F_{AD} = 1.5 \text{ kN} \rightarrow$

Resolve horiz. $\quad HR_C = F_{AD} = 1.5 \text{ kN} \leftarrow$

Resolve vert. $\quad VR_C = 2.5 \text{ kN} \downarrow$

Member AD is in compression and exerts a shear force on Pin A of $\underline{1.5 \text{ kN}} \leftarrow$



Consider member BC and spring

Take moments about B

$F_S \times 300 = 2.5 \times 400 + 1.5 \times 50$

$F_S = \underline{3.58 \text{ kN}} \uparrow$

Resolve horiz. $\quad HR_B = 1.5 \text{ kN} \rightarrow$

Resolve vert. $\quad VR_B + 2.5 = 3.58$

$VR_B = 1.08 \text{ kN} \downarrow$

Result. at B $= \sqrt{1.5^2 + 1.08^2}$

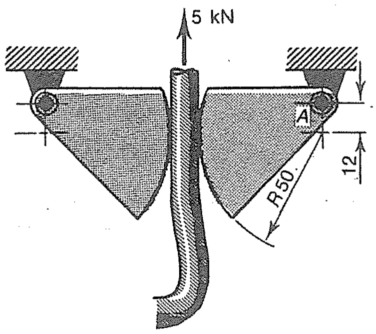
$= 1.85 \text{ kN}$

$\tan\theta = \frac{1.08}{1.5}$

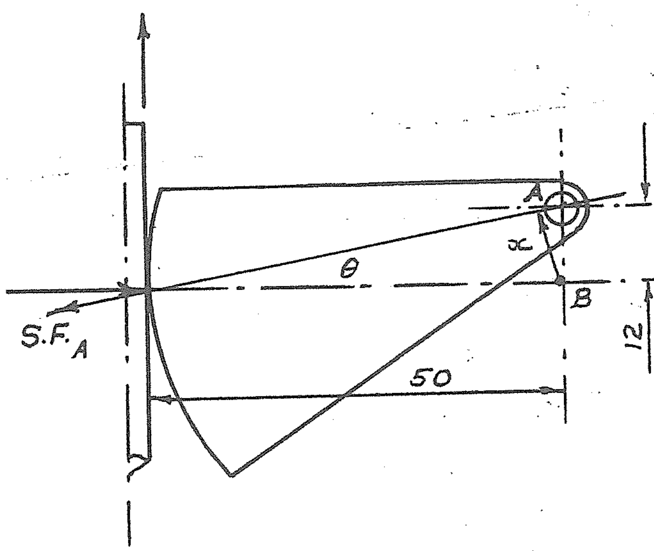
$= 0.72$

$\therefore \theta = 35.75°$

$\therefore$ S.F. on Pin B $= \underline{1.85 \text{ kN}}$ 35.75°



18/47
The device shown is commonly used on yachts, and grips a rope (a line) under tension because of the very large friction forces developed. Given that the rope has a 5-kN force in it and the coefficient of friction present is 0.4, determine the total shear force acting on the swivel pin at A for the position shown.



$\tan\theta = 12/50$

$= 0.24$

$\therefore \theta = 13.4957°$

$\sin\theta = x/50$

$x = 50 \times 0.2334$

$= 11.67 \text{ mm}$

Take moments about B

$S.F._A \times 11.67 = 2.5 \times 50$

$S.F._A = \underline{10.71 \text{ kN}}$

NOTES

NOTES